Approximations of the inverse wavelet transform for analogue circuits

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Abstract: We propose two approximations of the inverse wavelet transform implemented with a voltage adder and two analog filters. They work together with a set of scaled band-pass analog filters that perform the wavelet transform of a continuous time signal. With this approach an integrated circuit has been fabricated. On-chip measurements demonstrate signal to reconstruction error ratios up to 25.8 dB.

Keywords: wavelet transform, integrated circuits, analog circuits

Classification: Electron devices, circuits, and systems

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1 Introduction

The Wavelet Transform (WT) is commonly used as a tool to perform a simultaneous time-frequency analysis of transitory features in time signals. Some processing techniques that are applied when the signal is in the wavelet domain require the application of the Inverse Wavelet Transform (IWT) in order to take the processed signal back to time domain.

Even when most of the reported implementations of the WT and IWT have been numerically performed, there has also been interest in finding analog implementations. For example, in [1] a methodology is presented to design analog filters (intended for low power bioimplantable devices) whose Impulse Response (IR) is an approximation of a given mother wavelet. Moreover, in [2] it is shown that the IR of a biquadratic band-pass filter fulfils the admissibility and stability constraints to be considered as an exact wavelet. Thus, a complete WT system can be created with a set of biquad scaled filters, where the set of output signals is the wavelet domain representation of the input signal.

Now this work discusses the problem of recovering the original signal from the wavelet domain using an analog implementation of the IWT.

2 Ideal wavelet transform and inverse wavelet transform system

Fig. 1a shows an *ideal* system performing the WT and the IWT of a continuous time signal, expressed in the frequency domain. It contains an adder and two sets of scaled filters, $H(r^m\omega)$ and $X(r^m\omega)$, where $r > 1$ is the constant of scaling between adjacent filters, $m = \dots, -1, 0, 1, 2, \dots$, $\omega = 2\pi f$, and f is frequency in Hz. A necessary condition to perform the WT is that $H(\omega)$ must be a band-pass filter. $F(\omega)$ and $F_{out}(\omega)$ are the Fourier transforms of the continuous input signal, $f(t)$, and the output signal, $f_{out}(t)$, respectively. We will denote the IR of $H(r^m\omega)$ as $h_{r^m}(t)$, then we can obtain that

$$h_{r^m}(t) = (1/r^m) h_{r^0}(t/r^m). \quad (1)$$

The output of $H(r^m\omega)$, given by $F(\omega)H(r^m\omega)$, will be denoted in time domain as $W_{r^m}f(t)$. Then, $W_{r^m}f(b) = \int_{-\infty}^{\infty} f(t) h_{r^m}(b-t) dt$. This is the formal definition of the *dyadic* WT given in [3] (for the case $r = 2$), where $W_{r^m}f(b)$ is the WT at translation time b , with respect to the mother wavelet, $h_{r^0}(-t)$, scaled to the m -th scale. In this work we use the more general term of *semidiscrete* WT when the transform is continuous in translation and discrete in scale. This covers any value $r > 1$. In the system of Fig. 1a, the IWT is performed by the set of filters $X(r^m\omega)$ and an adder.

Note that a given system will produce an output signal equal to its input signal only if its Transfer function (TF) is equal to 1, i.e., the complex TF must have a magnitude of 1 and a phase angle of 0 over the system frequency range. In the case of the ideal WT-IWT system, this condition for perfect reconstruction is $\sum_{m=-\infty}^{\infty} H(r^m\omega) X(r^m\omega) = 1$, for $\omega \in (0, \infty)$ [3].

3 Approximations of the semidiscrete inverse wavelet transform

We present two approximations of the ideal WT-IWT system. The *simple* approximation is the system of Fig. 1b working with the switches opened. For circuital simplification, we have taken only a finite number of filters, $(M+1)$, and given a constant value to all filters $X(r^m\omega) = \chi$. This new system

Fig. 1. Frequency domain descriptions of semidiscrete WT-IWT systems (a) Ideal system (b) Proposed approximations

works only along a finite range of frequency covered by the band-pass filters $H(r^m\omega)$. We will denote this frequency range as L . For this approximation, the condition for perfect reconstruction is $\sum_{m=0}^M \chi H(r^m\omega) = 1$, for $\omega \in L$.

To validate the proposed approach, the simple approximation has been on-chip fabricated for $H(\omega)$, r , M , and χ given in Table I. In [2] it is shown that the IR of this $H(\omega)$ satisfies the mathematical conditions to be used as a mother wavelet. The χ value is selected to normalize the system gain. As can be seen in Fig. 2a, the gain is within 1.000 ± 0.005 (very close to 1) from 100 Hz to 10 kHz. We will take this frequency range as L . For its part, the phase response on L is within $0^\circ \pm 30.2^\circ$, far from the required zero phase shift (see Fig. 2b).

