LETTER

Two-Tone Distortion Products in Hardware-Efficient Cochlea Model based on Asynchronous Cellular Automaton Oscillator

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Abstract In this paper, we present a cochlea model whose nonlinear dynamics are described by an asynchronous cellular automaton. Our proposed model is demonstrated to enable the reproduction of two-tone distortion products. The proposed model is implemented on a field-programmable gate array (FPGA). It is subsequently demonstrated that our proposed model can be implemented using fewer hardware resources than a conventional cochlea model, which is a Hopf-type cochlea model, implemented on a digital signal processor performing a numerical integration.

key words: cochlea, field-programmable gate array, Hopf oscillator, asynchronous cellular automaton

Classification: Integrated circuits

1. Introduction

A laser Doppler velocimeter performed on live mammalian cochleas has observed that they have an active amplifier and essential nonlinearities [1]–[3]. For example, Fig. 1 shows typical nonlinear response characteristics of a mammalian cochlea measured in an anesthetized chinchilla [3]. In this figure, the basilar membrane (BM) is stimulated by two-tone frequencies f_1 and f_2 ($f_2 > f_1$), and it perceives tones with a combination of two frequencies, namely $n_1 f_1 \pm n_2 f_2$ ($n_1, n_2 \in \mathbb{Z}$), where the characteristic frequency (CF) in the stimulated site of the BM corresponds to $2f_1 - f_2$. Such responses to tones with frequencies not included in two-tone stimuli are called two-tone distortion products. Eguiluz *et al.* were the first to introduce a cochlea model based on Hopf oscillators to understand the nonlinearities of hearing [4]. Subsequently, Stoop *et al.* succeeded in demonstrating that the model enables the reproduction of a number of nonlinear hearing phenomena such as nonlinear compression, two-tone suppression, combination tone generation, and first (second) pitch shift [5]–[10]. Since then, there has been an increasing interest in implementing a cochlea model utilizing nonlinear oscillators on digital and analog electronic circuits [11]–[30]. Such an electronic circuit can be applied

Fig. 1. Spectrum of cochlear basilar membrane (BM) responses to two-tone stimuli with frequencies f_1 and f_2 ($f_2 > f_1$) measured in chinchilla adapted from [3]. The two-tone distortion products are perceived at frequencies not included in the stimulus (e.g., $2f_1 - f_2$, $3f_1 - 2f_2$, $2f_2 - f_1$, and $3f_2 - 2f_1$), where the characteristic frequency (CF) in the stimulated site of the BM corresponds to $2f_1 - f_2$.

to hearing aids and cochlear implants that incorporate nonlinear signal processing.

Traditional methods applicable to modeling and implementing nonlinear biological oscillators, including the cochlear amplifier, can be classified into the following three ways based on the continuousness of time and state.

- (i) The first method to model a nonlinear biological oscillator is by using an ordinary differential equation, which has a continuous time and state. Such a model is implemented by a nonlinear electronic analog circuit [11]–[14].
- (ii) The second method to model a nonlinear biological oscillator is by using a difference equation, which has discrete time and continuous states. Such a model is implemented by a switched-capacitor circuit [19]–[22].
- (iii) The third method to model a nonlinear biological oscillator is by using a discrete difference equation or a cellular automaton (CA), which has discrete times and states. Such models are implemented using a digital signal processor (DSP) that performs numerical integration or sequential logic [15]–[18].

Most nonlinear biological oscillators that have been studied belong to one of the above-mentioned three categories [11]–[18]. Recently, our group and a few other groups have designed a nonlinear biological oscillator using the below-mentioned fourth method.

- (iv) The fourth method to model a nonlinear biological oscillator is by using asynchronous CA, which has a continuous state transition time and discrete states. Such a model is implemented by an asynchronous sequential logic [23]–[30].

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Fig. 2. (a) Hardware configuration of the proposed cochlea model. (b) Schematic of asynchronous cellular automaton (CA) oscillator.

Inspired by the Hopf-type cochlea [4]–[10], our group has proposed a cochlea model designed by the fourth method and has shown that the model can reproduce the frequency-threshold tuning curves of a mammalian cochlea [27]. However, the reproduction of nonlinear response characteristics to two-tone stimuli remains to be demonstrated.

