Pure statistical indoor pathloss model

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Abstract: The pathloss model in the literature suffers from two major deficiencies: First, it is distance dependent, whereas the distance of an interior link is unknown to outsiders. Second, it suffers from line-of-sight (LOS) uncertainty with channel parameters fluctuating sharply between LOS and non-LOS conditions. In this paper, we address these problems by taking the distance as a random variable and by introducing a room-density index that combines the LOS and non-LOS conditions. We then modify the widespread IEEE 802.15.4a pathloss model into a unified pure statistical pathloss model that is a function of the room size and the room-density index. This pure statistical model supplements the conventional pathloss model and can help cognitive users infer pathloss information of the non-cognitive without knowing the latter's link distance.

Keywords: channel model, statistics of pathloss, pure statistical pathloss model

Classification: Wireless circuits and devices

References

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1 Introduction

Pathloss model plays a fundamental role in the design and evaluation of communication systems. The current pathloss models in the literature invariably treat the pathloss as a function of distance and shadowing [1], the

shadowing being modeled as a Gaussian random variable but the distance as deterministic. However, the distance itself is known inside the communicating terminals, but unknown to the outsiders. Besides, line-of-sight (LOS) and non-LOS (NLOS) uncertainties influence the pathloss dearly, leading to sharply different parameters. With the advancement of cognitive radio, devices are becoming increasingly intelligent. Cognitive users are required to be aware of their surrounding environment through sensing, learning, reasoning, and making decision [2]. For this purpose, a distance- and LOS-independent pure statistical pathloss model is more insightful and convenient. In this paper, we try to modify the widespread IEEE 802.15.4a pathloss model into such a statistical model that depends on room size and room-density index, both of which are commonly knowledge to all the users.

2 The Brief of the Conventional Channel Model

2.1 Pathloss

In the well-known IEEE 802.15.4a channel model, the pathloss L in decibels at some distance Z is modeled as,

$$L = l_0 + 10n \log_{10} Z + S \quad (1)$$

where l_0 is the pathloss at $z_0=1$ m, n is referred to as the pathloss exponent, and S is the log-normal shadow fading, namely $S \sim N(0, \sigma_s^2)$ in decibels.

This model suffers from two major deficiencies: First, the distance is unknown to the outsiders. Second and worse, the other parameters are uncertain with the following two uncertain channel conditions:

$$\text{LOS} : \{l_0, n, S\} \triangleq \{35.4, 1.63, N(0, 1.9^2)\} \quad (2)$$

$$\text{NLOS} : \{l_0, n, S\} \triangleq \{57.9, 3.07, N(0, 3.9^2)\} \quad (3)$$

A question is thus easily invited—Can we eliminate the uncertainties?

We address this problem by taking the parameters in (2) and (3), as well as the distance Z , as random variables. We claim that, by doing so, L remains Gaussian due to the following facts: First, the original parameters in (2) and (3) in general are averaged over the small-scale fading [1, p. 4] which is due to a large number of multiple signal paths between the transmitter and receiver. Second, the shadow fading or large-scale fading S in particular comes from sum effects of many obstacles absorbing or scattering the radio power. Third, L is Gaussian justified by the central limit theorem (CLT). Therefore, the addition of more random variables into L makes the CLT hold even more rigid. What left is to decide the moments of L .

2.2 The Randomizing of the Channel Parameters

Firstly, we randomize the channel parameters $\{l_0, n, S\}$ by introducing a room-density index, q , defined as the probability of occurrence of LOS condition, namely,

$$p(l_0, n, s) = \begin{cases} q & \text{if } \{l_0, n, S\} \in \text{LOS} \\ 1 - q & \text{if } \{l_0, n, S\} \in \text{NLOS} \end{cases} \quad (4)$$

