Precoder design with non-uniform power allocation for multimode precoded MIMO schemes with limited feedback

Rong Ran\textsuperscript{1a)}, Sungyoon Cho\textsuperscript{1}, Janghoon Yang\textsuperscript{2}, and Dongku Kim\textsuperscript{1b)}

\textsuperscript{1} The Dept. of Electrical and Electronic Engineering Yonsei University, Seoul, Korea. Tel/Fax: +82-2-2123-2877/+82-2-365-4504
\textsuperscript{2} The Department of Newmedia, Korean German Institute of Technology, Seoul, Korea
\textsuperscript{a)} sunny_rr@yonsei.ac.kr
\textsuperscript{b)} dkkim@yonsei.ac.kr

Abstract: This paper investigates the problem of designing a codebook with non-uniform power allocation for multimode precoded MIMO schemes (e.g., spatial multiplexing and STBC) with limited feedback in fading channels. The generalized Lloyd algorithm is employed for the design, and two methods with different complexities are addressed for the computation of the centroid which is formulated as an optimization problem. Numerical results show that the proposed design outperforms comparable algorithms which equally allocate the total transmit power to each data stream.

Keywords: MIMO, precoder design, limited feedback

Classification: Science and engineering for electronics

References

1 Introduction

It is well known that employing full channel state information (CSI) at the transmitter in wireless communication systems can yield large performance improvements [1]. However, the full CSI require significant feedback overhead, which is not practical. Therefore, various types of limited feedback systems have been designed [2, 3], which achieve the water-filling (WF) gain from the perspective of designing a codebook. However, the codebook contains only a set of columns of quantized eigen beamforming and the total power is equally allocated to the eigenmodes.

In this paper, we address a codebook design problem in the framework of multimode precoding for both spatial multiplexing and STBC systems with nonuniform power allocation. Compared to [2] and [3], we relax equal power allocation constraint and use a generalize Lloyd algorithm [4] to design a codebook. The difficulty comes from the computation of centroid of Lloyd algorithm which is formulated as an optimization problem. A straightforward solution is to use exhaustive search which requires as many as possible precoder candidates for optimization resulting in significant computing burden. Hence, an approximate closed-form solution is derived and shows the efficiency when negligible performance loss is allowed.

2 Overview of multimode precoded MIMO schemes

We consider a multimode precoded MIMO system with $M_t$ transmit and $M_r$ receive antennas. A set of input bits is demultiplexed into $M$ different bit streams for each channel. The value of $M$ is referred to the mode of the precoder and varies between 1 and $\lceil M_t/2^J \rceil$ where $J = 0$ or 1 indicates multimode precoded spatial multiplexing transmission (MM-SM) [2] or multimode precoded STBC transmission (MM-STBC) [3], respectively. The reason why $M_t$ is divided by 2 for MM-STBC is because that each bit stream is transformed to a $2 \times 2$ matrix by a Alamouti STBC [5]. Note that each bit stream is modulated with the same constellation, generating a spatial multiplexing symbol vector $\mathbf{s} = [s_1, s_2, \ldots, s_{2^J M}]^T$ with $E_{s_i}[|s_i|^2] = 1$. An $M_t \times 2^J M$ precoding matrix $\mathbf{P}^J$ maps $\mathbf{s}$ into an $M_t \times 2^J$ vector transmitted by $M_t$ antennas with $\text{tr}(\mathbf{P}^J H \mathbf{P}^J) \leq 2^J M$. The matrix $\mathbf{P}^J$ is selected from a set of precoder matrices $\mathcal{P}_M^J$ that is designed offline and known by both the transmitter and the receiver, where $M \in \mathcal{M}$ and $\mathcal{M}$ denotes a set of supported mode values, e.g., $\mathcal{M} = \{1, 2\}$ for a dual-mode precoding. When the number of precoders in $\mathcal{P}_M^J$ is $N_M$, the total feedback bits $B$ is $\lceil \log_2 (\sum_{M \in \mathcal{M}} N_M) \rceil$, where $\lceil x \rceil$ is the smallest integer which is larger than or equal to $x$.