In this paper, a cochlea model with nonlinear dynamics described by an asynchronous CA was studied. We demonstrated that the proposed cochlea model enables the reproduction of two-tone distortion products [3]. The proposed model was implemented on a field-programmable gate array (FPGA). Subsequently, we demonstrated that the proposed cochlea model can be implemented using fewer hardware resources than a Hopf cochlea-type cochlea model implemented on a DSP performing a numerical integration.

2. Cochlea Model based on Asynchronous CA Oscillator

In this section, a hardware-efficient cochlea model based on an asynchronous CA oscillator is proposed. Subsequently, we demonstrated that the proposed cochlea model enables the reproduction of two-tone distortion products. Fig. 2(a) shows a hardware configuration of the proposed cochlea model. As shown in the figure, the proposed cochlea model comprises a sigma-delta analog-to-digital converter (ADC) and an asynchronous CA oscillator.

2.1 Asynchronous CA oscillator

Fig. 2(b) shows a schematic of the asynchronous CA oscillator. As shown in the figure, the oscillator accepts the following clock.

$$C_{int}(t) \equiv \sum_{k=0}^{\infty} \delta_R(t - kT_{int}), \quad C_{int} \in \mathbb{B} \equiv \{0, 1\}, \quad (1)$$

where $t \in \mathbb{R}$ and $T_{int} \in \mathbb{R}^+ \equiv \{x \in \mathbb{R} \mid x > 0\}$ represent the continuous time and a period of the clock, respectively, and the function $\delta_R: \mathbb{R} \rightarrow \mathbb{B}$ represents the unit impulse.

$$\delta_R(x) \equiv \begin{cases} 1 & (x = 0), \\ 0 & (x \neq 0). \end{cases}$$

Fig. 3. Timing chart of asynchronous CA oscillator. The parameter values are $N = 2^7$, $M = 2^8$, $\mu = -10$, $\omega = 2\pi \times 10^3$, $l = 0.25$, $T_{int} = 2^{-23}$, $T_{ext} = 3 \times 10^{-6}$, $\omega_1 = 2\pi \times 10^3$, $\omega_2 = 2\pi \times 200$, $A_1 = 0.125$, $A_2 = 0.125$, and $B_0 = 2$.

Furthermore, as shown in Fig. 2(b), the oscillator has the following two discrete state variables,

$$X \in \mathbb{Z}_N \equiv \{0, \dots, N-1\}, \quad Y \in \mathbb{Z}_N,$$

and the following two discrete auxiliary variables

$$P_X \in \mathbb{Z}_M \equiv \{0, \dots, M-1\}, \quad P_Y \in \mathbb{Z}_M,$$

where $N \geq 2$ and $M \geq 2$ are integers determining the resolution of a state space $\mathbb{Z} \equiv \{(X, Y, P_X, P_Y) \mid X \in \mathbb{Z}_N, Y \in \mathbb{Z}_N, P_X \in \mathbb{Z}_M, P_Y \in \mathbb{Z}_M\}$ of the oscillator. To design a nonlinear vector field, the following two functions $f_X: \mathbb{Z}_N \times \mathbb{Z}_N \rightarrow \mathbb{Z}$ and $f_Y: \mathbb{Z}_N \times \mathbb{Z}_N \rightarrow \mathbb{Z}$ are introduced.

$$\begin{aligned} f_X(x, y) &\equiv \left\lfloor \frac{1}{T_{int}(\mu(x - \frac{N}{2}) - \omega(y - \frac{N}{2}) - l^2(x - \frac{N}{2})((x - \frac{N}{2})^2 + (y - \frac{N}{2})^2))} \right\rfloor, \\ f_Y(x, y) &\equiv \left\lfloor \frac{1}{T_{int}(\omega(x - \frac{N}{2}) + \mu(y - \frac{N}{2}) - l^2(y - \frac{N}{2})((x - \frac{N}{2})^2 + (y - \frac{N}{2})^2))} \right\rfloor, \end{aligned} \quad (2)$$

where $l \in \mathbb{R}^+$, $\mu \in \mathbb{R}$, and $\omega \in \mathbb{R}^+$ are the parameters. The derivation of the functions f_X and f_Y in Eq. (2) is presented in Appendix. Furthermore, $\lfloor \cdot \rfloor$ denotes the following floor function.

$$\lfloor x \rfloor \equiv \max\{l \in \mathbb{Z} \mid l \leq x\}, \quad x \in \mathbb{R}.$$

The functions f_X and f_Y are implemented in the lookup tables (LUTs). Fig. 3 shows the timing chart of the oscillator. As shown in Fig. 3, the clock C_{int} triggers the following transitions of the discrete state variables X and Y .

