Experimental verification of electromagnetic scattering via two-dimensional periodic array of small resonant apertures

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Abstract: The transmission characteristics of a frequency selective surface (FSS) consisting of small resonant circular apertures with a ridge are related to the transmission cross section of a single ridged aperture, and the performance is verified experimentally. The FSS structure with a ridged circular aperture has a lower resonant frequency, enlarged fractional bandwidth, and a larger wavelength-to-periodicity ratio compared to an FSS with a conventional circular aperture. Experiment results show that the resonant frequency of the proposed FSS structure can be lowered from 13.98 GHz to 7.30 GHz (47.8%) by adding the ridge.

Keywords: frequency selective surface, small resonant aperture, ridge, band pass filter, lower resonant frequency

Classification: Microwave and millimeter wave devices, circuits, and systems

References

dguide filter utilizing complementary split ring resonator (CSRR)," Prog. Electromagn. Res. 80 (2008) 107

1 Introduction

The problem of electromagnetic wave transmission through small apertures on an
infinite conductor plane has long been of interest since Bethe's theory of
diffraction by small holes [1]. As is well known, in this structure, Bethe
quantitatively demonstrated that transmission efficiency is very low; that is, the
amount of electromagnetic waves transmitted through a small aperture, compared
to the wavelength, is very small.

In order to obtain high transmission efficiency for a very small aperture,
compared with the wavelength, Harrington placed a capacitor across the midpoint
of the aperture, or transformed it into a structure having a capacitor characteristic,
thereby causing a transmission resonance phenomenon [2]. Here, the transmission
resonance phenomenon means a phenomenon in which the shape of the aperture is
deformed, despite the small actual aperture area, and the power of the wave
transmitted through the deformed small aperture is remarkably increased. Recently,
a lot of research has studied methods for increasing the transmission efficiency of a
small aperture in the design area of a nano-scale optical probe, i.e., in the design
area of a near-field scanning optical microscope (NSOM) probe, for near-field
optical applications such as optical data storage, nano-lithography, and
nano-microscopy [3-8].

On the other hand, a structure in which such small apertures are periodically
arranged two-dimensionally in a plane is called a frequency selective surface
(FSS). Such an FSS has a frequency filtering function that transmits all the specific
frequency bands incident on it, and simultaneously reflects all the other frequency
bands, and vice versa. In conventional FSSs, unit cells with circular, rectangular,
or loop shapes were used, but the resonance characteristic of the single unit cell
itself, except for a $\lambda/2$ (half wavelength) slot dipole on an infinite conducting plane,
seems to have never been related to a periodic array [9-10].
In this paper, we first investigate the transmission efficiency of small circular apertures with/without a ridge through transmission cross section (TCS) calculation, and compare the results of the calculation of the frequency response characteristics for the FSS structures in which the resonant circular aperture with a ridge and a conventional circular aperture are arranged in the conductor plane. Next, the FSSs with a resonant ridged circular aperture and an aperture without the ridge are fabricated, and their frequency characteristics are verified experimentally.

2 Transmission cross section of small apertures

If an electromagnetic wave is incident on a structure with a very small aperture, compared to the wavelength on an infinite perfect conductor plane that is very thin, the power of the wave passing through the aperture will be extremely small, as mentioned above. In order to maximize the transmission of electromagnetic waves through a small aperture, it is necessary to implement a resonance phenomenon by modifying the structure so as to have a capacitive component. As a practical example, Fig. 1(a) shows a circular aperture with a very small diameter compared to the wavelength, and Fig. 1(b) corresponds to a structure in which resonance occurs by adding a ridge.

When a resonant phenomenon occurs in a modified small aperture structure, as shown in Fig. 1(b), the incident wave is transmitted at maximum, and the TCS is transmitted to the maximum extent as $\frac{3\lambda^2}{4\pi}$ regardless of the shape of the aperture [2]. In this case, assuming that the power density of the plane wave incident perpendicularly to the conductor plane with the opening is $P$ [W/m²], the physical quantity of the TCS is defined as the total transmitted power through the small aperture to the $z > 0$ region divided by the incident power density of the plane wave incident on the $z < 0$ region. That is, the larger the TCS, the higher the transmission efficiency and the larger the total power of the transmitted wave passing through the aperture.

