Fast-super-twisting sliding mode speed loop control of permanent magnet synchronous motor based on SVM-DTC

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Abstract Aiming at the problems of frequent overshoot and slow response time in space vector direct torque control system of permanent magnet synchronous motor, a control strategy based on a fast-super-twisting algorithm was proposed. Based on the traditional super-twisting algorithm, the fast terminal sliding mode reaching law (FTSMRL) is introduced in the strategy, which improves the algorithm's robustness and convergence speed, suppresses the sliding mode chattering. The stability of the system is proved by using a quasi-quadratic Lyapunov function. Simulation results show that the proposed strategy can effectively improve the response speed and robustness of the system, and significantly reduce the system overshoot and sliding mode chattering.

key words: Direct Torque Control Space Vector Modulation (DTC-SVM), Fast-super-twisting algorithm sliding mode control (FSTA-SMC), Permanent magnet synchronous motor (PMSM).

Classification: Power devices and circuits

1. Introduction

PMSM has significant advantages such as high mechanical efficiency, high power factor, and large output. It has been applied to more and more high-performance occasions and has considerable development prospects. It has become a research hotspot in the field of electrical transmission in recent years [1-4]. DTC does not have the complex coordinate transformation of vector control, can directly control the torque, and has a good dynamic and static performance. Many scholars at home and abroad have conducted related research and have obtained significant research results [5-9]. Although DTC's application in asynchronous motors and induction motors is relatively mature, synchronous motors need further investigation.

In the traditional direct torque control, the Bang-Bang controller generates PWM signals by controlling the amplitude of torque and flux linkage and optimizes the inverter's switching state to obtain high dynamic torque performance. However, there are some problems, such as extensive flux linkage, torque ripple, and high-frequency noise caused by torque ripple. To overcome these drawbacks, a scheme of DTC system based on SVPWM is proposed in the literature [10-11]. In this scheme, the standard voltage vector calculation unit is used to replace the traditional Bang-Bang controller in the direct torque system. Compared with the previous SVM control, only two PI regulators of speed and torque are needed, and the control structure is optimized.

The above strategy is to achieve high-performance torque control. However, in the DTC-SVM speed regulation system, speed loop control is also significant. For example, in the non-automatic mechanical transmission system, electric vehicles' control performance significantly impacts the gear shifting process [12]. In direct torque control (DTC), the PI controller is generally used for speed loop, simple in structure, and easy to implement. However, the specific PI regulator parameters are often sensitive to the changes in motor parameters, speed and load, and the system's robustness [13].

Sliding mode variable structure control is an effective control method for nonlinear uncertain systems. It has strong robustness to parameter perturbation and external disturbance and has a simple structure and fast response [14]. Literature [15-16] introduces the sliding mode control strategy into direct torque control of PMSM. The speed loop SMC is used to replace the PI controller in direct torque control to enhance the system's robustness and rapidity. In [17-20], a new direct torque control method based on STA-SMC is proposed to reduce chattering. Although the scheme can suppress the chattering of sliding mode, since the scheme's proportional term is the square root calculation, it dramatically reduces the convergence speed and disturbance-rejection ability of the algorithm [21].

An FSTA-SMC speed loop controller based on Fast terminal sliding mode reaching law (FTSMRL) is designed in this paper to overcome the above drawbacks. This method increases the system's reaching speed and disturbance-rejection ability without affecting the second-order sliding mode characteristics. Finally, the effectiveness of the scheme is verified by simulation comparison.

2. Basic principle of DTC

The torque equation of PMSM can be expressed as the relation of load angle, namely:
\[ T_\varepsilon = \frac{3}{2} \frac{p_n}{L_d L_i} \left[ \psi^* i L_d \sin \delta + \frac{1}{2} \psi^* (L_q - L_d) \sin 2\delta \right] \] (1)

For surface PMSM, stator inductance \( L_d = L_q = L_s \), at this point, Equation (1) can be expressed as:

\[ T_\varepsilon = \frac{3}{2} \frac{p_n}{L_d} \psi_f \sin \delta = k_t \sin \delta \] (2)

Where: \( p_n \) represents pole pairs; \( L_d \) represents the inductance of d-axis stator winding; \( \psi_f \) and \( \psi_s \) represents rotor flux linkage and stator flux linkage, respectively; \( \psi^* \) is given value of stator flux linkage; \( \delta \) is load angle; \( k_t = \frac{3}{2} \frac{p_n}{L_d} \psi_f \) is a proportional coefficient.