Or equivalently,

$$p(l_0, n, s) = q\delta(l_0 - 35.4)\delta(n - 1.63)f_1(s) + (1 - q)\delta(l_0 - 57.9)\delta(n - 3.07)f_2(s) \quad (5)$$

where $\delta(\cdot)$ is the Dirac delta function, $f_1(s)$ and $f_2(s)$ are Gaussian functions with different parameters, namely,

$$f_1(s) \triangleq N(0, 1.9^2), f_2(s) \triangleq N(0, 3.9^2) \quad (6)$$

2.3 The pdf of Random Distance in a Square Room

Secondly, we calculate the statistics of the random distance. Let the two points $A(X_1, Y_1)$ and $B(X_2, Y_2)$ represent two communicating wireless terminals, uniformly distributed over the area of a square room with each side a . In other words, the coordinates X_1, Y_1, X_2 , and Y_2 are independently and uniformly distributed over the interval $[0, a]$ with $a > 0$. Then, the distance between A and B

$$Z = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} \quad (7)$$

is a random variable, whose distribution has been extensively studied in the literature. Borrowing from [3] and by simple manipulation, we can write the probability density function (pdf) of Z as follows,

$$f_Z(z) = \begin{cases} \frac{2z^3}{a^4} - \frac{8z^2}{a^3} + \frac{2\pi z}{a^2}, & 0 \leq z < a; \\ -\frac{2z^3}{a^4} + \left[-\frac{4}{a^2} + \frac{8\sin^{-1}(\frac{a}{z})}{a^2} + \frac{8\sqrt{z^2 - a^2}}{a^3} - \frac{2\pi}{a^2}\right]z, & a \leq z \leq \sqrt{2}a. \end{cases} \quad (8)$$

With this, we can derive the cumulative distribution function (CDF) as follows,

$$F_Z(z) = \begin{cases} 0, & z < 0; \\ \frac{z^4}{2a^4} - \frac{8z^3}{3a^3} + \frac{\pi z^2}{a^2}, & 0 \leq z < a; \\ -\frac{z^4}{2a^4} + \frac{4z^2}{a^2} \sin^{-1}\left(\frac{a}{z}\right) + \frac{4z}{a} \sqrt{1 - \frac{a^2}{z^2}} + \frac{8}{3} \frac{(z^2 - a^2)^{3/2} - (2a + \pi a)z^2}{a^3} + \frac{1}{3}, & a \leq z < \sqrt{2}a; \\ 1, & z \geq \sqrt{2}a. \end{cases} \quad (9)$$

2.4 The statistics of L

Thirdly, assuming that Z is independent of l_0, n , and S , and using (5), (6), and (8) we can calculate the first and second moments of L as

$$E(L) = \iiint_{D \in \mathbb{R}^4} \left[l_0 + \frac{10n \ln(z)}{\ln(10)} + s \right] p(l_0, n, s) f_Z(z) dl_0 dn ds dz \\ \approx 57.9 - 22.5q + (13.3 - 6.25q) \int_0^{\sqrt{2}a} \ln(z) f_Z(z) dz \quad (10)$$

$$\begin{aligned}
 E(L^2) &= \iiint\limits_{D \in \mathbb{R}^4} \left[l_0 + \frac{10n \ln(z)}{\ln(10)} + s \right]^2 p(l_0, n, s) f_Z(z) dl_0 dn ds dz \\
 &\approx 3367.62 - 2110.85q + (1543.94 - 1042.75q) \int_0^{\sqrt{2}a} \ln(z) f_Z(z) dz \\
 &\quad + (177.76 - 127.65q) \int_0^{\sqrt{2}a} [\ln(z)]^2 f_Z(z) dz
 \end{aligned} \quad (11)$$

Now, we embark on calculating the moments of $\ln(Z)$. With the aid of the formula manipulating software Maple [4] and integrating by parts, we get

$$\int_0^{\sqrt{2}a} \ln(z) f_Z(z) dz = [\ln(z) F_Z(z) - G(z)]_0^{\sqrt{2}a} \quad (12)$$

$$\int_0^{\sqrt{2}a} [\ln(z)]^2 f_Z(z) dz = \{ [\ln(z)]^2 F_Z(z) - 2 \ln(z) G(z) + 2H(z) \}_0^{\sqrt{2}a} \quad (13)$$

where $F_Z(z)$ is the CDF given previously by (9), $G(z)$ and $H(z)$ are two newly constructed auxiliary functions from the CDF as follows,

$$G(z) = \int_{-\infty}^z \frac{F_Z(x)}{x} dx, \quad z \in (-\infty, \sqrt{2}a] \quad (14)$$

$$H(z) = \int_{-\infty}^z \frac{G(x)}{x} dx \quad z \in (-\infty, \sqrt{2}a] \quad (15)$$

