Phase error analysis of the LC-Tank CMOS quadrature oscillators

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Abstract: Using an analytical approach, new estimation of phase error in LC-Tank quadrature oscillators is proposed. The mismatches between passive components of LC tanks are considered as the reasons of the phase error. Closed form equations relating LC mismatches to phase errors are presented. To evaluate the analysis, a 5 GHz quadrature LC-Tank oscillator is simulated using TSMC 0.18 μm model technology. The results confirm the simplicity and high accuracy of the proposed analysis.

Keywords: quadrature oscillator, phase error

Classification: Integrated circuits

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1 Introduction

One of the most significant sections of many communication systems are oscillators with quadrature outputs [1]. There are different ways to generate

quadrature outputs. A straightforward way is using a four-stage differential ring oscillator, but a more common with higher performance topology is two coupled LC-Tank CMOS oscillators. Different ways have been proposed to couple two CMOS LC-Tank oscillators. The first method is parallel coupling proposed by A. Rofougaran [2]. In this topology two additional transistors are added in parallel to one transistor of the oscillator. Using the gates of the parallel transistors, the oscillators are coupled as shown in Fig. 1.

Fig. 1. the parallel coupled quadrature oscillator

In fully match parts, applying Barkhausen's phase criterions implies the 90 degrees phase difference between outputs. Mismatches, like device mismatch, layout asymmetries, and parasitic effects cause phase error. The phase error affects the overall performance of the systems incorporating the quadrature oscillators [3]. This shows the significance of accuracy of the outputs' relative phase. Some references investigate the phase noise of the above oscillators and some other study the phase error using simulation. This tradeoff has been mentioned in several papers [4, 5]. Having simple accurate equations makes the optimal design straightforward. In this paper closed form equations relating phase error and mismatches are proposed. The rest of the paper is as follows: In section 2, for ideal case and without any mismatches, the quadrature oscillator is analyzed and some attractive aspect of the oscillator is explained. In section 3 the effect of mismatches on phase accuracy is investigated. Simulation results and comparisons are introduced in section 4.

2 LC-Tank quadrature oscillator

LC-Tank quadrature oscillators have the topology shown in Fig. 1. In this figure using coupling transistors, shown as M_C , the outputs of each oscillator are fed to the inputs of another. A simple analysis is performed under fully matched devices and components. In oscillation under conditions mentioned above, we can write equations 1 - 2 for four voltages shown in figure 1.

$$V_{I\pm} = \mp A \cos(\omega t) \quad (1)$$

$$V_{Q\mp} = \mp A \cos(\omega t + \theta) \quad (2)$$

In above equations, each block generates differential outputs so 180 degrees phase difference is considered for single ended outputs of each block. Because

of differential inputs each block experiences, the source of transistor have a constant voltage equivalent to a small signal ground. So the current of each transistor can be written as $I = G_m V$ where G_m is the “transconductance” of each transistor. In node V_{Q+} we write the following simple equation.

$$-V_{Q+} = (I_{I-} + I_{Q-}).Z \quad (3)$$

In above equation Z is the impedance of LC Tank at oscillation frequency. Writing equation (3) in term of phasors leaves to equation (4).

$$-e^{j\theta} = (G_{mc}e^{j0} - G_{ms}e^{j\theta}).Z \quad (4)$$

In (4) G_{mc} and G_{ms} are the transconductance of coupling and switching transistors of the oscillator. Equation (4) can be simply expressed as (5).

$$e^{j\theta} = \frac{G_{mc}.Z}{G_{ms}.Z - 1} \quad (5)$$

We can write similar equation for node V_{I+} as shown in (6) that is reduced to (8).

$$-V_{I+} = (I_{Q+} + I_{I-}).Z \quad (6)$$

$$-e^{j\pi} = (G_{mc}e^{j\theta} + G_{ms}e^{j0}).Z \quad (7)$$

$$e^{j\theta} = -\frac{G_{ms}.Z - 1}{G_{mc}.Z} \quad (8)$$

Multiplying (5) and (8) concludes the following significant equation, proving the quadrature nature of the single ended outputs. One can simply show that two differential output are also quadrature.

$$e^{j2\theta} = -1 \Rightarrow \theta = \pi/2 \quad (9)$$

3 Phase error caused by mismatch between LC-tanks

The above calculations were performed when two LC-Tanks have no mismatches. Under such a condition the phase difference between outputs was 90 degrees but simulations show that when two LC tanks are not exactly the same the phase difference between outputs deviates from 90 degrees. In this section the phase error is related to mismatches of LC tanks and a closed form equation is extracted. To perform this we suppose impedances Z_1 and Z_2 for two tanks as shown in figure 1.

