The Relation between the Formula for Solution and the Components of Drawings for Arithmetic Word Problem Solving of the First-Grade Students in Elementary School*

Masato KAWASAKI*¹, Taisuke MORITA*², Minoru UMEZAWA*¹ and Mamoru KOIKE*¹

*¹Teikyo University of Science, 2-2-1 Senjusakuragi, Adachi-ku, Tokyo, 120-0045 Japan
*²Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162-8601 Japan

Received for publication, April 15, 2013

In order to examine the teaching method of arithmetic word problems, we created investigation problem for first graders based on Riley's classification and analyzed the relation between the drawings and the formula for solution which the subjects drew in the problem solving process of word problems. As a result, it was found that, in order to answer arithmetic word problems correctly, comprehension of relation was required in problems of CHANGE 1 \(a + b = x\) and 2 \(a - b = x\), and comprehension of unknown value was required in problems of CHANGE \(3(a + x = b), 4(a - x = b), 5(x + a = b)\) and 6 \(x - a = b\), where \(a\) and \(b\) denote known values and \(x\) denotes unknown value. These findings suggested that it is important in instruction of arithmetic word problems to make problem structure grasped after clarifying the relation and the unknown value.

Key words: arithmetic education, arithmetic word problem, problem solving process, external representation, formula for solution

1. INTRODUCTION

The TIMSS (Trends in International Mathematics and Science Study) 2003 conducted by IEA (International Association for the Evaluation of Educational Achievement), it is pointed that the correct answer rate of arithmetic word problems is lower than that of numerical calculations (Kawamata 2007). It is very difficult for a child to answer arithmetic word problems correctly. Therefore, in instruction of arithmetic word problems, in order to make the problem structure grasped, diagrams, tape drawings, and line segment drawings are used in many cases. There are many researches on problem solving of arithmetic word problems focusing on drawing or external representation.

The question then is what kind of element contained in drawings which children draw in the arithmetic word problem solving process can help them answer correctly. Yoshino et al. (2008) studied fifth graders paying attention to the capability side, in which suitable drawings could not be drawn even though drawings were effective in the arithmetic word problem solving process. In this investigation, first, it investigated using the space grasp problem and the logical deduction problem. Then each subject was shown the solution which actually used the figure for the exercise on the blackboard, and was given explanation about the necessity for drawings. They were instructed to write a formula and an answer in advance of drawing figures. The researchers set up the standard of what kind of drawing is suitable, and analyzed the drawings. In accordance with that standard, three raters evaluated the drawings using a five-point scale. As a result, they found the existence of pupils who had a problem in understanding the text or whose drawings were not suitable. However, the standard of suitability used for the analysis of the drawings was set up for each problem. Therefore, it does not have versatility.

Sakamoto (1999) conducted research on two ratio word problems with decimals that could be solved with single calculation for fifth graders. The drawings made into the object of examination were extracted from the solution explanation of the children. She compared the types of drawing and analyzed the relation with the correctness of a formula for solution. The drawings were divided roughly into three categories: line segment

* This paper was originally published in Jpn. J. Educ. Technol., Vol.36, No.4, pp.351-360 (2013)
drawings, relation drawings, and unrelated drawings. Her analysis of the drawings was based on further type divisions within those categories. She pointed out that many of the drawings which the children voluntarily drew were different from the segment charts in the textbook, and that the correctness of formula for solution was greatly related to whether the difference of the problem structure of the arithmetic word problem was successfully reflected to the type of drawn figure. Sakamoto (1999) has indicated that the external representation of the arithmetic word problem about ratio requires three elements (a known value, an unknown value, ratio), and expression of a known value and an unknown value in association with ratio. However, in her analysis of drawings, not the existence of entry of these elements but the classification of the expression method was used.

This research analyzes children's drawings which they draw for solving arithmetic word problems by the existence of entry of the elements (known values, unknown values, and relation), and considers the relation between the drawings and the formula for solution in order to clarify the required elements for a drawing for problem solving and to examine the teaching method of arithmetic word problems. The investigation problem must be basic, and should be solved by simple calculation. Therefore, we created the investigation problem using the classification based on the semantic structure of arithmetic word problems of Riley et al. (1983). Riley et al. (1983) classified arithmetic word problems into "CHANGE", "EQUALIZING", "COMBINE", and "COMPARE", according to the semantic structure of arithmetic word problems, in order to investigate the developmental stage of children's arithmetic word problem solving capability. This classification is well used in research of the arithmetic word problem solving processes. Yoshida (1991) has argued that problems based on Riley et al.'s (1983) classification are desirable because using this classification has the advantage of becoming the foundation when studying an arithmetic word problem and because one may be unable to discover a process peculiar to arithmetic word problems if more complicated problems are used from the beginning of research.

