Practical criteria for the implementation of a hybrid FDTD-MoL technique used for electromagnetic analysis of microstrip paths

Alejandro Dueñas Jiménez^{a)}

Departamento de Electrónica, Universidad de Guadalajara

Av. Revolución 1500, Guadalajara, Jalisco, 44430 México

a) alejandro.duenas@red.cucei.udg.mx

Abstract: Several practical criteria to improve the performance of a mixed FDTD-MoL method of electromagnetic analysis are proposed. Concepts as the properly dimension selection of the waveguide “containing” the microstrip geometry, the true propagation speed on the microstrip, the suitability of the excitation pulse spread and the relation between the reflected and incident waveforms, the preservation of both, the time and space derivatives for the FDTD segmentation-discretization part, and the analytical incorporation of the boundary conditions are all discussed.

Keywords: hybrid electromagnetic analysis methods, microstrip tracks, high-speed interconnects, RFID

Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

The design of microstrip tracks and paths as high-speed interconnects and delay line based radio frequency identification (RFID) circuits, will be not nowadays conceivable without the aid of the powerful numerical techniques, the advanced electromagnetic analysis methods and the very fast computer resources. In effect, all these tools have been mixed to create hybrid methods that overcome the drawbacks of single techniques and advantage the time-consuming cut-and-try processes. Thus, the printed circuit board (PCB) designers and the electromagnetic simulation practitioners presently use more and more a combination of several commercial codes to get a final design. However, the creation of hybrid methods confronts challenges since the weaknesses of the single methods can combine to produce non-convergence and numerical dispersion among other disadvantages.

To minimize these inconveniences, a series of practical corrective measures conceived as some analysis criteria, are proposed for a FDTD-MoL technique in this article. The FDTD method is formulated with the Telegrapher or Maxwell wave equations, and the method of lines (MoL) [1] is stated with the rectangular-waveguide transverse-magnetic (TM) Maxwell's equations (all of them hyperbolic equations). When the Telegrapher equations are used the characteristics of a microstrip are totally and only defined by the elements of its equivalent circuit [2]. On the contrary, when the Maxwell equations are employed, the microstrip characteristics are strongly dependent of the defined geometry.

The rectangular-waveguide TM Maxwell's equations are given by [3]

$$-\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y}, \quad (1)$$

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \quad (2)$$

$$\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}, \quad (3)$$

Differently to the pure Mol method where the time derivative is usually reserved for the analytical process, the hybrid FDTD-MoL solves analytically for one space variable and leaves the others space and time variables for the segmentation-discretization procedure.

2 Waveguide dimensions

After define the line discretization of the x -variable, and the subsequent getting of the eigenvalues and eigenvectors for one of the four different combinations of Dirichlet (D) and Neumann (N) boundary conditions, (DD, DN, ND and NN), the solution for the i th line of the transformed electric field component $\bar{E}_z$ is given by [3]

$$\bar{E}_{zi}(y, t) = \sum_n (A_{ni} \cos \omega_{ni} t + B_{ni} \sin \omega_{ni} t) \sin \alpha_n y \quad (4)$$

where

$$\omega_{ni} = \frac{v_{pmc}}{\sqrt{(n\pi/b)^2 - \lambda_i^2/(\Delta x_{MoL})^2}}, \quad (5)$$

$$\begin{aligned} \lambda_i &= 2 \sin \frac{i\pi}{2(N+1)} && \text{for DD boundary conditions,} \\ \lambda_i &= 2 \sin \frac{(i-0.5)\pi}{2N+1} && \text{for DN boundary conditions,} \\ \lambda_i &= 2 \sin \frac{(i-0.5)\pi}{2N+1} && \text{for ND boundary conditions,} \\ \lambda_i &= 2 \sin \frac{(i-1)\pi}{2N} && \text{for NN boundary conditions,} \\ i &= 1, 2, \dots, N, \end{aligned} \quad (6)$$

$$\alpha_n = n\pi/b, \quad (7)$$

n is the mode number [4] and

$$\Delta x_{MoL} = \frac{a}{N+1} \quad (8)$$

is the spacing between discretized lines by considering a (the longest side of the waveguide) as the unity and N the number of lines (which coincides with n) for the MoL process only, since a is the width of the microstrip substrate for the FDTD process. In (5) and (7), b is the shortest side of the waveguide and v_{pmc} is the corrected propagation speed or phase velocity for a microstrip line, as will be explained in the next section. B_{ni} is given by [5]

$$B_{ni} = A_{ni} \tan(\omega_{ni} \Delta t / 2) \quad (9)$$

and A_{ni} can be obtained from (4) by knowing $\overline{E}_z$ from the mixed FDTD-Mol process and by considering the orthogonal property of the modal fields [4].