Since the amplitude of the rotor flux linkage of PMSM is constant, \( k_t \) is constant when the amplitude of stator flux is kept constant, and the torque of the motor is only related to the load angle. Therefore, the derivation of Equation (2) is performed to obtain the expression of the relationship between torque change and load angle change, as shown in Equation (3).

\[ \frac{d}{dt} T_\varepsilon = k_t \cos \delta \frac{d}{dt} \delta \] (3)

From Equation (2) and Equation (3), it is not difficult to see that the torque and load angle relationship is nonlinear. From Equation (3), the torque variation equation can be obtained

\[ \Delta T_\varepsilon = k_t \cos \delta \Delta \delta \] (4)

The control period is concise; the value of \( \cos \delta \) changes little, so the torque change is consistent with the load angle change. In a control cycle, the change of the stator flux vector and rotor permanent magnet flux vector is shown in Fig. 1, in which the amplitude of the flux vector remains unchanged.

\[ \psi^* \frac{\alpha\beta}{2} \sin \Delta\delta + \psi^* \frac{\alpha\beta}{2} \sin \Delta\delta + \psi^* \frac{\alpha\beta}{2} \sin \Delta\delta = k_t \sin \delta \] (5)

The relationship between the stator flux vector and the voltage vector is described:

\[ u_s = \frac{d}{dt} \psi_s + R_i s \] (6)

From Equation (6), the stator flux vector's change reflects the voltage vector's control effect on the flux vector. Therefore, the change of the stator flux vector can be compensated by precisely controlling the voltage vector. According to the voltage equation of the stator flux, the change of the stator flux vector can be obtained:

\[ \left\{ \begin{array}{l}
\psi^* = \frac{\psi^* \cos (\Delta \delta + \theta_t)}{T_s} - \frac{\psi^* \sin \delta}{T_s} + R_i a \\
\psi^* = \frac{\psi^* \sin (\Delta \delta + \theta_t)}{T_s} - \frac{\psi^* \sin \delta}{T_s} + R_i b
\end{array} \right. \] (7)

Where \( u^* s_a, u^* s_b \) is the desired voltage of \( \alpha\beta \) the axis, \( i_a, i_b \) is the current of \( \alpha\beta \) the axis, \( T_s \) is the control period.

**3. Design of Fast-super-twisting algorithm and Stability analysis**

### 3.1 Traditional Super-twisting algorithm

The general form of the Super-twisting algorithm [22-24] is as follows:

\[ \frac{dx}{dt} = -M^\frac{1}{3} |\tilde{x}| \text{sgn}(\tilde{x}) + y \] (8)

\[ \frac{dy}{dt} = -N \text{sgn}(x) + \phi(t) \]

Where \( x \) is the actual variable value; \( y \) is the output observation value of the algorithm. \( \phi(t) \) is the disturbance term; \( \text{sgn}(\cdot) \) is the switching function; \( \tilde{x} = x - \hat{x} ; M, N \) is the sliding mode gain coefficient, and the values of M and N are going to be:

\[ \begin{cases} M > 2 \\ N > \frac{M^3 + \eta^2 (4M - 8)}{M (4M - 8)} \end{cases} \]

For the proof of finite-time stability and the estimation of convergence time of Equation (8), see literature [25].
3.2 Design of Fast-super-twisting algorithm

