Design of a robust NTF for continuous-time $\Delta\Sigma$ modulators

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Abstract: This paper presents an analytical-empirical method for the design of continuous-time $\Delta\Sigma$ modulators. For this purpose, both the Schreier and SISO toolboxes are employed. This method improves the robustness of the modulator against imperfections such as excess loop delay and unity-gain bandwidth of the opamps.

Keywords: Delta-Sigma modulator, continuous-time

Classification: Integrated circuits

References

1 Introduction

The most usual method for designing continuous-time (CT) ΔΣ modulators is to utilize the Schreier toolbox [1, 2, 3]. In this method, making use of the Schreier toolbox and CLANS methodology, the noise transfer function (NTF) is obtained in z-domain, and consequently the loop-filter transfer function is determined. Finally, utilizing the discrete-time (DT) to continuous-time transformation, the equivalent CT loop-filter is derived [4].

This paper employs the Schreier toolbox and performs a few modifications on it to design high performance CT ΔΣ modulators. In this method, in order to find the loop-filter transfer function, both the maximum amplitude of NTF over the whole bandwidth and maximum amplitude of NTF over the signal bandwidth are taken into consideration.

In section 2, the proposed method for designing CTΔΣ modulators is explained. To reveal both the advantages and restrictions of the presented method, a few simulations are performed in section 3. Finally, the paper is summarized in section 4.

2 Proposed Method for Designing CT ΔΣ Modulators

There is an ADC inside the loop of each CT ΔΣ modulator, which injects a quantization noise to the input signal. In Fig. 1 (a), the power spectral density (psd) of the quantization noise of the internal ADC, $S_{ei}(f)$, is shown [5]. The psd of the output quantization noise of the modulator, $S_{eo}(f)$, is obtained through the following formula:

$$S_{eo}(f) = |NTF(f)|^2 \times S_{ei}(f)$$

If the maximum of the NTF within the signal bandwidth is called $A_x$, as illustrated in Fig. 1 (b), by substituting the value of $S_{ei}(f)$, for in-band frequencies, we will have:

$$S_{eo}(f) \cong A_x^2 \times \frac{\Delta^2}{12f_s} \text{ for } |f| \leq f_b$$

In this formula, $f_s$ denotes the sampling frequency and $\Delta$ represents the difference of two adjacent quantization levels, calculated as follows:

$$\Delta = \frac{V_{ref}}{2^N}$$

In the above formula, $N$ indicates the number of bits of the quantizer and $V_{ref}$ is the reference voltage of the quantizer. The power of the output noise is identical to the area of the underneath part of the output noise spectrum (Fig. 1 (c)) in the signal bandwidth. It is calculated through the following formula:

$$P_{eo} = \int_{-f_b}^{f_b} S_{eo}(f) \, df \cong A_x^2 \times \frac{\Delta^2}{12f_s} \times 2f_B \cong A_x^2 \times \frac{\Delta^2}{12} \times \frac{1}{OSR}$$

In this formula, $P_{eo}$ is the power of the output noise, $f_b$ is the bandwidth of the input signal and $OSR$ is the oversampling ratio ($OSR = \frac{f_s}{2f_b}$). The
relation between SNR and $P_{eo}$ is achieved through the following formula:

$$SNR_{max} = 10 \log \left( \frac{P_{smax}}{P_{eo}} \right)$$  \hspace{1cm} (5)

In the above formula, $P_{smax}$ is the maximum power of the input signal which can be obtained from:

$$P_{smax} = \frac{1}{2} \left( \frac{V_{ref}}{2} \right)^2$$  \hspace{1cm} (6)

Now by substituting equations (4) and (6) in (5) and performing a few simplifications, we will have:

$$SNR_{max} = 10 \log \left[ \frac{1}{2} \left( \frac{V_{ref}}{2} \right)^2 \right] \approx 10 \log \left[ \frac{1}{2} \left( \frac{V_{ref}}{2} \right)^2 \right]$$

$$= 10 \log \left[ \frac{1.5 \times 2^{2N}}{A_x^2 \times \frac{\Delta f}{Tf}} \times \frac{1}{OSR} \right]$$  \hspace{1cm} (7)

Finally, the relation between $SNR_{max}$ and $A_x$ can be summarized as follows:

$$SNR_{max} \cong 6N + 1.76 \, dB + 10 \log (OSR) - 20 \log (A_x)$$  \hspace{1cm} (8)

In other words, we will have:

$$-20 \log (A_x) \cong SNR_{max} - (6N + 1.76 \, dB + 10 \log (OSR))$$  \hspace{1cm} (9)

In practice, the values of $N$, $OSR$, and $SNR_{max}$ are known and we should design the NTF so that the following formula is satisfied. Note that the value of $A_x$ is smaller than 1.

