Improvement of the error characteristics of N-continuous OFDM system by SLM

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Abstract: N-continuous OFDM is a modulation technique that has a lower sidelobe than the original OFDM as a result of the continuous connection with its higher-order derivatives between the OFDM symbols. However, N-continuous OFDM has a high symbol error rate. In the present paper, we improve N-continuous OFDM without increasing the symbol error rate by using a selected mapping technique.

Keywords: N-continuous OFDM, sidelobe suppression, selected mapping

Classification: Science and engineering for electronics

References


1 Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted in several telecommunications technologies. The advantages of OFDM are fast data transmission and robustness against multipath fading. However, there is a problem with OFDM in that high sidelobes arise from the discontinuity of adjacent OFDM symbols. Various methods of sidelobe suppression have been proposed. The insertion of wide guard band or cancellation carriers [1, 2] is a useful approach for avoiding interference with adjacent bands but decreases spectral efficiency. The windowing technique [3] in the time domain and the adaptive symbol transition [4] reduce the data transmission speed due to the extended guard interval associated with the windowing or non-data time domain blocks for sidelobe suppression. The \( N \)-continuous OFDM [5] does not require the guard interval to be extended nor a wide guard band or cancellation carriers to be inserted. However, the \( N \)-continuous OFDM has a high symbol error rate.

To solve the above-described problem, we improve \( N \)-continuous OFDM using the selected mapping technique [6].

2 \( N \)-continuous OFDM

The key idea of \( N \)-continuous OFDM is precoding the set of transmit symbols in each OFDM symbols such that the baseband-equivalent OFDM signal becomes \( N \)-continuous. The \( i \)th \( N \)-continuous OFDM symbol \( s_i(t) \) satisfies (1) and (2),

\[
s_i(t) = \sum_{k \in K} \bar{d}_{k,i} e^{j2\pi \frac{k}{T_s} t}, \quad -T_g \leq t < T_s,
\]

\[
\frac{d^n}{dt^n} s_i(t) \bigg|_{t=-T_g} = \frac{d^n}{dt^n} s_{i-1}(t) \bigg|_{t=T_s},
\]

where \( \bar{d}_i = (\bar{d}_{k_0,i}, \ldots, \bar{d}_{k_{K-1},i})^T \) is a precoded transmit symbol \( d_i = (d_{k_0,i}, \ldots, d_{k_{K-1},i})^T \), \( K = \{k_0, \ldots, k_{K-1}\} \) is a set of data channel indices, \( K \) is the number of data channels, \( T_s \) is the OFDM symbol duration, \( T_g \) is the guard interval length, and \( n \in \{0, \ldots, N\} \).

From (1) and (2), J. van de Beek et al. proposed a method by which to precode the symbol \( d_i \) to \( \bar{d}_i \) as

\[
\bar{d}_i = d_i + w_i,
\]

\[
w_i = -P d_i + P \Phi^H \bar{d}_{i-1},
\]

where \( P = (A\Phi)^\dagger A\Phi \),

\[
A = \begin{bmatrix}
1 & 1 & \ldots & 1 \\
k_0 & k_1 & \ldots & k_{K-1} \\
\vdots & \vdots & \ddots & \vdots \\
k_0^N & k_1^N & \ldots & k_{K-1}^N
\end{bmatrix},
\]

\[
\Phi = \text{diag}(e^{j\phi k_0}, \ldots, e^{j\phi k_{K-1}}), \quad \phi = -2\pi \frac{T_g}{T_s}, \quad \text{and } (\cdot)^\dagger \text{ is the Moore-Penrose pseudoinverse of } (\cdot).
This precoding satisfies (1) and (2), and the inserted symbol \( w_i = (w_{k0,i}, \ldots, w_{kK-1,i})^T \) has the smallest power in a Euclidean sense.

### 3 Analysis and improvement by SLM

We evaluate the method whereby the system number of FFT points is 512, \( K = 300, T_g = \frac{9}{128} T_s \) and each of subcarriers is modulated using 16QAM and those parameters are compliant with LTE. We define \( K = \{-150, \ldots, -1, +1, \ldots, +150\} \) for each data channel and connect \( N = 5 \)th-order derivatives continuously between OFDM symbols. Figure 1 (a) shows \( E\{|w_{k,i}|^2\} \) and \( E\{|X_{k,i}|^2\} \), where \( E\{x\} \) is the expected value of \( x \), \( X_i = (X_{k0,i}, \ldots, X_{kK-1,i})^T = Pd_i \), and the half of the minimum distance between a constellation points is 1. Figure 1 (b) presents the symbol error rate for each channel. Based on these figures, the higher the powers of \( w_{k,i} \) and \( X_{k,i} \), the more the symbol error rate increases.

