Age-Related Changes in Performance and Strategies in an Area and Volume Comparison *

Masato KAWASAKI *1, Sayuri TAKESHITA *2 and Taisuke MORITA *3

*1 Teikyo University of Science, 2-2-1 Senjusakuragi, Adachi-ku, Tokyo, 120-0045 Japan
*2 PRECIOUS PARTNERS Co., Ltd., 1-22-2 Nishishinjuku, Shinjuku-ku, Tokyo, 160-0023 Japan
*3 Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162-8601 Japan

Received for publication, August 23, 2016

The purpose of this study was to examine children's age-related changes in performance and strategies in an area and volume comparison. Kindergarteners and pupils in the lower grades of an elementary school participated in this study. Pairs of rectangles or rectangular solids were presented to the participants, and they were asked to indicate which one was larger. Results showed that the percentage of correct answers in the area comparison tasks varied across age groups in a u-shaped manner, whereas the percentage of correct answers in volume comparison tasks increased in a linear manner. The results from the analyses of the comparison cues that children used revealed that the kindergarteners and first graders were unable to integrate differences in multiple dimensions, and that they utilized the differences as separated cues in the area and volume comparisons. However, second and third graders could integrate and utilize differences in multiple dimensions as comparison cues. These results support the view suggested by Bausano and Jeffrey (1975), that three-year-olds should outperform four- or five-year-olds in area comparisons because three-year-olds tend to compare areas and utilize the differences in the longest dimension as comparison cues.

Key words: judgment of size, area, volume, dimensions, arithmetic education

1. INTRODUCTION

The 2008 course of study (Ministry of Education, Culture, Sports, Science, and Technology 2008), which is the curriculum development guideline for schools, stipulates that teachers should enhance pupils’ abilities to acquire concepts for quantity and measurement through an abundance of experience to compare the lengths, areas, and volumes of objects around them.

Regarding area, pupils learn that they can compare the areas of objects by piling one object on top of another. They also learn that when the piling strategy is not available, they may compare areas by counting and comparing the numbers of plates that they spread on top of the compared objects. With volume, pupils learn that they may compare volumes of objects by putting a smaller box into a bigger box, pouring water into one container and out of another, or examining how many times they may ladle water in a container using a cup. The key question here is when do children acquire the above-mentioned strategies with which they compare areas and volumes.

The development of a child’s performance in various domains, such as concepts, language, cognition, and sociality tends to change in a u-shaped manner: rising, descending, and rising once again with age (SHIMIZU 2002; AOKI 1989). MARATSOS (1973) was the first to demonstrate that children’s ability to compare areas develops in a u-shaped manner. He asked children whose ages ranged from three to five years to compare the areas of a pair of rectangles. The results showed that the percentage of correct answers decreased with age. Given that adults have sufficient ability to compare the areas of rectangles, he suggested that the area comparison ability develops in a u-shaped manner.

SENA and SMITH (1990) conducted a replication of a study by MARATSOS (1973) with tasks that were identical in his experiment. The results of their study were partially consistent with those of Maratsos (1973). Children whose ages ranged from three to five years, exhibited age-related decreases in correct answer rates in the area.
comparison tasks, whereas adults exhibited higher correct answer rates. They also examined whether children’s area comparisons depend on the height of the rectangle or the overall area of the rectangle. The results showed that the percentage of children who compared the area of rectangles correctly increased with age, based on the height of the rectangles.

Shimizu (1996) suggested that there are three distinct hypotheses that may explain the finding that correct answer rates decrease in four- or five-year-old children. The three hypotheses are as follows: a) five-year-old children tend to focus their attention on the height of rectangles; b) in the course of language acquisition, five-year-old children tend to confuse the newly acquired word “high” with the already acquired word “big”; and c) five-year-old children tend to rely on analytical cognitive processing, whereas three-year-old children tend to rely on holistic cognitive processing. However, Shimizu (1996) implied that the findings that three-year-old children can compare areas of rectangles successfully are questionable. In addition, other studies suggested that three-year-old children compare areas of rectangles based not on the area, but on the length of the longest side of the rectangles (Baustano and Jeffrey 1975), and that there was no evidence showing that area comparison decision-making in three-year-old children depends on the area of the rectangles (Ravn and Gelman 1984).