If $C_{int}(t) = 1$, then

$$\begin{aligned} X(t_+) &:= \text{sat}_N(X(t) + \delta_Z(P_X(t)) \text{sgn}(\text{sat}_M(f_X(X(t), Y(t)))), \\ Y(t_+) &:= \text{sat}_N(Y(t) + \delta_Z(P_Y(t)) \text{sgn}(\text{sat}_M(f_Y(X(t), Y(t)))), \end{aligned} \quad (3)$$

Fig. 4. (a) Typical responses of asynchronous CA oscillator without input stimulus. The parameter values are $N = 2^7$, $M = 2^8$, $\mu = -10$, $\omega = 2\pi \times 10^3$, $l = 0.25$, and $T_{int} = 2^{-23}$. (b) Typical responses of asynchronous CA oscillator with two-tone input stimulus. The parameter values are the same as those chosen in Fig. 3

where the symbol “ t_+ ” represents “ $\lim_{\epsilon \rightarrow +0} t + \epsilon$ ”, the symbol “ $:=$ ” represents an “instantaneous state transition,” and $\text{sat}_N : \mathbb{Z} \rightarrow \mathbb{Z}_N$ and $\text{sat}_M : \mathbb{Z} \rightarrow \mathbb{Z}_M^\pm \equiv \{-(M-1), \dots, 0, \dots, M-1\}$ denote the following saturation functions.

$$\text{sat}_N(x) \equiv \begin{cases} N-1 & (x > N-1), \\ x & (0 \leq x \leq N-1), \\ 0 & (x < 0). \end{cases}$$

$$\text{sat}_M(x) \equiv \begin{cases} M-1 & (x > M-1), \\ x & (0 \leq x \leq M-1), \\ -(M-1) & (x < -(M-1)). \end{cases}$$

Furthermore, $\delta_Z : \mathbb{Z} \rightarrow \mathbf{B}$ denotes the following unit impulse function.

$$\delta_Z(x) \equiv \begin{cases} 1 & (x = 0), \\ 0 & (x \neq 0), \end{cases}$$

where $\text{sgn} : \mathbb{Z} \rightarrow \mathbf{Q} \equiv \{-1, 1\}$ denotes the following signum function.

$$\text{sgn}(x) \equiv \begin{cases} 1 & (x \geq 0), \\ -1 & (x < 0). \end{cases}$$

As shown in Fig. 3, the clock C_{int} triggers the following transitions of the discrete auxiliary variables P_X and P_Y .

If $C_{int}(t) = 1$, then

$$\begin{aligned} P_X(t_+) &:= P_X(t) + F_D(P_X(t), |\text{sat}_M(f_X(X(t), Y(t)))|), \\ P_Y(t_+) &:= P_Y(t) + F_D(P_Y(t), |\text{sat}_M(f_Y(X(t), Y(t)))|), \end{aligned} \quad (4)$$

Fig. 5. Schematic of sigma-delta ADC.

where $F : \mathbb{Z}_M^2 \rightarrow \mathbb{Z}_M^- \equiv \{-(M-1), \dots, 0, 1\}$ denotes the following function.

$$F_D(x, a) \equiv \begin{cases} 1 & (x < a), \\ -x & (x \geq a). \end{cases}$$

Fig. 4(a) shows a typical time waveform of the discrete state variables X and Y without an input stimulus.