The purpose of this research is to clarify the elements required to solve the arithmetic word problems, and to examine the teaching method of the arithmetic word problems. Therefore, we thought it more desirable than a special problem to make a general and basic problem, and decided to create an investigation problem based on Riley et al.'s (1983) classification. Riley et al. (1983) classified the model of 16: CHANGE 6, EQUALIZATION 2, COMBINATION 2, and COMPARISON 6. However, in this research, in consideration of the load to a child, we decided to deal only with CHANGE and created 12 arithmetic word problems for investigation about six models of CHANGE. In this investigation, the first graders were asked to answer 12 arithmetic word problems and six numerical calculations about CHANGE. In order to clarify the component of the drawings required for solving arithmetic word problems correctly, we asked for description of the formula for solution and investigated the relation between the description of explanation by drawings or words and the formula for solution for arithmetic word problem solving. By analyzing quantitatively the relation between the explanation by such drawings and words and the description of the formula for solution, the relation between the components of drawings which were drawn in the process and the formula for solution was examined. While not all of the problem solving process of arithmetic word problems becomes clear by dealing with the six models, the analysis method which we propose in this research can not only clarify the point where one should be careful in the arithmetic word problem instruction but also be applicable to analysis of the drawings drawn in the problem solving process of other arithmetic word problems. In addition, although both researches of Yoshino et al. (2008) and Sakamoto (1999) targeted fifth graders, first graders were targeted in this research. The reason is as follows. First, in addition to the knowledge of the four basic operations, the arithmetic word problems which Yoshino et al. (2008) used needed other knowledge (knowledge about multiple relations and elimination). Moreover, the arithmetic word problem which Sakamoto (1999) used needed the knowledge about ratios. In the investigation using the problem which needs two or more kinds of knowledge, it may be difficult to judge whether the cause of an incorrect answer is in the inability to draw the necessary elements of a drawing or in the misunderstanding of "multiple relations" etc. Therefore, in this research aiming at considering the relation of the drawing and the formula for solution which were drawn, it is desirable to use a basic question as an investigation problem, in which addition or subtraction is used once. However, such a basic problem may be very easy
for children of elementary school upper classes, and a drawing drawn by those children in such a problem solving process can be largely omitted. So, in this research, when examining the process which solves such a basic problem, we investigated first graders in an elementary school considered to have suitable capability. It is necessary to conduct the same investigation with children of grades other than the elementary school first grade, as will be stated later in the section of discussion.

2. PURPOSE AND METHOD

2.1. Purpose

The purpose of this research is to clarify the relation between the component of the drawing and the formula for solution which children draw in the problem solving process of an arithmetic word problem, and to acquire knowledge about the teaching method of arithmetic word problems.

2.2. Subjects

Participants were 28 first graders in an elementary school in Nagano prefecture. In advance of the investigation, we showed the principal and the homeroom teachers of the elementary school the investigation problem in October, 2010, and we explained the survey content. The teachers checked the following things.

- Since the drawing is not drawn on the exercise of a textbook, the teacher carried out instruction to use drawings in solving problems at the time of this investigation.
- This investigation which makes children answer 12 arithmetic word problems and six numerical calculations continuously would not impose a heavy load for them.

2.3. Creation of Investigation Problem

2.3.1. Riley et al.’s (1983) Classification

The investigation problem which we created is a

<table>
<thead>
<tr>
<th>Table 1. Riley et al.’s (1983) classification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHANGE</strong></td>
</tr>
<tr>
<td>Result Unknown</td>
</tr>
<tr>
<td>1. Joe had 3 marbles.</td>
</tr>
<tr>
<td>Then Tom gave him 5 more marbles.</td>
</tr>
<tr>
<td>How many marbles does Joe have now?</td>
</tr>
<tr>
<td>2. Joe had 8 marbles.</td>
</tr>
<tr>
<td>Then he gave 5 marbles to Tom.</td>
</tr>
<tr>
<td>How many marbles does Joe have now?</td>
</tr>
<tr>
<td><strong>CHANGE Unknown</strong></td>
</tr>
<tr>
<td>3. Joe had 3 marbles.</td>
</tr>
<tr>
<td>Then Tom gave him some more marbles.</td>
</tr>
<tr>
<td>Now Joe has 8 marbles.</td>
</tr>
<tr>
<td>How many marbles did Tom give him?</td>
</tr>
<tr>
<td>4. Joe had 8 marbles.</td>
</tr>
<tr>
<td>Then he gave some marbles to Tom.</td>
</tr>
<tr>
<td>Now Joe has 3 marbles.</td>
</tr>
<tr>
<td>How many marbles did he give Tom?</td>
</tr>
<tr>
<td><strong>Start Unknown</strong></td>
</tr>
<tr>
<td>5. Joe had some marbles.</td>
</tr>
<tr>
<td>Then Tom gave him 5 more marbles.</td>
</tr>
<tr>
<td>Now Joe has 8 marbles.</td>
</tr>
<tr>
<td>How many marbles did Joe have in the beginnimg?</td>
</tr>
<tr>
<td>6. Joe had some marbles.</td>
</tr>
<tr>
<td>Then he gave 5 marbles to Tom.</td>
</tr>
<tr>
<td>Now Joe has 3 marbles.</td>
</tr>
<tr>
<td>How many marbles did he have in the beginnimg?</td>
</tr>
<tr>
<td><strong>EQUALIZING</strong></td>
</tr>
<tr>
<td>1. Joe has 3 marbles.</td>
</tr>
<tr>
<td>Tom has 8 marbles.</td>
</tr>
<tr>
<td>What could Joe do to have as many marbles as Tom? (How many marbles does Joe need to have as many as Tom?)</td>
</tr>
<tr>
<td>2. Joe has 8 marbles.</td>
</tr>
<tr>
<td>Tom has 3 marbles.</td>
</tr>
<tr>
<td>What could Joe do to have as many marbles as Tom?</td>
</tr>
</tbody>
</table>