Shimizu (1996) pointed out a problem in experimental tasks that was adopted in previous studies: the height of the stimulus rectangle co-varied with its area, and its width co-varied with its area. To overcome this problem, he designed a new experimental task and asked children whose ages ranged from three to six years to perform the task. The results of the experiment showed that three- to five-year-old children compared rectangles by relying on the width of the rectangles, whereas six-year-old children did not. It was also shown that though previous studies considered children’s choosing a larger rectangle as evidence that they compare based on the area, some three- to five-year-old children exhibited comparison by width.

On the other hand, based on the fact that children rarely make mistakes in comparing the area of objects in everyday situations, Matsumoto and Midorikawa (2000) examined the possibility that the responses of children in a context in which they are asked to respond with the word “big” differs from their responses in a context in which they proactively compare areas of objects and express the result verbally. The results indicated several findings, as follows. Changing the context enhanced the accuracy of children’s usage of the word “big.” Younger age groups associated the word “big” with only a single concept. As children aged, they learned to differentiate the word from or mix it with other concepts, during the course of language acquisition. Eventually, they grasped the full range of meanings of the word “big.” Children could successfully choose larger rectangles when the experimenter did not use the word “big.”

The studies described above were on comparisons of area. How does the children’s abilities to compare volume change and develop?

Mori et al. (1980) examined the development of expressions that describe spatial volume in infants. As a result, it was shown that younger children tended to consider that higher or wider objects were larger in volume, irrespective of their actual volume. However, as their study was not on comparison of the spatial volume of three-dimensional objects, but on expressions of spatial volume, it is still unclear which elements children utilized as a cue to compare the volume of objects.

Ebersbach (2009) presented kindergartners, first graders, third graders, and adults with eight types of cuboids, and asked them to estimate the number of one cubic centimeter cubes contained by the cuboids. The results showed that the majority of kindergartners estimated the volume of cuboids by considering all three dimensions, and that 42% of kindergartners had multiplicatively integrated at least two dimensions. Ebersbach’s (2009) study could be considered as an examination not of the comparison of volumes of the conversion of the volume of given cuboids to the appropriate number of one cubic centimeter cubes.

As there have been few studies that have examined comparisons of spatial volumes in infants, it is unknown whether the ability to compare develops in a u-shaped manner, and which elements are used by infants to compare volumes. However, it is crucial to clarify the development of the ability to compare volumes in children ranging from infants to lower level graders in elementary school, because first graders in elementary school begin to learn how to compare volumes through various activities.

Therefore, this study examined the development
of this ability by presenting children with tasks that require the comparison of area and volume, and estimating which elements were utilized by children as cues, to compare the area and volume of objects.

This study contributed to uncovering children’s readiness to make comparisons of the area and volume of objects, which may help teachers in elementary school to provide instruction in how to consider area and volume deductively.

2. METHOD

2.1. Participants

Thirty kindergartners and 30 pupils in elementary school participated in this study. The kindergartners comprised three groups: junior, middle, and senior grade. The pupils were composed of three groups: first, second, and third grade. The number of participants in each group was ten. Table 1 shows the mean, range, standard deviation of ages, and the number of participants in each group. None of the participants had received instruction on how to calculate the area and volume of objects.

2.2. Materials and Tasks

2.2.1. Materials

Figure 1 depicts flat shapes and solid shapes presented in comparison tasks. The flat shapes were made from black thick paper without any letters on them. The solid shapes were transparent containers made of three-millimeter acrylic plates without any letters on them.

The solid shapes a and b have mark lines that are parallel to the bottom line of the shapes. All the lengths of the solid shapes in Figure 1 are of the outer diameter.

2.2.2. Comparison tasks

The comparison tasks of the flat shapes are shown in Table 2, and those for the solid shapes are shown in Table 3. “Depth” meant that the shape was placed so that the longest side of each shape was placed in the depth direction. “Width” meant that the shape was placed so that the longest side of each shape was placed in the width direction. “Length” meant that the task was to compare volumes of solid shapes, assuming that the shapes were filled with water up to the broken line.