![Fig. 1. Power spectral density of \( E\{|w_{k,i}|^2\} \) and \( E\{|X_{k,i}|^2\} \), and Symbol error rate for each channel (SNR = 16 dB).](image)

We propose that the selected mapping (SLM) technique be introduced before the precoding [6] in order to reduce peak power of \( w_i \) and \( X_i \) in the high-frequency area [7]. The transmit symbol \( d_i \) is multiplied element-by-element with the \( M \) phase rotate vectors \( q^{(m)} = (q_{k0}^{(m)}, \ldots, q_{kK-1}^{(m)})^T \),

\[
d^{(m)}_{k,i} = d_{k,i} \cdot q_k^{(m)} ,
\]

where \( m \in \{0, \ldots, M-1\} \) and \( q_k^{(m)} \in \{+1, -1\} \), and a set of \( M \) different symbols \( d^{(m)}_i \) is generated. We then select \( m^* \), which is derived from

\[
m^* = \arg \min_m \{Z_i^{(m)}\} ,
\]

\[
Z_i^{(m)} = \max_{k \in K} \{|X_k^{(m)}|^2\} ,
\]

where

\[
X_i^{(m)} = (X_{k0,i}^{(m)}, \ldots, X_{kK-1,i}^{(m)})^T = Pd_i^{(m)} .
\]

\( m^* \) is sent as side information for a receiver.
Next, the proposed method is analyzed theoretically. Here, in case of 16QAM, \(d_{k,i}^{(m)}\) has a complex uniform distribution, the average of which is zero and the variance of which is \(\sigma^2 = 10\). When the \((k,l)\)-th element of \(P\) is denoted as \(p_{kl}\), it is derived from the central limit theorem that \(X_{k,i}^{(m)}\) has a complex Gaussian distribution, the average of which is zero and the variance of which is \(\sigma^2 \sum |p_{kl}|^2 / 2\). Therefore, \(|X_{k,i}^{(m)}|^2\) has a Rayleigh distribution and the probability that \(|X_{k,i}^{(m)}|^2\) is less than a threshold power \(Y\) is

\[
\text{Prob}(|X_{k,i}^{(m)}|^2 \leq Y) = 1 - \exp\left(-\frac{Y}{\sigma^2 \sum |p_{kl}|^2}\right). \tag{10}
\]

Next, we have experimentally investigated existence probability that \(|X_{k,i}^{(m)}|^2\) of the \(k\)th channel is to be the maximum power among all channels. The experimental result has shown that the existence probability of all channels except for \(k = \pm 150\) is less than 0.2% and the maximum power is prominently detected at the \(\pm 150\)th channels.

Therefore, the probability that \(Z_i^{(m)}\) derived from (8) is less than \(Y\) is approximately equal to the probability that the both powers \(|X_{k,i}^{(m)}|^2\) at the \(\pm 150\)th channels are less than \(Y\).

\[
\text{Prob}(Z_i^{(m)} \leq Y) \approx \prod_{k \in \{\pm 150\}} \text{Prob}(|X_{k,i}^{(m)}|^2 \leq Y) \tag{11}
\]

And conversely, the probability that \(Z_i^{(m)}\) exceeds \(Y\) is \(\text{Prob}(Z_i^{(m)} > Y) \approx 1 - \prod_{k \in \{\pm 150\}} \text{Prob}(|X_{k,i}^{(m)}|^2 \leq Y)\). When all of \(Z_i^{(m)}\) for \(m = 0, \ldots, M - 1\) exceed \(Y\), the minimum of \(Z_i^{(m)}\), that is \(Z_i^{(m^*)}\), obviously exceeds \(Y\). Therefore we obtain the probability that \(Z_i^{(m^*)}\) exceeds \(Y\) as

\[
\text{Prob}(Z_i^{(m^*)} > Y) \approx \left(1 - \prod_{k \in \{\pm 150\}} \text{Prob}(|X_{k,i}^{(m)}|^2 \leq Y)\right)^M. \tag{12}
\]

Figure 2 presents the experimental and theoretical results in \(M = 1\) and 8. Figure 2 shows that the more \(M\) increases, the more \(Z_i^{(m^*)}\) decreases, and the experimental and theoretical results are consistent with each other.

![Fig. 2. Experimental and theoretical results of Prob(Z_i^{(m^*)} > Y).](image-url)
In order to evaluate the performance of the proposed method, we performed a number of numerical experiments. The power spectral density is estimated in Welch’s averaged periodogram method with a 4,096-sample Hanning window and a 512-sample overlap in four-times oversampling. Figure 3(a) shows the performance of the power spectral density at \( N = 5 \) and \( M = 8 \). As is shown in the figure, the proposed method has the same performance as the conventional \( N \)-continuous OFDM. Here, Figure 3(b) shows the performance of the symbol error rate in the AWGN channel. The performance of the symbol error rate can be concluded to be improved, and the symbol error rate degrades only slightly as compared with the original OFDM.

![Graph showing power spectral density and symbol error rate](image)

**Fig. 3.** Performance of power spectral density and symbol error rate in the AWGN channel.

### 4 Conclusion

We have improved \( N \)-continuous OFDM by using the selected mapping technique. From the numerical experiments, it has clarified that the proposed method has the same power spectral density as the conventional \( N \)-continuous OFDM. Moreover, the performance of the symbol error rate in the AWGN channel can be concluded to be improved, and the symbol error rate degrades only slightly as compared with the original OFDM.