The value of b (the shortest side of the waveguide) comes from the following empirical criterion. Since the problem of attain the frequency-domain scattering parameters of the paths or tracks is reduced to the problem of calculating the cutoff frequencies of various modes of the rectangular waveguide, then in principle, the dimension b can be derived from the cutoff-frequency equation given by [6]

$$f_{c_{mn}} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \quad (10)$$

However, for the TM waves, the lowest order mode that propagates is the TM_{11} (there are no TM_{00} , TM_{01} or TM_{10} modes) and so, for a given microstrip path operating at a central frequency down the $f_{c_{mn}}$ of the TM_{11} mode, the value of b becomes imaginary (evanescent modes) and not profitable. To avoid this difficulty, the ratio between the shortest and longest sides for the TE_{10} dominant mode of the waveguide fitting the width a of the microstrip, is advisable. Thus for instance, for an operation central frequency of 1.5 GHz (around L and R bands), the relation between b and a will be $b/a = 0.5$.

3 Propagation speed

The corrected propagation speed for a microstrip line [7] is of major importance in electromagnetic simulation tasks, since this parameter strongly impact on other design parameters as the resonant frequency and the propagation time which are of crucial matter for confident signal timing and patterning of tracks and paths in modern RFID and high-speed interconnects applications.

A general form of the corrected phase velocity is given by

$$v_{pmc} = v_{pm} \cdot f_l \quad (11)$$

where f_l is the lengthening factor expressed in terms of the microstrip phase velocity v_{pm} (which is taken as the velocity in the 50- Ω input and output lines) and the dielectric phase velocity v_{pd} , as follows

$$f_l = \frac{kv_{pm}^n}{v_{pd}^n}, \quad (12)$$

$$v_{pm} = \frac{c}{\sqrt{\varepsilon_{eff}}} = c \frac{1}{\sqrt{\varepsilon_{eff}}} = cv_{rp}, \quad (13)$$

$$v_{pd} = \frac{c}{\sqrt{\varepsilon_r}}, \quad (14)$$

k and n are positive numbers, $k = p/2$ (where p is the number of circuit ports), n is experientially chosen as a positive entire number, c is the speed of light in free-space, ε_{eff} is the effective dielectric constant, v_{rp} is the relative phase velocity and ε_r is the relative permittivity. When v_{pmc} is used in the formulation of FDTD-MoL simulations an easy convergence is assured provided all the other parameters of the analysis criteria are complied.

4 The pulse spread and the reflected-to-incident wave ratio

Another very important analytical criterion is that of the pulse spread and its relation to the reflected-to-incident wave ratio. A procedure for optimizing the Gaussian excitation function is presented in [8]. The pulse spread or pulsewidth for 2-D simulations is proposed like

$$spread \geq 1.278 \left(\frac{1}{\Delta t \cdot f_{\max}} \right), \quad (15)$$

where

$$\Delta t = \frac{\Delta x_{FDTD}}{\sqrt{2}c}, \quad (16)$$

$$f_{\max} = \frac{c}{4\Delta x_{FDTD}}, \quad (17)$$

and

$$\Delta x_{FDTD} = \frac{W v_{pm}^n}{m v_{pd}^n}, \quad (18)$$

is the cell size for the FDTD process [2], which is the ratio of the width of the strip (W) to the number of segments of the discretized strip (m), multiplied by the factor v_{pm}^n/v_{pd}^n . As the pulse propagates down the path or track it degrades and reach the end of the line where it is reflected entering to a back and forth trip. Fig. 1 shows the importance of a good assessment of the pulse spread. This behavior was simulated by using the FDTD process for a single microstrip line as in [2]. As can be seen from the figure, a pulse trail or wake appears or is more visible when the value of the *spread* parameter was changed from 8 to 4. The time in Fig. 1 is obtained by multiplying the number of time steps (200) by the simulation timestep $\Delta t = 1.7249e^{-12}s$.

Associated to the pulse spread is the reflected-to-incident wave ratio. This is a practical criterion that permits to establish the correct time points for simulation, corresponding to the observation planes which define the access and transfer functions for the scattering S parameters [9].

A graphical view of the pulse flight and its reflection is shown in Fig. 2 for the same microstrip line. As can be seen from this figure, the pulse starts to be settled down before 100 time steps and hit upon the reflected waveform before 500 time steps so, it seems advisable to define simulation time points for the incident and reflected waves at values close to these limits. The values of 70 for the entrant pulse and 465 for the reflected waveform were found to be a convenient choice. The time in Fig. 2 is obtained by multiplying the number of time steps (1000) by the simulation timestep $\Delta t = 1.7249e^{-12}s$.