First, we investigate the resonance phenomenon in small apertures having the
shape shown in Fig. 1 and determine their actual TCS. To calculate the exact TCS of the small apertures, the moment method using Rao-Wilton-Glisson basis functions was used [11], and the accuracy of the calculated results was verified by comparing them with those of the finite difference time domain method [7].

Fig. 2 shows the calculated TCS of the small circular aperture (D = 10 mm). From the result shown in Fig. 2, we can see that the TCS is very small at a low frequency band, and then, it approaches a maximum value of 128 mm$^2$ at 15.5 GHz as the frequency increases. Since the free space wavelength at 15.5 GHz is about 19.35 mm, and the diameter of the circular aperture (D = 10 mm) is about a half-wavelength, the circular aperture is not small in terms of the wavelength at 15.5 GHz.

Fig. 3 shows the TCS characteristics of the ridged circular aperture in Fig. 1(b) with the same diameter of D = 10 mm. Two different cases are considered. Fig. 3(a) shows the TCS variations of the ridged aperture for different ridge gaps (g = 0.2 mm, 0.5 mm, and 0.8 mm) when the ridge width is fixed at w = 3 mm, whereas those for different ridge widths w = 1.5 mm, 2 mm, and 3 mm with the fixed ridge gap g = 0.2 mm are shown in Fig. 3(b).

We can see from Fig. 3 that a resonance phenomenon where transmission
efficiency remarkably improves is observed, and the TCS at the resonance frequency is almost the same as $3\lambda^2/4\pi$, which is indicated as a dotted line in Fig. 3. In addition, the smaller the ridge gap, $g$, and the larger the ridge width, $w$, the lower the resonance frequency becomes. The reason is that as the ridge width becomes larger and the ridge gap becomes smaller, the capacitive component is greatly increased, and the resonance frequency is lowered.

3 Transmission characteristics of two-dimensional periodic arrays of small apertures

In this section, the frequency responses of the FSS structures are analyzed where the small circular apertures shown in Fig. 1 are periodically arranged two-dimensionally in a plane. In the numerical analysis of the FSS structures, the scattering wave is defined by the development of the Floquet modes, and the magnetic current, which is a tangential electric field component in the aperture region, is set as a rooftop basis function. Next, the integral equation is obtained by applying the boundary condition of the continuous tangential magnetic field component in the aperture, and the moment method by using the Galerkin testing technique is used to calculate the transmission coefficients of the FSS structures composed of small apertures. In order to verify the accuracy of the simulated results, they were compared with the results in [11] and measurement results.

First, the transmission coefficient characteristics of the FSS structure with the circular apertures shown in Fig. 4 are calculated for different periodicities, $T_x = T_y = 15$ mm, 20 mm, and 30 mm, when the diameter of the aperture is $D = 10$ mm, as shown in Fig. 5. It can be seen that all the waves are transmitted at frequencies of 9.88 GHz, 11.63 GHz, 14.08 GHz, and 18.11 GHz, and in the other frequency band, most of the incident waves are reflected to perform the filter function. Table I summarizes the resonant frequency, the 3 dB bandwidth, the fractional bandwidth, and the ratio of wavelength to periodicity of the FSS structure in Fig. 4. We can see that when the periodicity is 15 mm, the bandwidth is excessively large, but other cases have a relatively narrow bandwidth. Since the ratio of the wavelength to the periodicity is almost 1, a grating lobe is generated even if the wave is incident at a slight angle (a few degrees). In this
case, the condition where the grating lobe does not occur in the FSS structure is \( \lambda_0/T > (1 + \sin \theta_0) \) where \( \theta_0 \) is the incident angle of the wave. For this reason, it is not useful in practical applications, because it must be used only when it is vertically incident on the FSS plane.