Since we focus on the system reliability in this paper the vector symbol error rate (VSER) is used as a criterion for linear precoding design because it provides a fair ground for selecting modes which allows different degrees of
multiplexing depending on channel conditions. Taking approach in [2] and [3], we observe that the upper bound of the VSER for both MM-SM and MM-STBC with zero forcing (ZF) receiver is

\[
P_r(\text{Error}|\mathbf{H}) \leq C \cdot Q\left\{\sqrt{\text{SNR}_{\min}} \frac{d_{\min}(M, R)}{2}\right\}
\]

(1)

Where \(\mathbf{H}\) is \(M \times N\) i.i.d complex channel matrix with elements having a zero mean and unit variance and \(C\) is a scale factor related to the constellation and the number of substreams. \(d_{\min}(M, R)\) is the minimum Euclidean distance of the used constellation. \(\text{SNR}_{\min}\) denotes the minimum received SNR of streams after linear processing and is bounded by [6]

\[
\text{SNR}_{\min} \geq \lambda_{\min}(\mathbf{H}_{\text{eff}})^2 \frac{\rho}{2JMN_0}
\]

(2)

where \(\lambda_{\min}(\cdot)\) is the minimum eigenvalue of the effective channel and \(\mathbf{H}_{\text{eff}}^J = \mathbf{H}\mathbf{P}^0\) when \(J = 0\) for MM-SM and \(\mathbf{H}_{\text{eff}}^J = \mathbf{H}\mathbf{P}^1\mathbf{P}^1^\dagger \mathbf{D}\) when \(J = 1\) for MM-STBC with \(\mathbf{D} = I_M \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\) due to the characteristic of STBCs.

From this bound (2), we observe that minimizing VSER is equivalent to looking for an optimal precoder which maximizes the minimum eigenvalue of the effective channel.

3 Precoder designs with non-uniform power allocation

The problem of codebook design is essentially a vector quantization problem and hence the conventional Lloyd algorithm [4] is applied here to find a codebook that optimizes an average distortion. The design procedure is summarized in the following two-step algorithm.

**Step1** Determine the precoder set \(\{\mathbf{P}_1^J, \cdots, \mathbf{P}_{NM}^J\}\) given a certain partition \(\{\mathcal{H}_1, \cdots, \mathcal{H}_{NM}\}\).

\[
\mathbf{P}_q^J = \arg\min_{\{F_i^J\}} E_{\mathbf{H} \in \mathcal{H}_q} [d(\mathbf{H}, F_i^J)]
\]

(3)

where \(F_i^J\) denotes a set of precoder candidates, \(d(\mathbf{H}, \mathbf{P}_q^J) = -[\lambda_{\min}(\mathbf{H}_{\text{eff}})]\) and \(E_{\mathbf{H} \in \mathcal{H}_q}[x]\) denotes the conditional expectation on \(\mathbf{H} \in \mathcal{H}_q\).

**Step2** Determine the partition \(\{\mathcal{H}_q\}\) given a precoder set \(\{\mathbf{P}_q^J\}\) by the nearest neighbor rule

\[
\mathcal{H}_q = \{\mathbf{H} : d(\mathbf{H}, \mathbf{P}_q^J) \leq d(\mathbf{H}, \mathbf{P}_q^J), \forall j, q \in [1, \cdots, NM], j \neq q\}
\]

(4)

Step 1 and Step 2 are repeatedly applied until convergence.

3.1 An exhaustive search

One difficult part of the Lloyd algorithm is the computation of the centroid
because the distortion measure $d(H, P_q^J)$ is non-linear. One straightforward way is to randomly generate a large number of non-uniform power allocated matrices as well as a set of channel matrix $H$ according to a given channel statistics as the training sequence. Therefore, the precoder of (3) can be found through exhausted searching. The generation of non-unitnorm precoder candidates is presented as follows:

1) Generate $L$ number of vectors $p_l = [p_{l0}, p_{l1}, \ldots, p_{l2JM}]$ such that each element $p_{li}$ satisfies uniform distribution $(0, 1)$ and compute $ar{p}_{li} = p_{li} \cdot \left(2^{1/2JM}/\sum_{j=1}^{2JM} p_{lj}\right)$;

2) Let $\bar{p}_l = \{\bar{p}_{li}, 1 \leq i \leq 2^{J/M}\}$ and generate $K$ random unitary matrices $A_k$;

3) $F^J_i = A_k \cdot \text{diag}(\bar{p}_l)$, $i = K \ast (i - 1) + k$.

The number $L$ and $K$ should be so large that Lloyd algorithm can have a good performance. However, it causes large computational burden. With the introduction of a heuristic approximation to the optimization problem, a simple solution of (3) is therefore derived as described in the following section 3.2 An approximate closed-form solution for precoder design

Let $f_q = E_{H \in \mathcal{H}_q}[\lambda_{\text{min}}^2(H^\text{eff}_q)]$, Note that $\|B\|_F^2 \geq M \lambda_{\text{min}}^2(B)$ where $B$ is a $M_t \times M$ matrix. Thus,

$$f_q = E_{H \in \mathcal{H}_q}[\lambda_{\text{min}}^2(H^\text{eff}_q)]$$

$$\leq \frac{1}{2^{1/2JM}}E_{H \in \mathcal{H}_q} tr((H P_q^J)^H (H P_q^J) + J(H^* P_q^J B)^H (H^* P_q^J B))$$

$$= \frac{1}{M}E_{H \in \mathcal{H}_q} tr((H P_q^J)^H (H P_q^J))$$

$$\approx \frac{1}{M} [\log \det(I + (P_q^J)^H \Omega_q P_q^J)]$$

where $\Omega_q = E_{H \in \mathcal{H}_q}(H^H H)$ is a conditional average matrix for the $q$th channel partition. (8) comes from (7) according to the property of $tr(X) \approx \log(\det(I + X))$ and Jensen’s inequality.