2.2.3. Task development and presentation

The flat shape comparison tasks were composed of five tasks. In each task, a pair of flat shapes was presented. The flat shapes in each pair were created so that the length of one of the two sides of the paired shapes were identical, or that the length of the two sides of the shapes were identical to determine whether participants used the width or depth of the shapes as cues to compare shapes. SA3 was introduced in order to further examine strategies participants used in SA1 and SA2.

The solid shapes comparison tasks were composed of 11 tasks. In each task, a pair of solid shapes was presented. The solid shapes in each pair were created so that the length of two of the three sides of the paired shapes were identical, or that the length of all sides of the shapes were identical to determine whether participants used width, depth, or height of the shapes as cues to compare shapes.

Regarding SC2, the height of the broken line from the bottom side, and the depth and width of the two shapes in SC2 were identical, whereas the heights of the shapes were not identical. SC2 was designed to examine whether participants used the height of the shape or the height of the water surface as a cue to compare solid shapes when they were asked to compare the volumes of water with which two shapes were filled up to the broken line.

To reduce the cognitive load of participants by clearly showing which shapes were changed, one of two shapes of stimuli presented in the previous trial was presented again in the next trial. The order of presenting tasks was fixed across participants.

2.3. Procedure

The experiment was conducted in a room of a kindergarten or an elementary school. Each participant and the experimenter sat across a table. After building rapport, the experimenter explained three points: a) the experimenter would
ask them to compare pairs of shapes, b) they could touch or move the shapes, and c) it was completely acceptable to answer, “I cannot tell” in the tasks. The elementary school participants were also instructed that these tasks were not achievement tests and that their performance in the tasks would not affect their school records.

After the explanation, the experimenter placed a pair of shapes on the table with an interval of about one centimeter between them. The edges of the shapes were turned up. The experimenter hid the remaining shapes from participants. When the solid shapes with broken lines were presented in the task other than SC2, the experimenter placed the shapes so that the broken lines were invisible to the participants. In the flat shapes comparison tasks, the experimenter indicating this pair of shapes, asked participants, “Which shape is larger, this or that”? In the solid shapes comparison tasks, the experimenter asked participants, “Which shape can contain more water, this or that”? In the solid shapes comparison tasks, the experimenter asked participants, “Which shape can contain more water if these are filled with water up to these lines” (SC2), to make the tasks clearer, especially for the kindergartners.

After the explanation, the experimenter placed a pair of shapes on the table with an interval of about one centimeter between them. The edges of the shapes were turned up. The experimenter hid the remaining shapes from participants. When the solid shapes with broken lines were presented in the task other than SC2, the experimenter placed the shapes so that the broken lines were invisible to the participants. In the flat shapes comparison tasks, the experimenter indicating this pair of shapes, asked participants, “Which shape is larger, this or that”? In the solid shapes comparison tasks, the experimenter asked participants, “Which shape can contain more water, this or that”? In the solid shapes comparison tasks, the experimenter asked participants, “Which shape can contain more water if these are filled with water up to these lines” (SC2), to make the tasks clearer, especially for the kindergartners.

Table 3. Materials for solid shape comparison tasks

<table>
<thead>
<tr>
<th>Order</th>
<th>Task</th>
<th>Stimulus 1 (D×W×H)</th>
<th>Stimulus 2 (D×W×H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SC1</td>
<td>a:10×10×10 b:10×10×12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>SC2</td>
<td>a(L):10×10×10 b(L):10×10×12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>SC3</td>
<td>a:10×10×10 c(D):12×10×10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>SC4</td>
<td>a:10×10×10 c(W):10×12×10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>SC5</td>
<td>b:10×10×12 d(D):12×10×12</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>SC6</td>
<td>b:10×10×12 d(W):10×12×12</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>SC7</td>
<td>e:12×12×12 d(D):12×10×12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SC8</td>
<td>e:12×12×12 d(W):10×12×12</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>SC9</td>
<td>c(D):12×10×10 d(D):12×10×12</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>SC10</td>
<td>c(D):12×10×10 d(W):10×12×12</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>SC11</td>
<td>c(W):10×12×10 d(D):12×10×12</td>
<td></td>
</tr>
</tbody>
</table>