2.2 Input stimulus modulated by sigma-delta ADC

In this study, an input stimulus is assumed to be

$$u(t) \equiv A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t}, \quad u \in \mathbb{C}$$

where $A_1, A_2 \in \mathbf{R}^+$ and $\omega_1, \omega_2 \in \mathbf{R}^+$ represent the amplitude and angular frequency of the input stimulus, respectively. The input stimulus $u(t)$ is transformed into a Cartesian coordinate representation for applying to the asynchronous CA oscillator as follows.

$$\begin{aligned} u_X(t) &\equiv A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t), & u_X &\in \mathbb{R}, \\ u_Y(t) &\equiv A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t), & u_Y &\in \mathbb{R}. \end{aligned}$$

The input stimuli u_X and u_Y are converted by using a sigma-delta ADC, and a schematic of the ADC is shown in Fig. 5. As shown in this figure, for sampling an analog signal, the following clock is introduced.

$$C_{ext}(t) \equiv \sum_{k=0}^{\infty} \delta(t - kT_{ext}), \quad C_{ext} \in \mathbf{B}, \quad (5)$$

where $T_{ext} \in \mathbf{R}^+$ represents a period of the clock. Furthermore, the input stimuli u_X and u_Y are sampled by clock C_{ext} and integrated by the following signals $W_X, W_Y \in \mathbb{R}$.

If $C_{ext}(t) = 1$, then

$$\begin{aligned} W_X(t_+) &:= u_X(t) + W_X(t) - B_0 S_X(t), \\ W_Y(t_+) &:= u_Y(t) + W_Y(t) - B_0 S_Y(t), \end{aligned} \quad (6)$$

where $S_X(t) = q(W_X(t))$, $S_Y(t) = q(W_Y(t))$ and $B_0 \in \mathbb{R}$, and the function $Q : \mathbb{R} \rightarrow \mathbf{Q}$ denotes the following quantizer.

$$q(x) \equiv \begin{cases} 1 & (x \geq 0), \\ -1 & (x < 0). \end{cases}$$

Subsequently, the clock $C_{ext}(t)$ triggers the following transitions of the discrete state variables X and Y .

If $C_{ext}(t) = 1$, then

$$\begin{aligned} X(t_+) &:= X(t) + S_X(t), \\ Y(t_+) &:= Y(t) + S_Y(t). \end{aligned} \quad (7)$$

Fig. 6. Two-tone distortion products in the proposed cochlea model (see also Fig. 1). The parameter values are $N = 2^7$, $M = 2^8$, $\mu = -10$, $\omega = 2\pi \times CF = 2\pi \times 10^3$, $l = 0.25$, $T_{int} = 2^{-23}$, $T_{ext} = 3 \times 10^{-6}$, $\omega_1 = 2\pi \times 1052.63$, $\omega_2 = 2\omega_1 - (2\pi \times CF)$, $A_1 = A_2 = 0.3$, and $B_0 = 2$.

Fig. 4(b) shows the typical time waveforms of the asynchronous CA oscillator with a two-tone input stimulus. The transitions of the discrete state variables X and Y are triggered by the uncoupled clocks C_{int} and C_{ext} , as shown in Fig. 3, the oscillator can be regarded as an asynchronous CA.

2.3 Reproductions of Two-tone Distortion Products

Fig. 6 shows a frequency spectrum of a time waveform of the proposed cochlea model stimulated by two tones, where CF corresponds to $2f_1 - f_2$. The frequency spectrum herein was obtained via fast Fourier transform (FFT) using a Hanning window with an applied amplitude factor. The proposed cochlea model perceives tones with frequencies not included in the two-tone stimulus, for example, $2f_1 - f_2$. Hence, the FFT analysis results shown in Fig. 6 verified that our proposed cochlea model can generate two-tone distortion products (see also Fig. 1).