| **COMBINE**                                    |
| Combine value unknown                          |
| 1. Joe has 3 marbles.                         |
|    Tom has 5 marbles.                         |
|    How many marbles do they have altogether?  |
| **Subset unknown**                            |
| 2. Joe and Tom have 8 marbles altogether.     |
|    Joe has 3 marbles.                         |
|    How many marbles does Tom have?            |

| **COMPARE**                                   |
| Difference unknown                           |
| 1. Joe has 8 marbles.                        |
|    Tom has 3 marbles.                        |
|    How many marbles does Joe have more than Tom? |
| 2. Joe has 8 marbles.                        |
|    Tom has 5 marbles.                        |
|    How many marbles does Tom have less than Joe? |

| **Compared quality unknown**                 |
| 3. Joe has 3 marbles.                        |
|    Tom has 5 more marbles than Joe.          |
|    How many marbles does Tom have?           |
| 4. Joe has 8 marbles.                        |
|    Tom has 5 marbles less than Joe.          |
|    How many marbles does Tom have?           |

| **Referent unknown**                         |
| 5. Joe has 8 marbles.                        |
|    He has 5 more marbles than Tom.           |
|    How many marbles does Tom have?           |
| 6. Joe has 3 marbles.                        |
|    He has 5 marbles less than Tom.           |
|    How many marbles does Tom have?           |
problem of six models about CHANGE among Riley et al.'s (1983) classifications. Based on the semantic structure of problems, Riley et al. (1983) classified the problems into four models: CHANGE, EQUALIZATION, COMBINATION, and COMPARISON. This classification is shown in Table 1. Riley et al. (1983) suggested that this table shows the example of the kinds of arithmetic word problems included in various researches and that each problem describes a simple situation which is related to addition or subtraction. This classification includes CHANGE, COMBINATION, and COMPARISON which were used by the research of Heller and Greeno, although the labeling words differ. These classifications are typical categorical schemes used by researchers in analyzing simple addition and subtraction problems. Equalization is based on Carpenter and Moser. The table shows the criterion of the problems for analyzing arithmetic word problems about the simplified addition and subtraction.

2.3.2. Investigation Problem Outline

In consideration of the load of the subjects, the investigation problem was limited to six models especially about CHANGE in the classification of Riley et al. (1983) described in 2.3.1, and was taken as the arithmetic word problems of six models and the problems (hereafter "numerical calculations") which ask for the value applicable to squares. As shown in Table 2, the created arithmetic word problems are six kinds of the CHANGE 1–6 dealing with an exchange of pencils between two persons. The investigation problem consists of 12 questions according to the direction of exchange: 6 questions representing the relationship by the word "give" and 6 questions representing the relationship by the word "get". In the 12 arithmetic word problems, the following directions are indicated by Japanese cursive characters with word segmentation.

(1) Please read this problem and explain what you thought about the problem with brief drawings.

(2) Please give answers to the following questions.

Formula for solution:

Answer:

The numerical calculations are six questions. Each question can obtain the correct answer operationally without setting up the formula for solution. All the arithmetic word problems are