(D:Depth, W:Width, H:Height)

Table 4. Criteria for estimating strategies in flat shape comparison tasks

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Number of dimensions</th>
<th>SA1</th>
<th>SA2</th>
<th>SA3</th>
<th>SA4</th>
<th>SA5</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1</td>
<td>c</td>
<td>s</td>
<td>i</td>
<td>s</td>
<td>c</td>
</tr>
<tr>
<td>W</td>
<td>1</td>
<td>s</td>
<td>c</td>
<td>i</td>
<td>c</td>
<td>s</td>
</tr>
<tr>
<td>DWId</td>
<td>1</td>
<td>c</td>
<td>c</td>
<td>n.a.</td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td>DWlt</td>
<td>2</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>c</td>
</tr>
</tbody>
</table>

(D:Depth, W:Width, lt: Integration; Id: Independent, c: Correct, i: Incorrect, s: Same, n.a.: no answer or no idea)

2.4. Estimation of strategy and the number of dimensions

Given the difficulty of obtaining accurate verbal reports from kindergartners about their reasons for their decisions in the tasks, we estimated strategies and the number of elements that participants used to respond to the tasks based on the criteria shown in Tables 4 and 5.
2.4.1. Flat shapes comparison tasks

In SA1 and SA5, the participants may make aligning shapes, by rotating them 90 degrees. Table 4 shows the criteria we elicited from these task demands. We analyzed the responses of the participants and determines the strategies they used by estimating the elements based on the criteria shown in Table 4. “Depth” meant that participants made their responses by simply comparing depths of shapes. “Width” meant that participants made their responses just by comparing widths of shapes. “Depth and width” meant that participants made their responses by comparing both depths and widths. “Integration” meant that participants made their responses by integrating multiple elements, such as aligning sides of the same length of the shapes in SA3. “Independent” meant that participants made their responses by using multiple elements independently. “Correct” meant that participants correctly chose the larger shape. “Incorrect” meant that participants mistakenly chose the smaller shape or that they indicated that one of the shapes was larger than the other although the shapes were identical. “Same” meant that participants mistakenly answered that both shapes were identical in size though the shapes were not identical. “N.A.” meant that participants gave no answer or that they said that they had no ideas.

We estimated the number of dimensions based on these strategies. The patterns of answers other than those shown in Table 4 were as follows.

When participants could answer correctly in both SA1 and SA5, we counted the strategy as “Depth.” When participants could answer correctly in both SA2 and SA4, we counted the strategy as “Width.” The patterns other than these were counted as “Inconsistent” and the number of dimensions was set as zero.

2.4.2. Solid shapes comparison tasks

In SC3, SC5, and SC8, participants may make correct responses by simply comparing the depth of shapes. In SC4, SC6, and SA7, participants may make correct responses by simply comparing width of shapes. In SC1, SC2, and SA9, participants may make correct responses by simply comparing height (line height in SC2) of shapes. However, SC10 and SC11 require responses involving the integration of multiple elements to make correct responses. Table 5 shows the criteria we elicited from these task demands.

We analyzed the participants’ responses and determined the strategies they used by estimating the elements based on the criteria shown in Table 5. “Line” meant that participants made correct responses in SC1, SC2, and SC9. “Height” meant that participants made correct responses in SC1 and SC9, but failed in SC2 because they compared shapes based on the heights of shapes not on the heights of lines. We estimated the number of dimensions based on these strategies. The patterns of answers other than those shown in
Table 5 were as follows. When participants could answer correctly in SC3, SC5, and SC8, we counted the strategy as “Depth.” When participants could answer correctly in SC4, SC6, and SC7, we counted the strategy as “Width.” When participants could answer correctly in SC1, SC2, and SC9, we counted the strategy as “Line.” When participants could answer correctly in SC1 and SC9, but not in SC2 we counted the strategy as “Height.” The patterns other than these were counted as “Inconsistent” and the number of elements was set as zero. Meanings of “Depth and width,” “Integration,” and “Independent” are the same as those in Table 4.