Table I. Selected values for on-chip implementation of system in Fig. 1b, for a range $L = [100 \text{ Hz}, 10 \text{ kHz}]$

$H(r^m\omega) = \frac{5.978 \times 10^4 (jr^m\omega)}{(jr^m\omega)^2 + 5.978 \times 10^4 (jr^m\omega) + 7.146 \times 10^9}$	$r = \sqrt{2}$	$M = 15$
$H_\alpha(\omega) = \frac{(j\omega)^2 + 1.504 \times 10^5 (j\omega)}{(j\omega)^2 + 1.504 \times 10^5 (j\omega) + 1.584 \times 10^{10}}$	$\chi = 0.3794$	$\kappa = 0.78$
$H_\beta(\omega) = \frac{375.0(j\omega) + 9.842 \times 10^4}{(j\omega)^2 + 375.0(j\omega) + 9.842 \times 10^4}$	$j = \sqrt{-1}$	$N = 20$

However, for the values in Table I we have numerically observed that $\sum_{m=-N}^{M+N} \chi H(r^m\omega)$ holds a magnitude near 1, and flattens the phase close to 0° , on L , as the non-negative integer N grows, as shown in Fig. 2a and 2b for $N = 0, 2, 4$. Now, if we define $\kappa H_\alpha(\omega) \approx \sum_{m=-N}^{-1} \chi H(r^m\omega)$ and $\kappa H_\beta(\omega) \approx \sum_{m=M+1}^{M+N} \chi H(r^m\omega)$ over the range L , for a real κ , and a sufficiently large N , it is possible to corroborate that $\sum_{m=0}^M \chi H(r^m\omega) + \kappa H_\alpha(\omega) + \kappa H_\beta(\omega) \approx \sum_{m=-N}^{M+N} \chi H(r^m\omega) \approx 1$, for $\omega \in L$ with almost flat phase (see Fig. 2b). In this manner, the implemented *compensated* approximation (Fig. 1b with switches closed) overcomes the phase problem on L .

For the manufactured chip, the $H_\alpha(\omega)$, $H_\beta(\omega)$, κ , and N values are given

Fig. 2. Frequency response of approximations in Fig. 1b for the values in Table I (numerical simulations)
(a) Magnitude (b) Phase angle

in Table I. This compensated system has a gain of 1.000 ± 0.007 , and a phase angle of $0.0^\circ \pm 0.4^\circ$, over the range L (see Fig. 2a and 2b).

Note that we could select different $H(\omega)$, r , M and χ values than those given in Table I, but a good reconstruction can be done only if the TF of the system has an approximate gain of 1 and phase of 0° over the system frequency range, L . Moreover, we could select filters $H_\alpha(\omega)$ and $H_\beta(\omega)$ with different shapes than those given in Table I, but their coefficients have to be numerically fitted to approximate the above indicated partial sums on L , for a sufficiently large N to flat the phase in this frequency range.

4 Circuit implementation

The approximations of Fig. 1b (for the Table I values) have been fabricated in a $0.5\mu\text{m}$ N-well CMOS process. Filters $H(r^0\omega)$ to $H(r^{15}\omega)$ are implemented with the band-pass filters F_0 to F_{15} of Fig. 3a, with TF $\frac{v_k(\omega)}{v_{in}(\omega)} = \frac{C_{a,k}G_k(j\omega)}{C_{a,k}(C_{b,k}+C_{c,k})(j\omega)^2+C_{a,k}G_k(j\omega)+G_k^2}$, for $j = \sqrt{-1}$, $k = 0, \dots, 15$, capacitor values given in Fig. 3a and transconductance, $G_k = G_{a,k} = G_{b,k}$, set as follows. The central frequency of the band-pass filters, F_k , has to be exponentially scaled by a factor r^k , as required by $H(r^k\omega)$ given in Table I. It is achieved by scaling exponentially the transconductance G_k of the Operational Transconductance Amplifier (OTA) of Fig. 3b. This OTA has a differential input range of $\pm 29\text{mV}$ within 1% of linearity [2]. The OTA transistors have the following dimensions in $\mu\text{m}/\mu\text{m}$: $36/4.5$ for M_1 to M_4 , $17/4.5$ for M_5 and M_6 , and $10/4.5$ for M_7 and M_8 . Transistors are biased in subthreshold region, where the current I_b (and the OTA transconductance G that is proportional to I_b) has exponential dependence with respect to nearly linear changes in the bias voltage V_b [2]. The bias voltages V_{b0} to V_{b15} are produced with the help of a ladder of 15 poly resistors of nearly equal value, R_1 to R_{15} [2].