$$-20 \log (A_x) \geq SNR_{max} - (6N + 1.76 \, dB + 10 \log (OSR))$$  \hspace{1cm} (10)
The relation between NTF and loop-filter transfer function $H(z)$ is [6]:

$$NTF(z) = \frac{1}{1 + H(z)} \quad (11)$$

Therefore, by substituting $Ax$ in the above formula, the first design criterion of the loop-filter is obtained as follows where $H_{\text{in-band}}$ represents the minimum value of the loop-filter over the input-signal bandwidth. In Fig. 2(a), a typical loop-filter transfer function is depicted in which the parameter $H_{\text{in-band}}$ is shown. The designer should design the loop-filter so that the magnitude of the loop-filter over the input-signal bandwidth becomes greater than $H_{\text{in-band}}$.

$$H_{\text{in-band}} = \left| \frac{1 - Ax}{Ax} \right| \approx \frac{1}{Ax} \quad (12)$$

The maximum value of NTF is another parameter used in our method. This parameter is illustrated in Fig. 1(b) by $\|NTF\|_{\infty}$. It has a direct relation with $\text{SNR}_{\text{max}}$ and a reverse relation with stability [7]. In other words, the larger value of this parameter results in the higher SNR and the lower stability of the modulator, and vice versa. By substituting $\|NTF\|_{\infty}$ in formula (11), the second design criterion of the loop-filter is obtained as follows where $H_{\text{whole-band}}$ denotes the minimum value of the loop-filter over the entire bandwidth. In Fig. 2(a), a typical loop-filter transfer function is depicted in which the parameter $H_{\text{whole-band}}$ is shown.

$$H_{\text{whole-band}} = \left| \frac{1 - \|NTF\|_{\infty}}{\|NTF\|_{\infty}} \right| \quad (13)$$

In the proposed method, the denominator of the loop-filter transfer function is chosen identical to that obtained by the Schreier toolbox and CLANS instruction. Then the zeros of the loop-filter transfer function are adopted so that the constraints on $H_{\text{in-band}}$ and $H_{\text{whole-band}}$ are satisfied. For this purpose, the SISOTOOL toolbox of MATLAB is employed. This toolbox includes a graphical environment that shows the frequency response of $H(z)$. This toolbox allows adding zeros to $H(z)$. It also allows the graphical displacement of zeros. By displacing the zeros graphically, $H_{\text{whole-band}}$ and $H_{\text{in-band}}$ adopt new values that can be observed on a window of this toolbox. This trend is continued until the properly determination of zeros. In Fig. 2(b), a movie file is embedded in which the procedure to find a loop-filter is shown (loop-filter $A^\prime$, which is shown in Fig. 3(b)).

### 3 Simulation Results

To reveal the advantages of the proposed method, a few simulations are performed in MATLAB. For this purpose, a third order 5-bit $\Delta \Sigma$modulator with OSR=16 is designed. In order to simulate the modulator, the structure of Fig. 3(a) is employed where the loop-filter is realized by the cascaded-integrator feed-forward (CIFF) method [1, 2]. Moreover, the model of real integrators described in [8] is utilized. For very large values of UGBW (unity-gain bandwidth), the modulator acts as an ideal modulator. The decrease
of UGBW degrades the performance of the modulator. When the modulator becomes unstable, the related UGBW is called as the minimum required UGBW.

Making use of both the CLANS instruction and proposed method, a few loop-filters are designed as shown in Fig. 3 (b) where loop-filters A and B are achieved by using the CLANS instruction. Besides, loop-filters A’ and B’ are achieved through the proposed method.

In Fig. 3 (b), the minimum UGBW required for the stability of each modulator is extracted. In Fig. 3 (c), the SNR diagram is illustrated versus the feedback loop delay. All these simulation results reveal the benefits of the proposed method over the conventional method.

Making use of both conventional and proposed methods, several 3rd order modulators are designed for various values of OSR. The simulation results are depicted in Fig. 3 (d). We have also performed other simulations for the design of 4th and 5th order modulators. It is considered that the proposed method is not helpful for modulators with order greater than 4 because the number of zeros of \( H(z) \) is increased and consequently the empirical optimization of zeros will be difficult.

Fig. 2. (a) A typical loop-filter transfer function. (b) A movie file showing how to design the loop-filter.
Fig. 3. (a) The CIFF structure for realizing the loop-filter. (b) The simulation results for various loop-filters. (c) The SNR versus the feedback loop delay. (d) Comparison between two methods for various OSRs.

### 4 Conclusion

In this paper, a new method is presented to design modulators with improved stability and SNR. It uses $\|NTF\|_{\infty}$ as a measure for stability and the maximum of NTF within the signal bandwidth as a measure for SNR. This method utilizes the SISOTOOL toolbox of MATLAB to place the zeros of the loop-filter appropriately.