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>give</th>
<th>get</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHANGE 1</strong>&lt;br&gt;a+b=x</td>
<td>1. Hanako had 5 pencils. Then Taro gave her 4 more pencils. How many pencils does Hanako have now?</td>
<td>7. Hanako had 5 pencils. Then Taro got 4 more pencils from Taro. How many pencils does Hanako have now?</td>
</tr>
<tr>
<td><strong>CHANGE 2</strong>&lt;br&gt;a−b=x</td>
<td>2. Hanako had 6 pencils. Then she gave 2 pencils to Taro. How many pencils does Hanako have now?</td>
<td>8. Hanako had 6 pencils. Then Taro got 2 pencils from Hanako. How many pencils does Hanako have now?</td>
</tr>
<tr>
<td><strong>CHANGE 3</strong>&lt;br&gt;a=x+b</td>
<td>3. Hanako had 4 pencils. Then Taro gave her some more pencils. Now Hanako has 7 pencils. How many pencils did Taro give her?</td>
<td>9. Hanako had 4 pencils. Then Hanako got some more pencils from Taro. Now Hanako has 7 pencils. How many pencils did Hanako get from Taro?</td>
</tr>
<tr>
<td><strong>CHANGE 5</strong>&lt;br&gt;x+a=b</td>
<td>4. Hanako had some pencils. Then Taro gave her 5 more pencils. Now Hanako has 8 pencils. How many pencils did Hanako have in the beginning?</td>
<td>10. Hanako had some pencils. Then Hanako got 2 pencils from Taro. Now Hanako has 8 pencils. How many pencils did Hanako have in the beginning?</td>
</tr>
<tr>
<td><strong>CHANGE 4</strong>&lt;br&gt;a−x=b</td>
<td>5. Hanako had 7 pencils. Then she gave some pencils to Taro. Now Hanako has 3 pencils. How many pencils did she give Taro?</td>
<td>11. Hanako had 7 pencils. Then Taro got some pencils from Hanako. Now Hanako has 3 pencils. How many pencils did Taro get from Hanako?</td>
</tr>
<tr>
<td><strong>CHANGE 6</strong>&lt;br&gt;x−a=b</td>
<td>6. Hanako had some pencils. Then she gave 2 pencils to Taro. Now Hanako has 5 pencils. How many pencils did Hanako have in the beginning?</td>
<td>12. Hanako had some pencils. Then Taro got 2 pencils from Hanako. Now Hanako has 5 pencils. How many pencils did Hanako have in the beginning?</td>
</tr>
</tbody>
</table>
written in Japanese cursive characters with word segmentation. We ensured the appropriateness of the questions by consulting with a first-grade pupil who did not participate in this study. Asking the pupil to indicate what parts were difficult to understand, we modified the parts. This process was repeated until there was no part that was difficult to understand for first graders. For example, he indicated that the direction sentence "solve a problem" was difficult to understand. In the directions "draw a drawing and a picture", since he used colored pencils and crayons, etc. and drew a detailed drawing, we added the word "brief" to the directions sentence. Since the arrangement of the investigation problem gave priority to the order of operation in CHANGE 3-6, the orders of CHANGE 4 and CHANGE 5 have been reversed. In Table 2, "a" and "b" indicate the values given in the problem (hereafter the "known value") and "x" indicates the value that is required as an answer (hereafter the "unknown value"). The problems shown in Table 2 present "give" and "get" in bold letters.

2.4. Procedure
The investigation was administered in March, 2011. After distributing the investigation problem which used 12 arithmetic word problems (one question per page) and six numerical calculations (six questions per page) as a booklet and explaining the purpose of the investigation, the instructor told the pupils to write down their thoughts and considered things as they were.

Furthermore, as an answer to each question, the subjects were instructed to write the explanation by drawings, pictures, and words, along with the formula and answer. We did not limit the time and made sure that all the subjects had finished answering all the problems, judging from their appearance, before announcing the end of the session. All the subjects finished answering in about 25 minutes. In addition, as described in 2.2, the burden of the subjects of answering 12 arithmetic word problems and six numerical calculations continuously was considered. We interviewed the principal of the elementary school and the homeroom teachers for investigation, and checked that the burden would not be too heavy for the subjects.

2.5. Analysis
In the analysis of the drawings which the children drew, two of us judged whether the known value, the unknown value, and the relation were drawn, and all four of us examined the relationship between the judgment result and the formula for solution.

3. RESULT

3.1. Numerical Calculation
Table 4 shows the number of subjects who answered correctly, the number of subjects who answered incorrectly, and the examples of incorrect answers of investigation problem (six numerical calculations). As shown in Table 4, most subjects were able to answer the numerical calculations correctly. However, this problem requires the appropriate value and did not require the description of thoughts and calculations in the process. Therefore, what kind of strategies they used in solving the problems remains unknown.

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Problem Number</th>
<th>correct</th>
<th>incorrect</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>(1)</td>
<td>28</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>(2)</td>
<td>27</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>(3)</td>
<td>27</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>(4)</td>
<td>26</td>
<td>2</td>
<td>7,4</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>(5)</td>
<td>28</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>(6)</td>
<td>25</td>
<td>3</td>
<td>1,10</td>
</tr>
</tbody>
</table>
3.2. Word Problem

Table 5 shows the number of subjects who answered correctly, the number of subjects who answered incorrectly, and the percentage of correct answers of 12 arithmetic word problems. We judged as a correct answer what answered correctly to both the formula and the answer. We then tried to examine whether the difference of the words which show relationship, “give” and “get”, is related to the number of correct answer persons and the number of incorrect answer persons. When the null hypothesis “the difference in the words of ‘give’ and ‘get’ and the correctness of an answer are unrelated to each other” was constructed and the Chi-square test was performed, as shown in Table 6, a significant difference was not found in the number of correct answer persons and the number of incorrect answer persons by the difference in word. Therefore, the null hypothesis is not rejected. Since the difference in word, and the number of correct answer persons and the number of