3. FPGA Implementation and Comparison

3.1 FPGA implementation

The proposed cochlea model is implemented on a field-programmable gate array (FPGA) in a register transfer level (RTL) code using VHDL as follows: The discrete state variables X and Y are implemented by registers as n -bit unsigned integers, where $n = \lceil \log_2 N \rceil$. The discrete auxiliary variables P_X and P_Y are implemented by using registers as m -bit unsigned integers, where $m = \lceil \log_2 M \rceil$. The functions $\text{sat}_M(f_X(x, y))$ and $\text{sat}_M(f_Y(x, y))$ are implemented in lookup tables (LUTs) with an n -bit unsigned integer input and an $m + 1$ -bit signed integer output in the two's complement format. The dynamic equations in Eqs. (3), (4), and (7) are written by sequential statements triggered by the clocks C_{int} and C_{ext} in Eqs. (1) and (5), respectively. The RTL code was synthesized by Xilinx Vivado Design Suite v2020.1, and a generated bitstream file was downloaded into Xilinx FPGA Artix-7 XC7A100T-1CSG324C. Fig. 7(a) shows the resulting RTL schematic of our proposed cochlea model. The sigma-delta ADC is implemented by a switched-capacitor technique on a field-programmable analog array (FPAA) (Anadigm Single Apex 3.3 V Development Kit). The dynamic equation in Eq. (6) is written using Anadigm

Fig. 7. (a) Schematic of the FPGA for implementing the proposed cochlea model. (b) Schematic of the FPAA for implementing the sigma-delta ADC.

Fig. 8. Two-tone distortion products in the proposed cochlea model implemented on FPGA (see also Figs. 1 and 6). The parameter values are the same as those chosen in Fig. 6

Designer 2, where the resulting schematic of the sigma-delta ADC is shown in Fig. 7(b). Fig. 8 shows the frequency spectrum of the proposed cochlea model stimulated by two-tone stimuli implemented on the FPGA. The two-tone distortion product can be confirmed from the frequency spectrum (see also Figs. 1 and 6).

3.2 Comparison with Hopf-type cochlea model

For comparison, we implemented a Hopf-type cochlea model [4] [5] on the same FPGA, where the hardware configuration is shown in Fig. 9. The model comprises a sigma-

Fig. 9. Hardware configuration of the Hopf-type cochlea model. The parameter values are $\omega_{ch} = 2\pi \times CF = 2\pi \times 10^3$, $b = -200$, $\omega_1 = 2\pi \times 1052.63$, $\omega_2 = 2\omega_1 - (2\pi \times CF)$, $L = 33$, and $\Delta t = 2^{-17}$.

Table I. Comparisons of Hardware Resource

	asynchronous CA oscillator	Hopf oscillator
# Slices	1031	2237
# LUTs	3713	6666
# FFs	216	161

Note: The hardware resources of the Hopf oscillator do not include those of the IIR LPF. Artix-7 XC7A100T-1CSG324C has 15,850 slices; each slice contains four 6-input lookup tables (LUTs) and eight flip-flops (FFs). DSP slices and block RAM were not used.

delta ADC¹ and the following Hopf oscillator.

$$\dot{z} = (b + j)\omega_{ch}z - |z|^2z + F(t), \quad z \in \mathbb{C}, \quad (8)$$

where $b \in \mathbb{R}$ and $\omega_{ch} \in \mathbb{R}$ are parameters, and $F(t) \equiv E_1 e^{j\omega_1 t} + E_2 e^{j\omega_2 t}$ is an external input representing a two-tone stimulus. To implement an electronic circuit, the model in Eq. (8) is transformed into a Cartesian coordinate representation as follows.

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + F_1(t), \quad x_1 \in \mathbb{R}, \\ \dot{x}_2 &= f_2(x_1, x_2) + F_2(t), \quad x_2 \in \mathbb{R}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} f_1(x_1, x_2) &\equiv bx_1 - \omega_{ch}x_2 - x_1(x_1^2 + x_2^2), \\ f_2(x_1, x_2) &\equiv \omega_{ch}x_1 + bx_2 - x_2(x_1^2 + x_2^2), \\ F_1(t) &\equiv E_1 \cos(\omega_1 t) + E_2 \cos(\omega_2 t), \\ F_2(t) &\equiv E_1 \sin(\omega_1 t) + E_2 \sin(\omega_2 t). \end{aligned}$$

The Hopf oscillator in Eq. (9) is discretized using the forward Euler method, which is one of the simplest numerical integration methods as follows.