![Fig. 5](image)

**Fig. 5.** Transmission coefficient of the FSS with the circular apertures vs. periodicity.

<table>
<thead>
<tr>
<th>Periodicity (mm)</th>
<th>Transmission Resonance Frequency [GHz]</th>
<th>3dB BW [GHz]</th>
<th>Fractional BW (%)</th>
<th>Ratio of Wavelength to Periodicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>18.11</td>
<td>4.9507</td>
<td>0.27344</td>
<td>1.105</td>
</tr>
<tr>
<td>20</td>
<td>14.08</td>
<td>1.1876</td>
<td>0.08434</td>
<td>1.065</td>
</tr>
<tr>
<td>25</td>
<td>11.63</td>
<td>0.2626</td>
<td>0.02257</td>
<td>1.032</td>
</tr>
<tr>
<td>30</td>
<td>9.88</td>
<td>0.0508</td>
<td>0.00514</td>
<td>1.013</td>
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</table>

![Fig. 6](image)

**Table I.** 3 dB bandwidth, fractional bandwidth and the ratio of wavelength to periodicity of Fig. 5.

Fig. 6 shows an FSS structure in which ridged circular apertures are arranged in the conductor plane. Its transmission coefficient characteristics for different periodicities, \( T_x = T_y = 15 \) mm, 20 mm, 25 mm, 30 mm, and 35 mm,
when aperture diameter $D = 10$ mm, ridge width $w = 3$ mm, and ridge gap $g = 0.5$ mm are shown in Fig. 7. The resonance frequencies are 7.28 GHz, 7.43 GHz, 7.56 GHz, 7.7 GHz, and 7.83 GHz. Comparing the frequency response characteristics of Fig. 7 with those of Fig. 5 shows that the band pass filter characteristic is better equipped in Fig. 7, and the resonance frequencies become significantly reduced. The incident wave passes all around the resonance frequency, and in other frequency bands, most of them are reflected, so they have a filter function. The physical state when the band pass filter structure is implemented using the two-dimensional array structure of the resonance element can be explained as follows. In the case of a single resonant element, the electromagnetic energy incident on a very large TCS, compared to the actual aperture area, passes through the actual narrow aperture area as if passing through a funnel, and then transmits through the rear. When the resonant element is two-dimensionally arranged in a plane, they can make the transmission coefficient almost 1. Therefore, this resonant element is similar to the extraordinary optical transmission (EOT) problem which has been extensively studied in the optical field.

The resonant frequency, the 3 dB bandwidth, the fractional bandwidth, and the ratio of wavelength to periodicity of the FSS structure in Fig. 7 are summarized in Table II. Compared with Table I, we can see that the fractional bandwidth in Fig. 7 is considerably larger, except when $T_x = T_y = 15$ mm. In addition, when transmission resonance occurs, the wavelength-to-periodicity ratio of Table II is significantly increased, compared to the circular aperture of Table I, so that the total aperture area is reduced. At the same time, even when the incident angle deviates from the vertical, the possibility of the generation of the grating lobe is remarkably reduced. Therefore, since the FSS structure with the ridged circular aperture has a lower resonant frequency, enlarged fractional bandwidth, and a larger wavelength-to-periodicity ratio, it is possible to obtain better filtering performance, and it will be much easier to design and apply the FSS structure.

![Fig. 7. Transmission coefficient of the FSS with ridged circular apertures vs. periodicity.](image-url)
Based on the analysis of the transmission characteristics of the FSS structures of small circular apertures with and without the ridge, the prototypes of FSS structures consisting of circular apertures with and without the ridge for the periodicity $T_x = T_y = 20 \text{ mm}$ were fabricated on an flexible printed circuit board (FPCB) substrate based on polyimide ($\varepsilon_r = 3.5$ and $t = 0.04 \text{ mm}$), as shown in Figure 8.