For a general solution we assume below expressions for four single ended outputs.

$$V_{I-} = A \cos(\omega t) \quad (10)$$

$$V_{Q+} = A \cos(\omega t + \theta_1) \quad (11)$$

$$V_{I+} = A \cos(\omega t + \theta_2) \quad (12)$$

$$V_{Q-} = A \cos(\omega t + \theta_3) \quad (13)$$

The current voltage relation for V_{I+} is as (14)

$$-V_{I+} = (I_{Q+} + I_{I-}).Z_1 \quad (14)$$

$$I_{I-} = G_{ms}A \cdot \cos(\omega t), I_{Q+} = G_{mc}A \cdot \cos(\omega t + \theta_1) \quad (15)$$

$$I_{Q+} + I_{I-} = G_{ms}A(\cos \omega t + k \cos(\omega t + \theta_1)) , \quad k = \frac{G_{mc}}{G_{ms}} \quad (16)$$

The current of (16) passes through Z_1 and builds up V_{I+} . The first term of (16) has a real phasor of size 1 and angle of zero. The second term has a complex phasor of size k and angle of θ_1 . Looking at Fig. 2 one can simply find that the phase of (16) is as (17).

$$\angle(I_{Q+} + I_{I-}) = \frac{\theta_1}{2} + \alpha \quad (17)$$

As we know without any mismatch the phase error is zero and $\theta_1 = 90$. Mismatches causes θ_1 deviates from 90 degrees but it is close to 90 yet. To compute α we use the approximation of $\theta_1 \approx 90$ in following calculations.

In figure 2 (a) we can write

$$\frac{\sin y}{\sqrt{2}} = \frac{\sin \alpha}{k-1} \Rightarrow \sin \alpha = \frac{(k-1)}{\sqrt{2}} \frac{\tan y}{\sqrt{1 + \tan^2 y}}$$

Using $\theta_1 \approx 90$ approximation we will have

$$\tan y = \frac{1}{k} \Rightarrow \sin \alpha = \frac{k-1}{\sqrt{2(k^2+1)}} \Rightarrow \alpha = \sin^{-1} \left(\frac{k-1}{\sqrt{2(k^2+1)}} \right) \quad (18)$$

So the phase of (16) is as

$$\angle(I_{Q+} + I_{I-}) = \frac{\theta_1}{2} + \alpha \quad (19)$$

Fig. 2. (a) phasor diagram of (16). (b) the dependency between k and α

So considering (14), the relation of (20) holds for θ_1, θ_2 .

$$\theta_2 = \frac{\theta_1}{2} + \alpha + \varphi_1 + \pi , \quad \varphi_1 = \angle Z_1 \quad (20)$$

Using similar approach for V_{Q+} , V_{I-} and V_{Q-} we find following relations among different angles.

$$V_{Q+} : \quad \theta_1 = \frac{\theta_3}{2} + \alpha + \varphi_2 , \quad \varphi_2 = \angle Z_2 \quad (21)$$

$$V_{I-} : \quad \pi = \frac{\theta_2}{2} + \frac{\theta_3}{2} + \alpha + \varphi_1 \quad (22)$$

$$V_{Q-} : \quad \theta_3 = \frac{\theta_1}{2} + \frac{\theta_2}{2} + \alpha + \varphi_2 + \pi \quad (23)$$

Angle of impedance of tanks at oscillation frequency is as (24) and (25).

$$\varphi_1 = \tan^{-1} \left[\frac{R_{p1}(1 - L_1 C_1 \omega^2)}{L_1 \omega} \right] \quad (24)$$

$$\varphi_2 = \tan^{-1} \left[\frac{R_{p2}(1 - L_2 C_2 \omega^2)}{L_2 \omega} \right] \quad (25)$$