As $F_Z(z)$ is continuous over the interval $(-\infty, \infty)$ and when $z \rightarrow 0$, $F_Z(z) \sim o(z^2)$, we are sure that $F_Z(z)/z$ is integrable, such that $G(z)$ exists. Again $G(z)/z$ is also integrable, so $H(z)$ exists.

By substituting (9) into (14) and integrating piecewise, we get

$$G(z) = \begin{cases} 0, & z < 0; \\ \frac{z^4}{8a^4} - \frac{8z^3}{8a^3} + \frac{\pi z^2}{2a^2}, & 0 \leq z < a; \\ -\frac{z^4}{8a^4} + \left(\frac{2}{a^2} \sin^{-1} \left(\frac{a}{z} \right) - \frac{\pi}{2a^2} - \frac{1}{a^2} \right) z^2 \\ + \frac{2z}{a} \sqrt{1 - \frac{a^2}{z^2}} + \frac{4}{3} \tan^{-1} \left(\frac{a}{\sqrt{z^2 - a^2}} \right) \\ + \frac{13}{36} + \frac{8}{9} \frac{(z^2 - a^2)^{3/2}}{a^3} + \frac{4}{3} \frac{\sqrt{z^2 - a^2}}{a} \\ + \frac{1}{3} \ln(z) - \frac{2\pi}{3} - \frac{1}{3} \ln(a), & a \leq z < \sqrt{2}a; \\ \frac{25}{12} - \frac{\pi}{3} + \frac{\ln(2)}{6}, & z = \sqrt{2}a; \end{cases} \quad (16)$$

which shows that $G(0)=0$ and $G(\sqrt{2}a) = \frac{25}{12} - \frac{\pi}{3} + \frac{\ln(2)}{6} \approx 1.151660312$. In turn, substituting (16) into (15) and integrating piecewise, we get

$$H(0) = \int_{-\infty}^0 G(x)/x dx = 0 \quad (17)$$

$$H(\sqrt{2}a) = \int_{-\infty}^{\sqrt{2}a} G(x)/x dx \approx 0.8595400026 \quad (18)$$

Substituting the obtained $G(0)$, $G(\sqrt{2}a)$, $H(0)$, and $H(\sqrt{2}a)$ back into (12) and (13), and using the fact that $F_Z(0)=0$, $F_Z(\sqrt{2}a)=1$, $\lim_{z \rightarrow 0} \ln(z)F_Z(z)=0$, $\lim_{z \rightarrow 0} [\ln(z)]^2 F_Z(z)=0$, and $\lim_{z \rightarrow 0} \ln(z)G(z)=0$, we have

$$\int_0^{\sqrt{2}a} \ln(z) f_Z(z) dz \approx \ln(a) - 0.805086722 \quad (19)$$

$$\int_0^{\sqrt{2}a} [\ln(z)]^2 f_Z(z) dz = 1.040923161 - 1.610173444 \ln(a) + [\ln(a)]^2 \quad (20)$$

Substituting (19), (20) back into (10), (11) and retaining only two decimal digits for simplicity, we finally obtain the statistics of the pathloss as follows,

$$E(L) \approx (13.3 - 6.25q) \ln(a) + 47.19 - 17.47q \quad (21)$$

$$\begin{aligned} E(L^2) \approx & (177.76 - 127.65q) [\ln(a)]^2 + (1257.71 - 837.21q) \ln(a) \\ & + (2309.65 - 1404.22q) \end{aligned} \quad (22)$$

$$\begin{aligned} Var(L) = & E(L^2) - [E(L)]^2 \\ \approx & 39.11q(1-q) [\ln(a)]^2 + 218.45q(1-q) \ln(a) + 85.03 - 305.03q^2 + 243.29q \end{aligned} \quad (23)$$

