Practical approach in estimating inertial navigation unit’s errors

C.H. Lim\textsuperscript{a)}, W.Q. Tan\textsuperscript{b)}, T.S. Lim\textsuperscript{c)}, and V.C. Koo\textsuperscript{d)}

Faculty of Engineering & Technology, Multimedia University
Jalan Ayer Keroh Lama, Melaka 75450, Malaysia
a) chlim@mmu.edu.my
b) Weiqiang4896@gmail.com
c) tslim@mmu.edu.my
d) vckoo@mmu.edu.my

Abstract: Inertial navigation unit (INU), which is commonly composed of three orthogonally aligned accelerometers and gyros, is well known for its short term measurement accuracy in position, velocity and attitude. However, such measurement accuracy degrades with time due to various types of errors. In this paper, a practical approach is proposed to estimate both the deterministic and random errors of an INU. The deterministic errors, which include bias and scaling errors, can be estimated through a simple experimental setup; while the random noise is modeled using Allan Variance (AV) analysis method. The empirical values of the errors are then fed into the INU’s system model for error correction using Kalman filtering. Finally, the calibrated INU shows promising results in preserving long term accuracy of the motion sensor.

Keywords: inertial navigation unit, calibration, Allan Variance (AV)

Classification: Sensing hardware

References

1 Introduction

Inertial navigation is a process of measuring the inertial force and orientation to achieve position, velocity, and altitude determinations [1]. With recent advancement in Micro Electronic-Mechanical System (MEMS), inertial navigation has become possible with the “strapdown” configuration of inertial navigation unit (INU) [2]. A typical six degree-of-freedom strapdown INU consists of three orthogonally aligned accelerometers and gyroscopes fixed in a pre-specified positions. It is successfully applied in various applications such as land vehicle navigation [3], unmanned aerial vehicle (UAV) navigation [4], and autonomous underwater vehicle (AUV) navigation [5] for position, velocity, and orientation measurements.

The INU measurements are usually degraded by various noises, which include deterministic noise such as bias error and scaling error, and random noise such as random walk and bias instability [2, 6]. Since the navigation algorithm integrates the INU measurements, these errors will be integrated as well which results in quick diverges in navigation solution if left unattended. Therefore, in order to obtain an accurate estimation of navigation data, it is of great necessity to first model and acquire the INU’s measurement error components.

This work focuses on the development of a practical calibration and error modeling processes for INU. Standard six-point static calibration method [2] is adopted to measure the deterministic errors. Such method, although can be easily realized with adequate instrument, is not able to predict random errors. Hence a time-domain analysis technique, known as Allan Variance [6], is proposed to predict the random errors and to overcome the shortage of the above mentioned calibration method. A relatively accurate INU measurement can thus be achieved with the combination of the calibration method and the Allan Variance analysis.

The rest of the paper is organized as follow. Section 2 outlines the noise model of a typical INU. Section 3 provides an analysis on Allan Variance and how it is applied in the modeling of INU’s random errors. Section 4 outlines the calibration results obtained from experiments. Section 5 presents the field experimental results of a calibrated GPS aided INU system. Finally, section 6 concludes the paper.

2 System modeling

The conventional measurements of an INU with 3-axis accelerometers and 3-axis gyroscopes can be expressed in terms of the summation of its true...
value and noises, as shown below [7]:

\[
\begin{align*}
  a_{mi} &= a_{ti} + b_{ai} + S_{ai}a_{ti} + \eta_{ai} \\
  \omega_{mi} &= \omega_{ti} + b_{\omega i} + S_{\omega i}\omega_{ti} + \eta_{\omega i}
\end{align*}
\]

\[i = x, y, z\]  

(1)

where \(a_{mi}\) and \(\omega_{mi}\) are the accelerometers and gyro measurement values, \(a_{ti}\) and \(\omega_{ti}\) are the accelerometers and gyro true values, \(b_{ai}\) and \(b_{\omega i}\), \(S_{ai}\) and \(S_{\omega i}\), \(\eta_{ai}\) and \(\eta_{\omega i}\) are the bias errors, the scaling errors, and the random errors of accelerometers and gyroscopes, respectively.