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Problem Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>correct</td>
</tr>
<tr>
<td>CHANGE 1</td>
<td>(1)</td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>(2)</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>(3)</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>(4)</td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>(5)</td>
</tr>
<tr>
<td>CHANGE 7</td>
<td>(6)</td>
</tr>
<tr>
<td>CHANGE 1</td>
<td>(7)</td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>(8)</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>(9)</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>(10)</td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>(11)</td>
</tr>
<tr>
<td>CHANGE 7</td>
<td>(12)</td>
</tr>
</tbody>
</table>

Table 6. Chi-square test value

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Problem Number</th>
<th>Test Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>(1)(7)</td>
<td>$x^2 (10) = 0.113$</td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>(2)(8)</td>
<td>$x^2 (10) = 0.163$</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>(3)(9)</td>
<td>$x^2 (10) = 0.113$</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>(4)(10)</td>
<td>$x^2 (10) = 0.100$</td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>(5)(11)</td>
<td>$x^2 (10) = 0.084$</td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>(6)(12)</td>
<td>$x^2 (10) = 0.819$</td>
</tr>
</tbody>
</table>

incorrect answer persons are independent, we will perform the following analyses by adding together the data of (1) and (7), (2) and (8), (3) and (9), (4) and (10), (5) and (11), and (6) and (12).

3.2.1. Analysis Procedure

We describe below the process until it determines the analysis method of the drawings which the children drew. First, two raters described the explanation of each drawing which the subjects drew. For instance, descriptions such as: “An arrow from Taro who has 5 pencils to Hanako who has four pencils. Hanako with seven pencils. An expression ‘Taro gave n pencils to Hanako.’” Next, the two raters examined the sentences of explanation to all the drawings and extracted the matters expressed in common in two or more sentences. As a result, the following facts were extracted.

- A number, a sign (a circle or a square), the number of the pencil expressed in the form of the pencil
- The relation expressed by a word (give or get) or an arrow
- The number of pencils expressed by words (the number of pencils the characters get or give) or a question mark

From the above, we decided to pay attention only to the number of pencils and the relation, without classifying the elements into concrete things, abstract things, and words. All the arithmetic word problems used for the investigation ask the number of pencils which Hanako has after Hanako and Taro exchanged pencils. Therefore, there are two kinds concerning the number of pencils. One is the number of pencils specified by the problem sentence. Another is the number asked for as an answer. Thus we analyzed the following three as the components of the problem.

- Known value: the number of pencils specified by the problem sentence
• Unknown value: the number of pencils for which each problem asks as an answer
• Relation: the move direction of the pencils between two persons.

Based on the criterion of Table 7, the two raters judged independently whether or not any of these elements (hereafter "components") was expressed in the children's drawings. When they were not in agreement, the two raters talked. The coincidence rate between the raters was 97.4% (match=982, mismatch=26).

3.2.2. Factors relevant to Correct Answer and Incorrect Answer

In order to examine the factors relevant to a correct answer and an incorrect answer, we classified the problems into two groups according to the existence of a component, and counted the number of correct answer persons and the number of incorrect answer persons for each group. The independency of a component and an answer was authorized. The null hypothesis "a component and an answer are independent" was authorized with Fisher's exact test. Tables 8, 9, and 10 show the existence of a component, the number of subjects who answered correctly and the number of subjects who answered incorrectly, and the probability. In addition, in order to examine the factor which had a large influence on the rejection of this null hypothesis, the value of (observed-expected)/expected is shown in parenthesis in tables 8, 9 and 10. As a result of the Chi-square test, the following was shown:

• In CHANGE 1, 3, 5, and 6, the existence of known value's expression is not significantly independent from the correctness.

Table 7. The criterion about drawings

<table>
<thead>
<tr>
<th>Known value</th>
<th>The number of pencils that are specified in the problem statement is represented by a character or figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknown value</td>
<td>It is indicated by characters and signs that the number of pencils for which it asks as an answer is unspecified. For example, &quot;How many&quot;, &quot;?&quot;, &quot;How many were given?&quot;, &quot;How many were got?&quot;, etc.</td>
</tr>
<tr>
<td>Relation</td>
<td>It is expressed by characters. For example, an arrow showing the movement of a pencil, &quot;gave&quot;, &quot;got&quot;, etc.</td>
</tr>
</tbody>
</table>