In the traditional STA, the sliding mode surface in the proportional term is calculated by the square root. The gain of the proportional term directly affects the ability to follow the disturbance. In order to improve the convergence speed and robustness of the system, this paper proposes an FSTA based on the fast terminal sliding mode reaching law (FTSMRL) [26], namely:

\[
\frac{dx}{dt} = -M \Phi_1(\tilde{x}) + y
\]
\[
\frac{dy}{dt} = -N \Phi_2(\tilde{x}) + \phi(t)
\]

(10)

Where,

\[
\Phi_1(\tilde{x}) = \frac{1}{2} \epsilon^T \text{sgn}(\tilde{x}) + k \tilde{x}
\]
\[
\Phi_2(\tilde{x}) = \frac{1}{2} \epsilon^T \text{sgn}(\tilde{x}) + \frac{3}{2} k \epsilon^T \text{sgn}(\tilde{x}) + k^2 \tilde{x}
\]

(11)

Where \( k > 0 \).

On the premise of not affecting the algorithm's stability, the proportion term \( \Phi_1(\tilde{x}) \) is replaced by the FTSM, which significantly improves the algorithm's convergence speed and disturbance-rejection ability.

3.3 Stability analysis

For the stability analysis of STA, literature [27-29] puts forward a relatively simple proof method by using the quasi quadratic Lyapunov function and analyzes the STA convergence in the case of constant value disturbance and time-varying disturbance. The constant value disturbance can be regarded as a particular case of time-varying disturbance. Therefore, from the perspective of time-varying disturbance, the Lyapunov function constructed in literature [27-29] is used to prove the system's stability shown in Equation (10).

Theorem 1:

For the system (10), there are: If \( |\phi(t)| \leq \eta, \ \forall t \geq 0 \), then the values of M and N, k satisfy:

\[
\begin{align*}
M & > 2 \\
N & > \frac{M^2 + \eta^2 (4M - 8)}{M (4M - 8)} \\
k & > 0
\end{align*}
\]

(12)

The system (10) can converge to the origin \((0, 0)\) in finite time.

Proof of Theorem 1:

Taking positive definite symmetric matrix

\[
P = \begin{bmatrix}
\frac{4N + M^2}{2} & -\frac{M}{2} \\
-\frac{M}{2} & 1
\end{bmatrix}
\]

is as follows: \( V(x, y) = \xi^T P \xi + \xi^T = \Phi_1(\tilde{x}) y \).

Let be \( A = \begin{bmatrix} -M & 1 \\ -N & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \),

\[
\tilde{\phi}(t) = |\xi| \phi(t), \quad \xi = \frac{1}{\Phi_1(\tilde{x})}.
\]

Using inequality \( \frac{d|x|}{dt} = x \text{sign}(x) \), and the derivative of \( \xi \):

\[
\dot{\xi} = \begin{bmatrix}
\frac{1}{2} \left( -1 + k (-M \left( |\tilde{x}|^\frac{1}{2} \text{sgn}(\tilde{x}) + k \tilde{x} \right) + y \right) \\
-N \left( \frac{3}{2} \epsilon^T \text{sgn}(\tilde{x}) + \frac{k}{2} \epsilon^T \text{sgn}(\tilde{x}) + k^2 \tilde{x} \right) + \phi(t)
\end{bmatrix}
\]

(13)

The derivative of V along the trajectory of the system (10) is obtained:

\[
\dot{V}(x, y) = \xi^T P \xi + \xi^T P \xi = \Phi_1(\tilde{x}) \left[ 2 \xi^T A^T + 2 \phi B^T \right] P \xi
\]

\[
\leq \Phi_1(\tilde{x}) \left[ 2 \xi^T A^T P \xi + 2 \phi B^T P \xi + \eta^2 |x| - \phi^2 \right]
\]

\[
= \Phi_1(\tilde{x}) \left[ 2 \xi^T A^T P \xi + 2 \phi B^T P \xi + \eta^2 \xi^T C^T C \xi - \phi^2 \right]
\]

\[
\leq \Phi_1(\tilde{x}) \left[ 2 \xi^T A^T P \xi + 2 \phi B^T P \xi + \eta^2 \xi^T C^T C \xi + \phi^2 \right]
\]