Filters $H_\alpha(\omega)$ and $H_\beta(\omega)$ were implemented with the Gm-C filters F_α , and F_β shown in Fig. 3a, with TF $\frac{v_\alpha(\omega)}{v_{in}(\omega)} = \frac{C_d C_e (j\omega)^2 + C_e G_\alpha (j\omega)}{C_d C_e (j\omega)^2 + C_e G_\alpha (j\omega) + G_\alpha^2}$, and $\frac{v_\beta(\omega)}{v_{in}(\omega)} = \frac{C_g G_\beta (j\omega) + G_\beta^2}{C_g (C_h + C_\beta)(j\omega)^2 + C_g G_\beta (j\omega) + G_\beta^2}$, respectively, where $G_\alpha = G_{a\alpha} = G_{b\alpha}$, and $G_\beta = G_{a\beta} = G_{b\beta}$.

Fig. 3. Integrated circuit implementation (a) Schematic circuit (b) Operational Transconductance Amplifier (OTA) (c) Typical (black) and MC (gray) simulations of circuit in Fig. 3a with EN active (d) Microphotograph of system in manufactured chip

A five-transistor operational amplifier (M_a to M_e in Fig. 3a) is used in inverter adder configuration, where $C_{c,k}$, C_α , and C_β are the input impedances, and C_f is the feedback impedance [4]. Transistors M_a to M_e , with aspect ratio $100\mu\text{m}/0.9\mu\text{m}$, work in the saturation region. Transistor M_f ($0.9\mu\text{m}/0.6\mu\text{m}$) is used in subthreshold region as a high-value resistor to hold the DC voltage level of C_f near zero [4]. The voltage follower is implemented with a MOSFET-P in source follower configuration. The pass transistor M_g is controlled by the logic signal EN . The adder operation is given by $v_{out}(t) = \sum_{k=0}^{15} (-C_{c,k}/C_f) v_k(t)$ when $EN = 0$, and by $v_{out}(t) = \sum_{k=0}^{15} (-C_{c,k}/C_f) v_k(t) - (C_\alpha/C_f) v_\alpha(t) - (C_\beta/C_f) v_\beta(t)$ when $EN = 1$.

To explore the effect of the process variations on the circuit response, Fig. 3c shows 100 Monte Carlo (MC) AC transistor level simulations of node v_{out} with respect to v_{in} for the compensated approximation case. 68% of the MC simulations had a magnitude within 1.000 ± 0.065 and a phase angle of $0.0^\circ \pm 4.6^\circ$ on L . The typical simulation is also shown in Fig. 3c. It is very close to the compensated approximation of Fig. 2a and 2b.

5 Experimental results

Fig. 3d shows a microphotograph of the manufactured chip. The system blocks cover an area of 0.66mm^2 and have a power consumption about $14\mu\text{W}$.

A probe signal made up of 4 sinusoidal cycles of period T , ± 10 mV of amplitude and a sinusoidal envelope of $8T$ period has been prepared. It was periodically applied to node v_{in} . Averages over 512 measures of $v_k(t)$ and $v_{out}(t)$ were taken to reduce the environmental noise effect. Fig. 4a shows the probe signal for $T = 4$ ms, and its WT plotted with the absolute value of the measured voltages $v_0(t)$ to $v_{15}(t)$. Fig. 4b shows the measured reconstructed signal $v_{out}(t)$ for both simple and compensated approximations, for $T = 4$ ms.

Fig. 4. On chip tests (a) Probe signal for $T = 4$ ms and its wavelet transform components v_0 to v_{15} (b) Reconstructions of probe signal for $T = 4$ ms (c) SER for various $1/T$ frequencies

It can be verified, that most of the probe signal energy spectrum is around frequency $1/T$. In both approximations, the probe signal was applied for, $T = \{2^{-4}, 2^{-3}, \dots, 2^4\}$ ms. We have to mention that the system gain was intentionally set to a value slightly greater than 1. Then we take amplification, A , as the ratio between the peak-to-peak voltages of the output and input signals. The measured average amplification for all the T values was $\bar{A} = -1.08$. Now, we define the Signal to (normalized) reconstruction Error Ratio as $SER = 10 \log_{10} \left(\int_0^{8T} |v_{in}(t)|^2 dt / \int_0^{8T} |v_{out}(t) / \bar{A} - v_{in}(t)|^2 dt \right) (\text{dB})$. The interval of integration used, $8T$, is explained in Fig. 4b. Fig. 4c plots the SER vs. $1/T$. Both approximations have a similar maximum SER that is limited due to the effect of the process variations. The compensated approximation holds the SER near 25 dB all along L . However, the simple approximation has its maximum only at 1 kHz, where the numerical model presents a zero phase angle shift (see Fig. 2b). Below and above 1 kHz, the increase in the phase absolute value produces a decrease in the SER.

6 Conclusion

An approach for implementing the inverse wavelet transform using simple analog circuits has been presented. The theoretical concept has been validated by means of a fabricated chip. Signal to reconstruction error ratios up to 25 dB have been measured.