Table 8. Existence of known value expression and correctness

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Existence</th>
<th>correct</th>
<th>incorrect</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>Yes</td>
<td>42(0.175)</td>
<td>7(0.716)</td>
<td>0.0223**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>3(1.225)</td>
<td>4(5.011)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>Yes</td>
<td>47(0.049)</td>
<td>5(0.346)</td>
<td>0.0720*</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2(0.643)</td>
<td>2(4.5)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>Yes</td>
<td>7(3.393)</td>
<td>11(0.830)</td>
<td>0.0266*</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>4(1.608)</td>
<td>34(3.932)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>Yes</td>
<td>11(9.842)</td>
<td>8(2.976)</td>
<td>0.0000**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2(5.055)</td>
<td>35(1.528)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>Yes</td>
<td>13(0.186)</td>
<td>25(0.081)</td>
<td>0.5353ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>4(0.392)</td>
<td>14(0.171)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>Yes</td>
<td>13(1.402)</td>
<td>22(0.513)</td>
<td>0.0306*</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2(2.336)</td>
<td>19(0.855)</td>
<td></td>
</tr>
</tbody>
</table>

ns: no significance, "":0.5<p<0.1, "":p<0.05, "":p<0.01

Table 9. Existence of unknown value expression and correctness

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Existence</th>
<th>correct</th>
<th>incorrect</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>Yes</td>
<td>0</td>
<td>0</td>
<td>1.0000ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>45</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>Yes</td>
<td>11(0.125)</td>
<td>48(0.000)</td>
<td>1.0000ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>0</td>
<td>7(0.002)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>Yes</td>
<td>2(3.051)</td>
<td>6(12.486)</td>
<td>0.003**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>5(0.509)</td>
<td>5(2.080)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>Yes</td>
<td>2(1.608)</td>
<td>5(5.320)</td>
<td>0.0123**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>0</td>
<td>40(0.268)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>Yes</td>
<td>1(1.769)</td>
<td>4(4.058)</td>
<td>0.0259**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>13(0.173)</td>
<td>38(0.390)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>Yes</td>
<td>2(1.304)</td>
<td>4(3.563)</td>
<td>0.0384*</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>11(0.156)</td>
<td>39(0.428)</td>
<td></td>
</tr>
</tbody>
</table>

ns: no significance, "":0.5<p<0.1, "":p<0.05, "":p<0.01

Table 10. Existence of Relation expression and correctness

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Existence</th>
<th>correct</th>
<th>incorrect</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>Yes</td>
<td>43(0.334)</td>
<td>6(1.365)</td>
<td>0.0021**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2(2.336)</td>
<td>5(9.557)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>Yes</td>
<td>48(2.89)</td>
<td>4(8.413)</td>
<td>0.0000**</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1(3.440)</td>
<td>50(24.083)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>Yes</td>
<td>32(0.091)</td>
<td>10(0.371)</td>
<td>0.2576ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1(1.114)</td>
<td>13(0.272)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>Yes</td>
<td>30(0.129)</td>
<td>12(0.109)</td>
<td>0.6682ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1(0.586)</td>
<td>8(0.172)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>Yes</td>
<td>30(0.129)</td>
<td>16(0.297)</td>
<td>0.2532ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>1(1.365)</td>
<td>9(0.595)</td>
<td></td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>Yes</td>
<td>32(0.001)</td>
<td>12(0.004)</td>
<td>1.0000ns</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>3(0.014)</td>
<td>9(0.005)</td>
<td></td>
</tr>
</tbody>
</table>

ns: no significance, "":0.5<p<0.1, "":p<0.05, "":p<0.01
• In CHANGE 3, 4, 5, and 6, the existence of unknown number's expression is not significantly independent from the correctness.
• In CHANGE 1 and 2, the existence of relationship's expression is not significantly independent from the correctness.

Next, from the value of 
\((\text{observed-expected})/\text{expected},\)
we compared the degrees of expected frequency and observed frequency, and examined the factor which affected the rejection of the null hypothesis.

Concerning the expression of the known value, it was found that b had the highest degree of deviation of "no expression and a incorrect answer"; CHANGE 3 and 5 had the highest degree of deviation of "an expression and a correct answer". Moreover, although the null hypothesis was rejected, CHANGE 6 did not have much difference in the degree of deviation concerning the existence of an expression of a correct answer person's known value.

3.2.3. Grasp of Key Factor relevant to Correct Answer and Incorrect Answer

In CHANGE 1, 2, 3, 5, and 6, the statistical significance was accepted with two or more components. In order to examine the elements which closely relate to the correctness of answers, we analyzed CHANGE 1, 2, 3, 5, and 6 by Quantification II. The known value expression and the relation expression were set as explanatory variables, and the correctness was set as the response variable n in CHANGE 1 and 2. The known value expression and the unknown value expression were set as explanatory variables, and the correctness was set as the response variable in CHANGE 3, 5, and 6. Quantification II is the method of conducting discriminate analysis with the value which quantifies the rate scale of each evaluation item as a category, and synthesizes those (Yamamoto et al. 1998).

The range of an explanatory variable, a partial correlation coefficient, a right distinction rate (rate which judge s a correct answer to be correct, or an incorrect answer to be incorrect), and a correlation ratio are shown in Table 11. As shown in Table 11, in CHANGE 1 and 2, the weight range and partial correlation coefficients were known value < relations, and the weight range and partial correlation coefficient were known value < unknown value in CHANGE 3 and 6. In CHANGE 5, the weight range and a partial correlation coefficient were unknown value < known value. From these results, it can be guessed that grasping a relation in CHANGE 1 and 2, unknown value in CHANGE 3 and 6, known value in CHANGE 5 is a key factor in obtaining a correct answer.