Here $R_{p1,2}$ are equivalent parallel resistors in the LC tank. Equations (20) to (25) form a six unknown equations system. Simple investigations show that we have $\theta_2 = \pi$ and $\theta_3 = \theta_1 + \pi$. Indeed under unbalance conditions, the single ended output save their differential nature. Applying this, (20) is reduced to (26).

$$\theta_1 = -2\varphi_1 - 2\alpha \quad (26)$$

To continue, assume that the difference of angles of two impedances be $\Delta\varphi$ as defined in (27).

$$\Delta\varphi = \varphi_2 - \varphi_1 \quad (27)$$

Summing (20) to (23), leaves equation (28).

$$\varphi_1 + \varphi_2 = -\frac{\pi}{2} - 2\alpha \quad (28)$$

From (27) and (28) we can express φ_1, φ_2 as (29) and (30).

$$\varphi_1 = -\frac{\pi}{4} - \alpha - \frac{\Delta\varphi}{2} \quad (29)$$

$$\varphi_2 = -\frac{\pi}{4} - \alpha + \frac{\Delta\varphi}{2} \quad (30)$$

From (29) and (30) equation (26) can be written as (31).

$$\theta_1 = -2\varphi_1 - 2\alpha = \frac{\pi}{2} + \Delta\varphi \quad (31)$$

Equation (31) shows that the phase error under mismatch is the difference between angles of two impedances. Substituting φ_1, φ_2 from (29) and (30) in (24) and (25) leads to (32) and (33).

$$\tan \left(\frac{\pi}{4} + \alpha + \frac{\Delta\varphi}{2} \right) = -\frac{R_{p1}(1 - L_1 C_1 \omega^2)}{L_1 \omega} \quad (32)$$

$$\tan \left(\frac{\pi}{4} + \alpha - \frac{\Delta\varphi}{2} \right) = -\frac{R_{p2}(1 - L_2 C_2 \omega^2)}{L_2 \omega} \quad (33)$$

$\Delta\varphi$ and ω are the solutions of equations system of (32) and (33). The direct solution of the system is very complicated. So we introduce an approximate method. We assume $\beta_1 = \alpha + \Delta\varphi/2$ and $\beta_2 = \alpha - \Delta\varphi/2$. We can write,

$$\tan \left(\beta_{1,2} + \frac{\pi}{4} \right) = \frac{\sin \left(\beta_{1,2} + \frac{\pi}{4} \right)}{\cos \left(\beta_{1,2} + \frac{\pi}{4} \right)} = \frac{\sin(\beta_{1,2}) + \cos(\beta_{1,2})}{\cos(\beta_{1,2}) - \sin(\beta_{1,2})} \quad (34)$$

$\beta_{1,2}$ have generally small values, so it is reasonable to write $\cos(\beta) = 1$ and $\sin(\beta) = \beta$, so from (34) we will have following equation.

$$\tan\left(\beta_{1,2} + \frac{\pi}{4}\right) = \frac{1 + \beta_{1,2}}{1 - \beta_{1,2}} \quad (35)$$

Using above approximation (35) is simplified to equation (36)

$$\frac{1 + \beta_{1,2}}{1 - \beta_{1,2}} = -\frac{R_{p1,2}(1 - L_{1,2}C_{1,2}\omega^2)}{L_{1,2}\omega} \quad (36)$$

From (36) can be written as (37).

$$\frac{1 + \beta_1}{1 - \beta_1} - \frac{1 + \beta_2}{1 - \beta_2} = \frac{R_{p2}(1 - L_2C_2\omega^2)}{L_2\omega} - \frac{R_{p1}(1 - L_1C_1\omega^2)}{L_1\omega} \quad (37)$$

Substituting $\beta_1 = \alpha + \Delta\varphi/2$ and $\beta_2 = \alpha - \Delta\varphi/2$, the left hand of (37) can be rewritten as (38)

$$\begin{aligned} \frac{1 + \beta_1}{1 - \beta_1} - \frac{1 + \beta_2}{1 - \beta_2} &= \frac{2\beta_1 - 2\beta_2}{1 - \beta_1 - \beta_2 + \beta_1\beta_2} \\ &= \frac{2\Delta\varphi}{1 - 2\alpha + \alpha^2 - \frac{\Delta\varphi^2}{4}} \cong \frac{2\Delta\varphi}{1 - 2\alpha + \alpha^2} = \frac{2\Delta\varphi}{(1 - \alpha)^2} \end{aligned} \quad (38)$$