By using these values, a handy form to define the S_{11} parameter is as follows

$$S_{11}(\omega) = \frac{E_{ref}(\omega, z_1)}{E_{inc}(\omega, z_1)} \exp(2 \cdot \gamma \cdot I_{op} \cdot \Delta x_{FDTD}), \quad (19)$$

where

$$\gamma = \alpha + j \frac{2\pi f}{v_{pm}}, \quad (20)$$

α is the total attenuation constant (dielectric attenuation plus conductor attenuation) as calculated from [6, 10, 11, 12] in chapter 3 of [2], f is the frequency band and I_{op} is the input observation plane corresponding to z_1 .

Fig. 1. Propagation of a Gaussian pulse with two different spreads on a microstrip track. When the spread is changed from 8 (top figure) to 4 (bottom figure) a large wake or trail emerges.

Fig. 2. Analysis of the Gaussian pulse flight to determine the simulation time points and the location of the observation planes defining the scattering S parameters.

A similar procedure can be carried out to define a usable equation for the S_{21} parameter. The S_{11} scattering parameters for the microstrip line as obtained from circuit analysis, FDTD simulation, measurement [2], and FDTD-Mol simulation using the criteria presented in this paper with a NN boundary condition, are all shown in Fig. 3 for comparison. As can be observed from this figure the agreement among them is good enough as to validate the procedure.

Fig. 3. The S_{11} parameter of a microstrip track obtained by circuit analysis (black), FDTD simulation (blue), measurement (red), and FDTD-MoL simulation (green).

5 Segmentation and discretization

As mentioned in section 1, the segmentation-discretization procedure is leaved for the FDTD portion of the hybrid routine which is highly efficient. On the contrary, the analytical solution for the remaining space variable could be very time consuming if the line-discretization is increased [13]. One example of a microstrip circuit requiring a large line-discretization is that of a multisection quarter-wave impedance matcher since it uses a cascade of several individual quarter-wave transformers of different widths to match properly two distinct impedances. This problem can be reduced by using a combined nonequidistant-discretization and modal-complexity-reduction procedure [13, 14].

6 Analytical boundary conditions

The perfectly matched layer (PML) absorbing boundary conditions (ABCs) [15] are the most popular edges in FDTD Maxwell equations based simulations, where a displacement current flux density is considered. These conditions permit to enclose the microstrip electromagnetic problem inside a tetragonal area (2-D) or a hexahedral volume (3-D) such a way that the computational task is limited to manageable dimensions. On the contrary,

the FDTD Telegrapher equations based simulations, profit the fact that the propagation is performed via a conduction current flux density and hence the limits of the microstrip conductor strips are natural boundaries between the side conductor-dielectric transitions and the top (conductor-air) and bottom (conductor-dielectric) transitions requiring only input/output or source/load circuit terminations. These efficient boundary conditions require however be amended each time the geometry of the simulation circuit is varied or the case of study is changed. In contrast, the mixed Dirichlet-Neumann boundary conditions of the Mol method are leaved to be settled down in the analytical part of the simulation process which means extra degrees of freedom since they are independent of the variations and complexity of the circuit geometry. Fig. 4 shows the timestep at which the Gaussian pulse reach the last cell of the segmented microstrip line where it is almost entirely absorbed because of the analytical NN boundary condition.

Fig. 4. Advance of a Gaussian pulse on a single microstrip line when is reaching the output port at the last line segment.

7 Conclusion

Despite the line-discretization of the hybrid method may have an excessive consumption of time for complicated structures, and slightly fails at the left side of the frequency band, and at the phase angle transitions of the S_{ii} scattering parameters (from 180° to -180° in Fig. 3), its performance is good enough as to be implemented for solving problems considering multi-layer and multiple-line tracking microstrip circuits and delay or meander line based RFID circuits. The conjunction of some simple practical criteria, as the empirical selection of the waveguide b dimension, the use of the corrected propagation speed, the correct estimation of the pulse spread, the proper location of the observation planes, the enhancement of the computational efficiency by nonequidistant-discretization and modal-complexity-reduction, and the analytical incorporation of the boundary conditions, makes this pos-

sible.

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The author is with the Departamento de Electrónica, División de Electrónica y Computación, Centro Universitario de Ciencias Exactas e Ingenierías, Universidad de Guadalajara, Apartado Postal 52-14, 45030 Guadalajara, Jalisco, México.