2.5 The Pure Statistical Pathloss Model

In summary, we obtain a unified Gaussian pathloss model $L \sim N(m_L, \sigma_L^2)$ where

$$m_L = (13.3 - 6.25q) \ln(a) + 47.19 - 17.47q \quad (24)$$

$$\begin{aligned} \sigma_L^2 = & 39.11q(1-q) [\ln(a)]^2 + 218.45q(1-q) \ln(a) \\ & + 85.03 - 305.03q^2 + 243.29q \end{aligned} \quad (25)$$

As opposed to the original (1) which is a function of distance and is uncertain about LOS or NLOS conditions, ours is shown to be a function of the room size a and the room density index q , and is advantageous in the following facts: First, the room size a is a common knowledge. Second, the q better characterizes the uncertain LOS or NLOS condition. For instance, when $q=1$, an LOS connection is experienced. When $q=0$, an NLOS connection is experienced. If a half-probable LOS/NLOS connection is experienced, we can let $q=\frac{1}{2}$. More situations can be included by letting q change continuously between 0 and 1. Obviously, the single q embraces the extreme LOS and NLOS conditions while adding something in between. Therefore, our model can be a good supplement to the original IEEE 802.15.4a channel model in the determination of distance-independent pathloss while eliminating LOS and NLOS uncertainties. In the later section, we will demonstrate how a cognitive user can infer pathloss information of the other non-cognitive users via our proposed model.

3 Numerical Results

In the numerical analysis, we consider two room sizes: $a=10$ m and $a=20$ m, and three room density indices: $q=1$, $q=0$, and $q=\frac{1}{2}$. With graphs, we compare the pdf of our model with that of the original.

In Fig. 1, the pdf's of the original pathloss model are illustrated by solid lines, where the same distance $z=5.2$ m is assumed but with different LOS and NLOS channel conditions. As seen, the pdf's are functions of distance, and with respect to LOS or NLOS channel conditions their mean value differs sharply by a gap of 33 dB.

The pdf's of the our model are illustrated by dashed, dashdot, and dotted lines respectively. The dashed lines corresponds to $q=1$, denoting pure LOS; the dashdot lines to $q=0$, denoting pure-NLOS; the dotted lines to $q=\frac{1}{2}$, denoting half-LOS and half-NLOS condition. As seen, the pdf's of our model are functions of the room size and the room-density index, eliminating the uncertainties about the LOS and NLOS conditions by the introduction of the room-density index.

What price to pay is the larger variances and longer tails in our model: When the room-density index is chosen as $q=\frac{1}{2}$, the standard variance σ_L reaches almost 20 dB, as indicated by the dotted lines. Nonetheless, this is still better than the 33-dB gap of the original model.

Next, we consider an example. If the room size $a=10$ m, the room-density index $q=\frac{1}{2}$, and a 95% coverage is guaranteed to primary users, what is their maximum pathloss predicted by a cognitive user?

From (24) and (25) we get $m_L=62$ dB and $\sigma_L^2=308$. By consulting the Q -function that calculates the tails under the pdf of normal Gaussian distribution, we know that with significance level of $p=5\%$, $x \approx 1.645$, where

$$x = \frac{L - m_L}{\sigma_L} \quad (26)$$

By solving (26) for L , the cognitive user infers that the primary users' maximum pathloss is $L=90.9$ dB, without knowing their maximum link distance.

Fig. 1. pdf of the pathloss in a square office room

4 Conclusion

By taking the distance between two indoor mobile terminals as random variable and by introducing a room-density index, we remodel the IEEE 802.15.4a pathloss model into a unified and pure statistical model, which eliminates the dependence on the unknown link distance as well as the uncertainty of LOS and NLOS channel conditions. The probability distribution function underlying our model is still Gaussian, but is shown to be a function of the room size and room-density index, which are common knowledge to all users located in the same region. Our proposed model can supplement the widely used IEEE 802.15.4a channel model in situations where cognitive users are required to learn the pathloss information of the non-cognitive without knowing the latter's link distance.

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