Ignoring $\Delta\varphi^2$ (37) is reduced to (39).

$$\frac{2\Delta\varphi}{(1 - \alpha)^2} = \frac{R_{p2}(1 - L_2C_2\omega^2)}{L_2\omega} - \frac{R_{p1}(1 - L_1C_1\omega^2)}{L_1\omega} \quad (39)$$

(39) is the relation between phase error and components of two LC tanks. Assuming the same value for the resistance of two tanks as R_p , (39) can be written as (40).

$$\begin{aligned} \Delta\varphi &= \frac{R_p(1 - \alpha)^2}{2\omega} \left[\frac{(1 - L_2C_2\omega^2)}{L_2} - \frac{(1 - L_1C_1\omega^2)}{L_1} \right] \\ &= \frac{R_p(1 - \alpha)^2}{2\omega} \left(\frac{\Delta L}{L_1L_2} + \Delta C\omega^2 \right) \end{aligned} \quad (40)$$

Where $\Delta C = C_1 - C_2$ and $\Delta L = L_1 - L_2$. Using quality factor of tanks, above expression can be written in a better form as follows:

$$Q = R_p\sqrt{\frac{C}{L}} \Rightarrow R_p = QL\omega_0 = \frac{Q}{C\omega_0} \quad (41)$$

Replacing R_p from (41) in (40) the phase error is represented as (42)

$$\Delta\varphi = \frac{Q}{2} \left(\frac{\omega_0}{\omega_{osc}} \frac{\Delta L}{L} + \frac{\omega_{osc}}{\omega_0} \frac{\Delta C}{C} \right) (1 - \alpha)^2 \quad (42)$$

We suppose $\omega_{osc} \cong \omega_0$ that concludes the below simplified expression for phase error.

$$\Delta\varphi = \frac{Q}{2} \left(\frac{\Delta L}{L} + \frac{\Delta C}{C} \right) (1 - \alpha)^2 \quad (43)$$

As (43) shows, decreasing Q and increasing α causes phase error be less sensitive to mismatches in LC tanks. On the other hand phase noise is

inversely proportional to Q , where $L(\Delta\omega) \propto 1/Q^2$ and directly proportional to coupling factor (k), so there is a tradeoff between phase noise and phase error [6, 7]. (Notice to Fig. 2 (b)).

As mentioned, decreasing phase error by changing Q or the relative ratio of coupling transistor and main transistor results in increasing phase noise. As we know, phase error is equal to the phase difference of two LC tank impedances. If we decrease phase difference of LC tanks' impedance without any change in the Q or coupling factor circuit, phase error will decrease without any increasing in phase noise. From (39), adding a small! Controlled parallel resistor to one of LC tanks changes R_p and consequently decreases phase error.

4 Simulation results

To evaluate the proposed analytical results, a 5 GHz oscillator is designed and simulated using 0.18 μm CMOS TSMC model and different experiments have been performed. The LC tank comprises of a 2-nH inductor and 0.5-pF capacitor. Transistors M_C and M_S have the same aspect ratio $[W/L]$ of 10 $[\mu\text{m}]/0.18 [\mu\text{m}]$. To verify the dependency of phase error to the oscillator coupling factor, the current of main transistors is kept constant at 1 mA, while the current of coupling transistors is varied from 1 mA to 2.5 mA. Dotted

Fig. 3. (a) Phase error versus Q ; $k = 1$ and 1% mismatches in inductors and capacitors. (b) Phase error versus k ; 5% mismatches in inductors. (c) phase error versus mismatches in inductors and capacitors; $k = 1$

curves in Fig. 3 (a) shows the simulated phase errors versus quality factor for a mismatch of 1% on tank capacitance and inductance compared with result of equations (20) to (25) and approximate equation (43). Fig. 3 (b) shows the phase error versus coupling factor for 5% mismatches in inductors with a Q of 10. Fig. 3 (c) shows phase error versus mismatches in inductors and capacitors.

As one can see the proposed analysis exactly meet the simulation results.

5 Conclusions

A new approach to analysis the phase error of the parallel coupled quadrature LC tank oscillators has been proposed and new closed form equation was introduced. All behavioral aspects seen in simulations have been estimated by the proposed model. For more study we are focusing to design a self phase error compensated quadrature oscillator.