3. RESULTS

3.1. Percentage of correct answers in each grade

The percentage of correct answers in the shape comparison tasks in each grade are shown in Figure 2.

We conducted a mixed design two-way analysis of variance (ANOVA) with grades of participants as the between-subject variable and types of shapes (flat shapes vs. solid shapes) as the within-subject variable to examine the effects of these factors on the performance in the tasks. The results of the ANOVA are tabulated in Table 6.

The analysis indicated a significant main effect of grades of participants ($F(5, 54) = 10.167, p < .01$). Pair-wise comparisons were conducted using multiple comparisons (Tukey’s method), which showed that the overall scores of the junior, middle, and senior kindergarteners, and the first graders in the elementary school were significantly smaller than those of the second and third graders in the elementary school (Table 7).

3.2. Strategies for shape comparison and the number of dimension

3.2.1. Strategies for flat shape comparison

The number of participants who used each strategy (see Table 8) was estimated based on the criteria shown in Table 4. The number of participants who used the length of the depth side as a cue increased in the middle kindergarteners and then decreased in the senior kindergarteners and the pupils. The number of participants who used the length of the width side was highest in the first graders. The number of participants who used the integrated cues increased with age in the senior kindergarteners and the pupils.

3.2.2. Strategies for solid shape comparison

The numbers of participants used each strategy (see Table 9) were estimated based on the criteria shown in Table 5. In Table 9, the number of participants who used “L” and those who used “H” were merged because these strategies were
Age-Related Changes in Performance and Strategies in an Area and Volume Comparison

141

We removed strategies that none of the participants used from Table 9. Although comparing solid shapes requires participants to integrate three lengths (width, depth, and height) of shapes, very few kindergartners compared shapes using all of them, while all the second graders did. SC2 was designed to examine whether participants use the height of the shape or the height of the water surface as a cue to compare solid shapes.

The numbers of participants who used “H” or “L” strategies were tabulated in Table 10. The table shows that the junior graders compared shapes using the heights of shapes as cues. It also shows that the third graders did use the heights of the water surface as cues.

3.2.3. Number of dimensions

We determined the number of dimensions that participants used as cues in the tasks using the strategies estimated based on the criteria shown in Tables 4 and 5. Figure 3 shows the mean number of elements the participants used in the tasks.

Maratosos (1973) pointed out that children’s ability to compare areas develops in a u-shaped manner. In contrast, Bausano and Jeffrey (1975) and Shimizu (1997) questioned whether three-year-old children could compare shapes considering the area of the shape.

The goal of this study was to examine the development of the ability to compare areas and volumes of shapes by estimating the number of dimensions that children utilized as cues to compare the area and volume of objects. If the participants understood how to obtain areas and volumes by calculation, it could be assumed that the number of dimensions used for the area comparison would be larger than that for the volume comparison, because they understand that area comparison requires two lengths (width and depth), and that the volume comparison requires three lengths (width, depth, and height). Without this understanding, however, the number of dimensions for area comparison might not be significantly larger than that for volume comparison, because the participants would compare shapes by focusing on the length of any side, or by considering visual impressions of shapes.

Thus, we conducted a mixed design two-way ANOVA with grades of participants as the between-subject variable, and types of shapes (flat shapes vs. solid shapes) as the within-subject variable, to examine the effects of these factors on the number of elements the participants used in the tasks.

Table 9. Numbers of participants who exploited each strategy in solid shape comparison tasks in each grade

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Junior</th>
<th>Middle</th>
<th>Senior</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>DWd</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DHl</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WHd</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WHlt</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>DWHlt</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Ucl</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>lc</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 10. Numbers of participants who exploited “Line” strategy and “Height” strategy in solid shape comparison

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Junior</th>
<th>Middle</th>
<th>Senior</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>H</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

(D:Depth,W:Width,L:Length)