### Table II. 3 dB bandwidth, fractional bandwidth and the ratio of wavelength to periodicity from Fig. 7.

<table>
<thead>
<tr>
<th>periodicity (mm)</th>
<th>transmission resonance frequency [GHz]</th>
<th>3dB BW [GHz]</th>
<th>fractional BW (%)</th>
<th>ratio of wavelength to periodicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7.83</td>
<td>1.6425</td>
<td>0.20990</td>
<td>2.556</td>
</tr>
<tr>
<td>20</td>
<td>7.70</td>
<td>0.8535</td>
<td>0.11084</td>
<td>1.948</td>
</tr>
<tr>
<td>25</td>
<td>7.56</td>
<td>0.5159</td>
<td>0.06821</td>
<td>1.587</td>
</tr>
<tr>
<td>30</td>
<td>7.43</td>
<td>0.3359</td>
<td>0.04520</td>
<td>1.346</td>
</tr>
</tbody>
</table>

### 4 Experiment results

Based on the analysis of the transmission characteristics of the FSS structures of small circular apertures with and without the ridge, the prototypes of FSS structures consisting of circular apertures with and without the ridge for the periodicity $T_x = T_y = 20 \text{ mm}$ were fabricated on an flexible printed circuit board (FPCB) substrate based on polyimide ($\varepsilon_r = 3.5$ and $t = 0.04 \text{ mm}$), as shown in Figure 8.

![Photographs of the fabricated FSSs with (a) circular apertures and (b) ridged circular apertures.](image)

The experiment setup for measuring the transmission characteristics of the FSS structures is illustrated in Fig. 9. When the transmission characteristics of the FSS structures are measured, the S parameters are measured by using two broadband double-ridged horn antennas under normal incidence, an Agilent E5071C vector network analyzer (VNA), and an SHF100APP broadband amplifier made by SHF Communication Technologies. The antennas were placed 500 mm away from the FSS structure so as to satisfy the far field condition. The pyramidal absorbers were placed around the FSS structure to prevent any unexpected reflected wave.
The calculated and measured transmission coefficients of the fabricated FSS structures with small circular apertures with and without the ridge are plotted in Fig. 10. For the FSS structure with circular apertures only, the calculated results with \( t = 0.4 \text{ mm} \) and without \( t = 0 \text{ mm} \) the substrate are compared. It is seen from Fig. 10(a) that the resonant frequency of the FSS slightly shifts toward a low frequency from 14.08 GHz to 13.98 GHz due to the loading effect of the substrate, and the measured transmission coefficient agrees well with the calculated results. However, when the ridge is added to the circular aperture, the resonant frequency of the FSS moves more toward a low frequency from 7.70 GHz to 7.30 GHz. In this case, good agreement is also observed between the calculated and measured results.

**Fig. 9.** Illustration of the experiment setup.

**Fig. 10.** Comparison of the calculated and measured transmission coefficients of the fabricated FSS structures with (a) circular apertures and (b) circular apertures with a ridge.

### 5 Conclusion

The relation between the small resonant circular aperture on an infinite conducting plate and an FSS structure with a resonant aperture unit cell has been analyzed. The TCS of the small circular aperture can be considerably enhanced by
appending a ridge to the aperture. The smaller the ridge gap and the larger the ridge width, the lower the resonance frequency becomes, because the capacitive component is greatly increased. When the small ridged aperture is applied as a unit cell of the FSS structure, the band pass filter characteristic is better equipped, and a lower resonant frequency, an enlarged fractional bandwidth, and a larger wavelength-to-periodicity ratio are achieved, compared to the conventional FSS with a circular aperture. In addition, the possibility for generation of the grating lobe is remarkably reduced when the incident angle deviates from the vertical.

Prototypes of the FSS structures consisting of circular apertures with and without the ridge were fabricated on an FPCB substrate, and their transmission characteristics were measured. It was demonstrated experimentally that the resonant frequency of the FSS structure with the ridged circular aperture can be reduced by 47.8%, compared to the FSS without a ridge.

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