3.2.4. Incorrect Formula for solution

The examples of incorrect answers of the arithmetic word problem are described in Table 12.

Although the error of the formula for solution of using addition for the problem of subtraction or subtraction for the problem of addition was seen in CHANGE 1 and 2, all the values used for solution were known values. However, in CHANGE 3, 5, and 6, there were errors using the incorrect formula for solution and errors using the values which were not given in the problem. Moreover, CHANGE 4 is a subtraction problem, and since its answer was drawn by using subtraction, there were few mistakes in operation. However, there was an error using the values which were not given in the problem like CHANGE 3, 5, and 6. These values that were not given as a problem were values of the answer in question, and almost all incorrect formula for solution was formula what is called a checking formula. CHANGE 3, 5, and 6 are addition reverse subtraction problems, and addition reverse subtraction problems are considered to be difficult for children in the lower grades (Yoshida 1991). However, when a concrete

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>Range</th>
<th>partial correlation</th>
<th>distinction rate</th>
<th>correlation ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Known Value</td>
<td>Unknown Value</td>
<td>Relation</td>
<td>Known Value</td>
</tr>
<tr>
<td>CHANGE 1</td>
<td>0.161</td>
<td>0.319</td>
<td>0.228</td>
<td>0.421</td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>0.042</td>
<td>0.418</td>
<td>0.085</td>
<td>0.710</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>0.116</td>
<td>0.326</td>
<td>0.301</td>
<td>0.555</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>0.233</td>
<td>0.176</td>
<td>0.585</td>
<td>0.373</td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>0.217</td>
<td>0.347</td>
<td>0.395</td>
<td>0.402</td>
</tr>
</tbody>
</table>
Table 12. Mistaken formula for solution and its number

<table>
<thead>
<tr>
<th>Type of Word Problems</th>
<th>correct Problem Number</th>
<th>incorrect sample (number of children)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE 1</td>
<td>5+4</td>
<td>(1) 5-4(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7) 5-4(2), 5+2(1), 5+5(1), 4+4(1)</td>
</tr>
<tr>
<td>CHANGE 2</td>
<td>6-2</td>
<td>(2) 6+2(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8) 6+2(4)</td>
</tr>
<tr>
<td>CHANGE 3</td>
<td>7-4</td>
<td>(3) 4+3(21), 7-2(1), 7-3(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9) 4+3(17), 4+7(3), 4+2(1), 7+3(1)</td>
</tr>
<tr>
<td>CHANGE 5</td>
<td>8-2</td>
<td>(4) 2+6(17), 3+6(2), 8+2(1), 10-2(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10) 6+2(18), 8+2(2), 5+3(1), blank(1)</td>
</tr>
<tr>
<td>CHANGE 4</td>
<td>7-3</td>
<td>(5) 7-4(17), 7-3(1), 7-2(1), 7+4(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11) 7-4(15), 7-5(1), 8-4(1), 0+3(1), 1+2(1)</td>
</tr>
<tr>
<td>CHANGE 6</td>
<td>5+2</td>
<td>(6) 7-2(12), 3+2(3), 4+1(1), 5-2(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12) 7-2(15), 5-2(4), 2+3(2), 4+1(1)</td>
</tr>
</tbody>
</table>

thing is used in the problem, or the children themselves are involved in the problem, even children in the senior class of kindergarten can get a correct answer (Uemura 2010). The kindergarten children's solution obtains a correct answer by the operation of concrete things and does not depend on the calculation by using a formula for solution. As shown in Table 4, to the problem which asks only for the number applicable to a square, most of the subjects were able to answer correctly. However, the subjects could not show the formula for solution for obtaining a correct answer in the arithmetic word problem which asks for description of the answer and the formula for solution. The checking formula was described as a formula for solution of each problem.

4. DISCUSSION

4.1. Components Required for the Arithmetic word Problem Solving

As shown in Tables 8, 9, and 10, in the problem like CHANGE 1 or CHANGE 2, in the case which the known value and the relation are not grasped, the number of incorrect answers increases significantly. On the contrary, if they are grasped, the number of correct answers will increase. This might be because the process of operational solving corresponded to the formula for solution. For example, the operation in problem number (1) of “adding the pencils got from Taro to the pencils which Hanako had first” and the formula (the number of pencils which she had first + the number of pencils which she gets = the number of pencils which she has now) are in agreement. Therefore, it is easy for a child to describe the formula. However, when a known value and an unknown value are grasped in addition reverse subtraction problems like CHANGE 3, 5, and 6 or the unknown is grasped in CHANGE 4, the number of correct answers increases significantly. Furthermore, as shown in Table 11, from these problems, it turns out that a major factor for problem solving is to grasp the unknown. For example, in problem 4, the number of pencils at the start and that at the end are operationally compared based on the situation “the number of pencils which Hanako had first changed since some pencils were got from Taro,” and the number of pencils which she got can be a known value. Moreover, when one grasps clearly what should be answered and understands that the change can be calculated by subtraction, a formula for solution can be connected to operation and a correct answer can be obtained. However, when it is not clear what one should answer, it is difficult to connect operation and a formula for solution. It is supposedly a situation where a low grader describes: “Although I know the answer, I don’t know the formula”. From these results, in arithmetic word problems like CHANGE 3, 4, 5, and 6, one should not only obtain a correct answer from a problem sentence operationally, but also must grasp what should be answered; and further, if the operation for obtaining a correct answer and a formula for solution are not associated, it is probable that a correct answer cannot be obtained.