The static calibration procedure involves accurately mounting the INU on a leveled device with proper alignment for each sensitive axis of the INU pointing alternatively up and down with respect to the center of Earth. By taking a relatively long period (> 1 hour) of data collections with such setup for each sensitive axis in both up and down position, and assuming zero mean random noise, then eq. (1) can be rewrite as follows:

\[
\begin{align*}
  Q_{up}^{mi} &= b_{Q i} + (1 + S_{Q i})K \\
  Q_{down}^{mi} &= b_{Q i} - (1 + S_{Q i})K
\end{align*}
\]

\[\{Q = a, \omega; K = g, \omega_e; i = x, y, z\} \]  

(2)

where \(g\) is the gravitational constant and \(\omega_e\) refers to the Earth’s rotational rate. Both parameters are constants, with \(g \approx 9.80665\ m/s^2\) and \(\omega_e \approx 7.292115 \times 10^{-5}\ rad/s\).

Hence, the bias errors and the scaling errors for both accelerometers and gyroscopes can be obtained from eq. (2):

\[
\begin{align*}
  b_{Q i} &= \frac{Q_{up}^{mi} + Q_{down}^{mi}}{2} \\
  S_{Q i} &= \frac{Q_{up}^{mi} - Q_{down}^{mi} - 2K}{2K}
\end{align*}
\]

\[\{Q = a, \omega; K = g, \omega_e; i = x, y, z\} \]  

(3)

### 3 Random noise estimation using Allan Variance

Allan Variance (AV) was originally developed in 1960s to analyze the frequency stability of oscillator. This time-domain analysis technique was soon adapted to characterize random drift of various types of devices. Overall AV is a relatively compendious method to analyze and model various random errors in INU as compared to power spectral density (PSD) approach [6].

Consider a continuous \(N\) measurement data from the INU with \(t_0\) sample time. It is possible to form a group of \(n\) consecutive data from \(N\)-data points with \(n < N/2\). Each member of the group is termed as a cluster, with each cluster’s duration equal to \(nt_0\), or simply \(T\). Assuming the instantaneous output rate of the INU is \(\Omega(t)\), then the cluster average can be expressed as:

\[
\Omega_k(T) = \frac{1}{T} \int_{t_k}^{t_k+T} \Omega(t)\,dt = \frac{1}{T} \int_{t_k}^{t_k+1} \Omega(t)\,dt
\]

(4)

where eq. (4) represents the average of the cluster’s output rate starts from \(k^{th}\) data with \(n\) data points. The subsequent cluster’s average can thus be
expressed as:

$$\Omega_{k+1}(T) = \frac{1}{T} \int_{t_{k+1}}^{t_{k+1}+T} \Omega(t)dt = \frac{1}{T} \int_{t_{k+1}}^{t_{k+1}+T} \Omega(t)dt$$  \hspace{1cm} (5)$$

The difference of two adjacent cluster’s average can thus be formed as:

$$\xi_{k+1,k} = \Omega_{k+1}(T) - \Omega_k(T)$$  \hspace{1cm} (6)$$

Hence the AV of the INU can be defined as the variance of the difference of all possible adjacent clusters that can be formed with respect to length $T$ [6], as shown in eq. (7):

$$\sigma^2(T) = \frac{1}{2(N-2n)} \sum_{k=1}^{N-2n} (\xi_{k+1,k})^2$$  \hspace{1cm} (7)$$

It is worth mentioning that there exists a unique relationship between $\sigma^2(T)$ and the PSD of the INU’s random errors, as shown below:

$$\sigma^2(T) = 4 \int_0^\infty df * S_\Omega(f) * \frac{\sin^4(\pi ft)}{(\pi ft)^2}$$  \hspace{1cm} (8)$$

where $S_\Omega(f)$ denotes the PSD of $\Omega(t)$, with $f$ represents the frequency. Eq. (8) shows that the AV $\sigma^2(T)$ is a function of $T$ with the PSD of $\Omega(t)$ be a known parameter. On the contrary, since $\sigma^2(T)$ is measurable from the INU data, it is thus possible to create a log-log plot of $\sigma(T)$ versus $T$ to provide a direct analysis on the random processes that existed in the data. Therefore, AV is an efficient method to quantify and identify various random noise terms existed in the INU data [6].