(14)

Let be \( Q = \left[ A^T P + PA + \eta^2 C^T C + PBB^T P \right] \xi \), then

\[
\dot{V} \leq \left| \frac{1}{\xi^T} \right| \xi^T Q \xi
\]

(15)

Here:

\[
Q = \begin{bmatrix}
3MN + M^3 - \frac{M^2}{4} - \eta^2 & \frac{M}{2} - N - M^2 \\
\frac{M}{2} - M^2 - N & M - 1
\end{bmatrix}
\]

(16)

If \( Q > 0 \), then \( \dot{V} < 0 \), according to the property of Shur complement [30], the value range of M and N for favorable timing \( Q \) is obtained as Equation (12). Using Theorem 1, we know that the system (10) can converge to the origin in finite time, and the estimation of convergence time is

\[
T = 2V^2(x, y) / \gamma(P, Q), \quad \gamma(P, Q) = \lambda_{max}(Q) / \lambda_{max}(P)
\]

This theorem 1 is proved.
4. Design based on FSTA-SMC Controller

The rotor motion equation of the PMSM is expressed as:

\[ T_e - T_L - B\omega = J \frac{d\omega}{dt} \]  \hspace{1cm} (17)

Where \( T_e \) is the electromagnetic torque; \( T_L \) is the load torque; \( J \) is the equivalent moment of inertia of the motor; \( \omega \) is the rotor angular velocity; \( B \) is the viscous friction coefficient.

Let \( \omega^* \) be the given speed, and define the sliding surface function as \( S_\omega = \chi = \omega^* - \omega \).

According to Equation (10), the speed controller can be designed as:

\[ T^* = M \left( |S_\omega|^{\frac{1}{3}} \text{sgn}(S_\omega) + kS_\omega \right) + T_e \]

\[ \frac{dT_e}{dt} = \frac{1}{2} \text{sgn}(S_\omega) + \frac{3}{2} k |S_\omega|^{\frac{1}{3}} \text{sgn}(S_\omega) + k^2 S_\omega \]  \hspace{1cm} (18)

Where \( M \) and \( N \) satisfy the stability condition (12).

### 4. Simulation Research

To verify the effectiveness of the direct control method proposed in this article. According to Fig. 2, use MATLAB/Simulink to build a simulation model for comparison. The motor parameters in the simulation experiment are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Parameters of the PMSM.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>Stator resistance</td>
</tr>
<tr>
<td>Pole-pairs number</td>
</tr>
<tr>
<td>Rotor flux</td>
</tr>
<tr>
<td>Q-axis induction Lq</td>
</tr>
<tr>
<td>D-axis induction Ld</td>
</tr>
<tr>
<td>Viscous friction coefficient</td>
</tr>
<tr>
<td>DC bus voltage</td>
</tr>
<tr>
<td>Rotational inertia</td>
</tr>
<tr>
<td>Stator flux give value</td>
</tr>
</tbody>
</table>

The parameters of the speed controller designed in this paper are \( M = 1, N = 600, k = 0.03 \). In addition, PI, two kinds of SMC with different parameters, traditional STA-SMC and the proposed strategy are compared. The PI parameter configuration is \( kp = 0.08, ki = 10 \); the parameter configuration of sliding mode controller is: \( k = 200, q = 1000 \) (SMC1); \( k = 200, q = 300 \) (SMC2); the parameter configuration of traditional STA-SMC is: \( M = 1, N = 600 \).

To verify the speed regulation performance and to chatter suppression performance of the proposed strategy, PMSM speed tracking experiments under disturbance is carried out. When the system's speed is set at 600 r/min, the system starts with no load, suddenly applies 8 N \cdot m load at 0.1 s, changes to 600 r/min at 0.2 s, and drops to 300 r/min at 0.3 s. The following figure shows the speed response diagram of PMSM under different control methods.
the system in the case of minor overshoot. To further compare the proposed strategy with traditional STA-SMC, traditional SMC, and PI, this paper lists the dynamic and static performance of the system under various control methods, as shown in Table 2. (overshoot 1 is the starting overshoot; overshoot 2 is the load disturbance overshoot; overshoot 3 is the acceleration overshoot; overshoot 4 is the deceleration overshoot; regulation time 1 is the load disturbance adjustment time; regulation time 2 is the acceleration adjustment time; regulation time 3 is the deceleration adjustment time.)