$$\begin{aligned} x_1(t + \Delta t) &= x_1(t) + \Delta t(f_1(x_1(t), x_2(t)) + F_1(t)), \\ x_2(t + \Delta t) &= x_2(t) + \Delta t(f_2(x_1(t), x_2(t)) + F_2(t)), \end{aligned} \quad (10)$$

where $\Delta t \in \mathbb{R}$ denotes a discretized time step. Furthermore, $F_1(t)$ and $F_2(t)$ are modulated by the sigma-delta ADC, which is the same as that shown in Fig. 7(b). As with Section 3.1, the discretized Hopf oscillator in Eq. (10) is implemented on the FPGA in an RTL code using VHDL, where the state variables and parameters are represented by fixed-point numbers in the two's complement format. Furthermore, as shown in Fig. 9, an infinite impulse response (IIR) low-pass filter (LPF) is implemented on the FPGA for demodulation of a sigma-delta ADC signal. Note that this digital filter is not required in the proposed cochlea model. The bit-lengths

¹The sigma-delta modulator is not necessarily chosen for the ADC in the Hopf-type cochlea model.

Fig. 10. (a) Forward Euler method. (b) Our method. t_0 and x_0 are constant.

L of all variables are reduced to be as short as possible under the condition that the model can generate two-tone distortion products. The RTL code is synthesized using the same development environment as that used in Section 3.1. Table I summarizes the comparison results of hardware resources. As shown in this table, the proposed cochlea model can be implemented using fewer hardware resources than the Hopf oscillator.

4. Conclusions

In this paper, a cochlea model based on an asynchronous CA oscillator is proposed. Our proposed cochlea model enables the reproduction of two-tone distortion products. The proposed cochlea and Hopf-type cochlea models were implemented on the same FPGA. The comparison revealed the following advantages of the proposed cochlea model: (i) the asynchronous CA oscillator can be implemented using fewer hardware resources than the Hopf oscillator and (ii) the proposed cochlea model does not require a digital filter for the demodulation of the sigma-delta ADC signal. Hence, this study contributes to the development of hearing aids and cochlear implants implemented in small-scale circuits. Our future work will include (i) extensive analyses of the two-tone distortion products in the proposed cochlea model and (ii) reproduction of other types of nonlinear response characteristics in a biological cochlea such as two-tone suppression and first (second) pitch shift.

Appendix: Derivation of functions f_X and f_Y

The proposed cochlea model is based on the Hopf-type cochlea model in Eq. (8). The forward Euler formula for the model in Eq. (10) without the input term is rewritten as follows.

$$\begin{aligned} x_1 \left(t + \left\lceil \frac{\Delta x}{f_1(x_1(t), x_2(t))} \right\rceil \right) &= x_1(t) + \Delta x \overline{\text{sgn}}(f_1(x_1(t), x_2(t))), \\ x_2 \left(t + \left\lceil \frac{\Delta x}{f_2(x_1(t), x_2(t))} \right\rceil \right) &= x_2(t) + \Delta x \overline{\text{sgn}}(f_2(x_1(t), x_2(t))), \end{aligned} \quad (11)$$

where $\Delta x \in \mathbb{R}$ denotes a discretized state step and the function $\overline{\text{sgn}} : \mathbb{R} \rightarrow \mathbf{Q}$ denotes $\overline{\text{sgn}}(x) = 1$ for $x \geq 0$ and $\overline{\text{sgn}}(x) = -1$ for $x < 0$. In the left-hand side of Eq.

(11), $|\Delta x/f_1(x_1(t), x_2(t))|$ and $|\Delta x/f_2(x_1(t), x_2(t))|$ represent the amounts of time advance per unit distance Δx . Furthermore, in the right-hand side of Eq. (11), the amounts of state transitions of x_1 and x_2 are restricted to Δx or $-\Delta x$. Fig. 10 shows the relationship between the forward Euler method and our method. In the proposed cochlea model, the discrete state variables $\{X, Y\}$ and the functions $\{f_X, f_Y\}$ correspond to $\{x_1, x_2\}$ and $\{\Delta x/f_1(x_1(t), x_2(t)), \Delta x/f_2(x_1(t), x_2(t))\}$ in Eq. (11), where $\Delta x = 1$. Also, the discrete auxiliary variables $\{P_X, P_Y\}$ whose dynamic equations are defined in Eq. (4) work as state-dependent frequency dividers using the functions $\{f_X, f_Y\}$ (see also the bent black arrows in Fig. 3).

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