4.2. Existence of Component and Formula for solution

In order to consider the relation between the first graders' drawings and strategies for the problems of CHANGE 3, 4, 5, and 6, the children's drawings that answered correctly all of the four questions are compared with the children's drawings that answered incorrectly all of
the four questions. Judgments concerning the drawing, formula for solution and answer, and a component of drawings about CHANGE 3～6 are shown in Fig. 1. A circle indicates that the two raters identified respective components in the drawings, and a cross indicates that they did not identify respective components in the drawings. As shown in Fig. 1, the correct answering children might judge the formula which should be used from the value calculated operationally and the known value when they grasped for what it should ask by expressing an unknown as □ in a drawing. Some other correct answering children grasped the unknown value and expressed it by “?” or “how many having been given”. However, in the incorrect answering children’s drawings, there is no expression by the drawings or words which shows an unknown value, but an obscure value not given as a known value is expressed. It can be thought that these values are the values equivalent to a correct answer and that the checking formula is described in the column of “formula for solution” while the calculation result of a checking formula is described in the column of “answer.” If the children had grasped what should be described as an answer, such an error would not have occurred. From these results, it is clear that instruction which makes children grasp an unknown exactly and which makes them understand that the change is calculated by subtraction is required in the problems where the change or the start is unknown such as CHANGE 3, 4, 5, and 6.

4.3. Instruction of Arithmetic Word Problem

From the analysis result of this investigation, it is thought that instruction focusing on the following three points is required.

(1) In order to solve correctly the problem which asks for the result value such as CHANGE 1 and 2, it is necessary to understand the relation and the known value. Therefore, it is important to understand the known value and the relation.

(2) In order to solve correctly CHANGE 3, 5, 6, and 4, where the CHANGE or the start is unknown, it is necessary to understand the relation and the known value as well as the unknown value.

(3) It is necessary in original teaching (Ernst 1995) to determine the start by subtracting the change from the result. In addition, it is necessary to understand the subtraction as the

<table>
<thead>
<tr>
<th>CHANGE 3</th>
<th>CHANGE 5</th>
<th>CHANGE 4</th>
<th>CHANGE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Drawing" /></td>
<td><img src="image2.png" alt="Drawing" /></td>
<td><img src="image3.png" alt="Drawing" /></td>
<td><img src="image4.png" alt="Drawing" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>correct</th>
<th>incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Formula" /></td>
<td><img src="image6.png" alt="Formula" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHANGE 3</th>
<th>CHANGE 5</th>
<th>CHANGE 4</th>
<th>CHANGE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Formula" /></td>
<td><img src="image8.png" alt="Formula" /></td>
<td><img src="image9.png" alt="Formula" /></td>
<td><img src="image10.png" alt="Formula" /></td>
</tr>
</tbody>
</table>

Fig. 1. Drawing and formula and answer which correctly answered child and incorrectly answered child.
difference calculation (Ernst 1995) where the change can be determined by subtracting the start from the result.

5. SUMMARY

The purpose of this research was to clarify the relation between the components of drawings used for solution and the formula for solution, and to examine the method of instruction of arithmetic word problems. We investigated first graders in an elementary school. The investigation problem consists of 12 problems about CHANGE based on the six models which Riley et al. (1983) classified from the semantic structure of arithmetic word problems and 6 questions of numerical calculation which asks for an applicable number. The result reveals that it is necessary to understand the relation and the known value to solve problems where the result is unknown. However, in teaching of the problems in which the change or the start is unknown, it was shown that in addition to the grasp of known value and the relation, the teaching that makes children understand the unknown value is necessary. In addition, this study indicated that it is necessary to understand the subtraction as the difference calculation. It is worth noting as a limitation of this study that the sequence of the survey questions used in this study has been fixed. Therefore, there is a possibility that the effect of training and fatigue affected the results of this study. Future study should explore the effect of the order of the survey questions. Furthermore, for graders other than the first graders, the same investigation should be conducted and compared with this finding. We will conduct research related to the teaching method for making the components of drawings consistent with the formula for solution in arithmetic word problem solving process.

ACKNOWLEDGEMENTS

We are deeply grateful to the elementary school principal and the homeroom teachers who cooperated in the implementation of this research.

REFERENCES