Table 2. PMSM system performance.

<table>
<thead>
<tr>
<th>Result</th>
<th>PI</th>
<th>SMC1</th>
<th>SMC2</th>
<th>STA-SMC</th>
<th>FSTA-SMC</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Time</td>
<td>50</td>
<td>50</td>
<td>55</td>
<td>4.5</td>
<td>3.5</td>
<td>ms</td>
</tr>
<tr>
<td>OverShoot 1</td>
<td>11</td>
<td>16.3</td>
<td>10.42</td>
<td>0.462</td>
<td>0.5</td>
<td>%</td>
</tr>
<tr>
<td>OverShoot 2</td>
<td>13.2</td>
<td>11.2</td>
<td>22.5</td>
<td>8.2</td>
<td>6.7</td>
<td>%</td>
</tr>
<tr>
<td>OverShoot 3</td>
<td>3.3</td>
<td>5.3</td>
<td>3.1</td>
<td>0.51</td>
<td>0.6</td>
<td>%</td>
</tr>
<tr>
<td>OverShoot 4</td>
<td>16</td>
<td>53.3</td>
<td>15.7</td>
<td>1.2</td>
<td>0.83</td>
<td>%</td>
</tr>
<tr>
<td>Regulation time 1</td>
<td>40</td>
<td>30</td>
<td>42</td>
<td>14</td>
<td>13</td>
<td>ms</td>
</tr>
<tr>
<td>Regulation time 2</td>
<td>60</td>
<td>30</td>
<td>62</td>
<td>5</td>
<td>4</td>
<td>ms</td>
</tr>
<tr>
<td>Regulation time 3</td>
<td>50</td>
<td>30</td>
<td>52</td>
<td>6</td>
<td>5</td>
<td>ms</td>
</tr>
</tbody>
</table>

From Fig. 3 and Table 2, it can be concluded that SMC2 can effectively improve the response speed and disturbance-rejection performance of the PMSM speed regulation system compared with PI and SMC1, but also increase the overshoot during starting and changing speed. Although the traditional STA-SMC can solve this problem, it has shortcomings in disturbance-rejection and response speed because its scale term is square root calculation. The proposed control strategy improves the system's response speed and robustness.

The chattering produced by sliding mode control is often shown in the system's speed tracking error; from Fig. 4 and Table 3, the proposed control strategy dramatically reduces the system's chattering based on improving the fast response performance and robustness.

Table 3. Speed tracking error.

<table>
<thead>
<tr>
<th>Result</th>
<th>PI</th>
<th>SMC1</th>
<th>SMC2</th>
<th>STA-SMC</th>
<th>FSTA-SMC</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>-2.3</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-0.15</td>
<td>-0.15</td>
<td>rpm</td>
</tr>
</tbody>
</table>

6. Conclusion

To achieve high-performance control of permanent magnet synchronous motor, a speed loop controller based on FSTA-SMC is proposed in this paper. Based on the original STA, FTSMML is introduced to improve the robustness and rapidity of the system. Through simulation, the proposed strategy was compared with PI, traditional SMC, and traditional STA-SMC, and the following conclusions were obtained:

1. The FSTA-SMC strategy is adopted to reduce the system response time and effectively solve the problems of low robustness of PI control, the contradiction between system chattering, reaching speed and robustness existing in traditional SMC, and the insufficient robustness and tracking the
performance of traditional STA-SMC;

2. In the sudden change of system speed and load, the proposed strategy can achieve an ideal control effect without adjusting controller parameters and has more robust adaptability to different working conditions.

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References