This approximation reduces the design of the precoder to

$$P_q^J = \arg \max_{P_q^J} [\log \det(I_M + (P_q^J)^H \Omega_q P_q^J)]$$

$$\text{s.t. } tr((P_q^J)^H P_q^J) \leq 2^{J/M}$$

by performing eigenvalue decomposition of $\Omega_q = W_q \Lambda_q W_q^H$, the linear precoder solution of this optimization problem is given by water-filling:

$$\Phi_F(i, i) = \begin{cases} \left[\frac{1}{k} - \Lambda_q^{-1}(i, i)\right]^+, & \Lambda_q(i, i) > 0 \\ 0, & \Lambda_q(i, i) \leq 0 \end{cases}$$

where $k > 0$ is a constant computed from the trace constraint, diagonal elements of $\Lambda_q$ are arranged in decreasing order of magnitude and $[x]^+$ stands for $\max(x, 0)$.
Consequently, the corresponding solution $P^J_q$ of step 1 is a matrix constructed from the first $2^J M$ columns of $W_q \cdot \Phi_{1/2}$. Practically, the approximate solution reduces the complexity significantly by removing the necessity of candidate vectors with different power allocations.

## 4 Simulation results

In this section, we provide simulation results to illustrate the performance of the proposed precoder design as derived in Section III. We consider a MIMO system with four transmit and two receiver antennas, where a ZF receiver is applied and the overall data-rate $R$ is fixed at 8 bps/Hz such that the corresponding constellations are 256QAM and 16QAM for Mode 1 and Mode 2 respectively when a dual-mode system $M = \{1, 2\}$ is applied.

Fig. 1 illustrates performance gains of the proposed designs with respect to the unitary precoder design in terms of average vector symbol error rate (VSER). The number of feedback bits is assumed to be three, which results in $N_1 = 4$, $N_2 = 4$ precoder allocation of modes for the MM-SM by using the feedback bit allocation criterion in [2]. Since the MM-STBC can save one bit for full diversity compared to the MM-SM [3], at least two $M_t \times 2$ precoders are allocated to Mode 1 and the remainder $M_t \times 4$ precoders are allocated to Mode 2, i.e., $N_1 = 2$, $N_2 = 6$ for MM-STBC. Results show the exhaustive search design (III-B) achieves around 1 dB and 0.6 dB performance gain at the error rate $10^{-3}$ compared to the unit-normalized precoder design for the MM-SM [2] and MM-STBC [3], respectively. Furthermore, the approximate closed-form precoder design (III-C) performs as well as the exhaustive search one in the MM-STBC system. Whereas, it suffers very marginal performance loss in the MM-SM system.

The performance of the precoded MIMO systems with different precoder designs is illustrated in Fig. 2 when four feedback bits are assumed. The

![Fig. 1. Performance comparison of the multimode precoded MIMO systems with different precoder designs for 3 bits feedback](Image)
precoder allocations for Mode 1 and 2 are $N_1 = 2$ and $N_2 = 14$ in the MM-STBC system, while $N_1 = 4$ and $N_2 = 12$ for the MM-SM. We observe that the exhaustive search design (III-B) still achieves the best performance and outperforms the unit-normalized design with about 0.7 dB gain both in the MM-STBC and MM-SM system. Meanwhile, the approximate closed-form precoder design (III-C) achieves the same performance as the exhaustive search one particularly in the MM-STBC system, and loses an acceptable performance in MM-SM system. Note that the MM-SM turns to perform better than the MM-STBC for 4 bits feedback, that is because it benefits more beamforming gain as the number of feedback bits is increased.

5 Conclusion

We have considered the problem of designing precoders with nonuniform power allocation for a multimode precoded MIMO system with limited feedback. The Lloyd algorithm is utilized to design the codebook with respect to maximizing the minimum eigenvalue of the effective channel. In each iteration of the Lloyd algorithm, the codebook can be constructed by an exhaustive search method which shows the best performance but causes significant computation burden. Therefore, an approximate closed-form solution is derived to reduce complexity with a tolerable performance loss compared to the exhaustive search one. Numerical results showed that the proposed precoder design with non-uniform power allocation outperforms the uniform precoder design reported in the literatures.

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