4 INU sensor calibration

The VN-100 from VECTORNAV is a miniature, low power inertial sensor suitable for embedded applications. It is chosen for the calibration and error modeling experiments. Both experiments were carried out in the Precision Measurement Lab, Multimedia University under room temperature. One-hour static data were recorded for each sensitive axis pointing up and down towards the center of Earth during the deterministic noise calibration. The mean of gyroscopes during static test is approximately zero.

Table I shows the accelerometers bias error and scaling error based on the static data collected during calibration procedure. A total of 5 sets of static data were collected to calculate the deterministic errors. Table I also summarized various random errors extracted from the static data based on AV analysis. A total of 7 sets 8-hours static data were collected under room temperature for the random errors modeling. The results indicate a fine estimation on each noise terms with standard deviation less that 15% from the mean value.

Fig. 1 shows one of the AV log-log plots of accelerometers’ and gyros’ static data respectively. Various random noise terms can be estimated from
Table I. Errors Estimation for VN-100

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<thead>
<tr>
<th></th>
<th>Bias Error</th>
<th>Scaling Error</th>
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<td></td>
<td>Mean</td>
<td>Std</td>
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<tr>
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<td>$a_x$</td>
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<td>Random Walk</td>
<td>Bias Instability</td>
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<tr>
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<tr>
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<tr>
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<td>$\omega_z$</td>
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Fig. 1. For instance, the random walk of both accelerometers and gyroscopes were estimated by fitting a straight line through the slope of $-0.5$ gradient and its values can be extracted at $T = 1$. Similarly, bias instability appears to be the flat region around the minimum on the plot. Rate random walk and rate ramp appear on the AV plot with a straight fitting line of gradient $+0.5$ at $T = 3$, and gradient $+1$ at $T = 2^{1/2}$, respectively. Notice that not all random noise terms can be estimated from the AV log-log plot such as
the gyros’ AV plot. The only noise term that can be examined from gyro’s AV plot is random walk, and it appeared to be the only dominant noise term from the AV plot for $T > 10$.

5 Field experiments

The deterministic and random errors obtained from previous experiments were applied in the subsequent for motion sensing experiments.

Fig. 2 shows the static test experiment’s result obtained with various processing methods. A total of 300 seconds of motionless inertial data was collected using the VN-100 inertial sensor. It is shown in Fig. 2 that the uncompensated position outputs diverged from zero in less than a minute. By implementing the bias errors and scaling errors into the process, the compensated position outputs show better results, but was diverged for long run as well. Hence, to improve further the position accuracy, Kalman filtering was implemented on the compensated output with the knowledge of non-deterministic errors obtained from AV plot, which converge the static position output to zero.

Fig. 3 shows one of the field test results obtained by applying both the deterministic errors compensation and GPS aided INU Kalman filtering estimation based on the random errors estimated from AV plot. The map with yellow trace shown in Fig. 3 (b) indicates the 5 Hz GPS position measurement, while the plot shown in Fig. 3 (a) compares the 5 Hz GPS position measurement with the 50 Hz INU prediction aided by the GPS using Kalman filter. The results clearly indicate the accuracy in terms of position prediction and measurement using the VN-100 inertial sensor.

Fig. 2. Static test results of Measured, Compensated, and Kalman Filter output
Fig. 3. Kalman filter estimated position and GPS position from field test

6 Conclusion

This paper presents a practical approach to model and calibrate both deterministic and random errors of an inertial navigation unit. The standard six-point static calibration method is adopted to estimate the deterministic errors, and Allan variance is applied to predict the random errors. The results show the applicability of the proposed method to be applied in motion sensing applications.