Fig 3. Mean number of dimension used by participants in each grade

Table 11. Analysis of variance results for the numbers of elements used

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>91.967</td>
<td>119</td>
<td>0.815</td>
<td></td>
</tr>
<tr>
<td>Grade</td>
<td>29.767</td>
<td>5</td>
<td>5.953</td>
<td>13.284**</td>
</tr>
<tr>
<td>Area×Volume</td>
<td>6.533</td>
<td>1</td>
<td>6.533</td>
<td>13.781**</td>
</tr>
<tr>
<td>Grade×(Area×Volume)</td>
<td>25.600</td>
<td>5</td>
<td>1.173</td>
<td>2.475*</td>
</tr>
<tr>
<td>Error</td>
<td>0.748</td>
<td>54</td>
<td>0.014</td>
<td></td>
</tr>
</tbody>
</table>

** p<.01 * p<.05

Table 4 and 5. Figure 3 shows the mean number of elements the participants used in the tasks.
The results of the ANOVA are tabulated in Table 11. The analysis revealed a significant main effect of participants’ grades, showing that their grades influenced the number of dimensions.

The analysis also revealed a significant main effect of the types of shapes, showing that whether the presented shapes were flat or solid influenced the number of dimensions. In addition, the interaction between the grades and the types of shapes was significant. As the interaction was significant, we carried out simple main effect tests.

The simple main effect of grades on the number of dimensions used in the flat shape comparison tasks was significant ($F(5, 108) = 3.015, p<.05$), in that the number of dimensions the junior and middle kindergartners used was significantly smaller than what the second and third graders used, and the number that senior kindergartners used was significantly smaller than what the third graders used. The simple main effect of grades on the numbers of elements used in the solid shape comparison tasks was significant ($F(5, 108) = 12.441, p<.01$), in that the number of dimensions the junior, middle, and senior kindergartners, and the first graders used were significantly smaller than what the second and third graders used, and that the number the second graders used was significantly smaller than what the third graders used. In addition, the number of dimensions that the junior and middle kindergartners, and the second and third graders used in the flat shape comparison tasks was significantly larger than what was used in the solid shape comparison tasks.

The difference of the number of dimensions between senior kindergartners and the first graders used in the flat shape comparison tasks, and those used in the solid shape comparison tasks, was not significantly different.

4. DISCUSSION

4.1. Correct Responses

4.1.1. Comparison of areas

Sena and Smith (1990) and Maratsos (1973) demonstrated that the correct responses in area comparison tasks decreased from three to five years of age. The participants in this study were kindergartners and elementary school pupils. We examined grade–related differences in performance of the tasks that considered the progression of teaching in elementary school.

As shown in Figure 2, the correct response rate decreased from the junior kindergartners to the middle kindergartners, and then increased with age. The correct response rates in three-, four-, and five-year-old children in the participants of this study were 0.700, 0.560, and 0.580, respectively. These results were in line with those of previous studies, though the experimental tasks in this study were different from those of previous studies. However, the results of multiple comparisons for grade–related differences yielded no significant differences among the junior, middle, and senior kindergartners.

The correct response rate of the junior kindergartners was not significantly larger than that of the middle and senior kindergartners. When we examined age–related differences of the correct response rates, there was a significant difference between the correct response rates of three– and five-year-olds. However, the differences between those of three–year-olds and four–year-olds, and the differences between those of four–year-olds and five–year-olds were not significant. To summarize, the area comparison ability apparently seemed to develop in a u-shaped manner, from kindergartners to pupils in elementary school, even though there was no significant effect of participants’ grades on the rate of correct responses.

4.1.2. Comparison of volumes

The results for correct response rate in the volume comparison tasks did not show a u-shaped change. The rate increased linearly from the junior kindergartners to the second graders in elementary school, then decreased in the third graders. To compare volumes of cubes, it is impossible to compare by considering only single faces of each cube. This requires considering another element in addition to the top face, front face, or the lengths of two sides. This difficulty in the comparison of volumes might prevent the junior kindergartners from outperforming the middle and senior kindergartners in task performance.

4.2. Number of dimensions

As shown in Figure 3, the number of dimensions used in the area and volume comparison tasks increased linearly with age. In particular, there were large increases between the junior kindergartners and the middle kindergartners, and between the first graders and the second graders. If the participants understood that the comparison of flat shapes and solid shapes required integrating the width, depth, and height of shapes, it is
natural to assume that there is a difference between the number of elements used in the area comparison tasks and the volume comparison tasks. As no participants in this study had received instruction on how to calculate the area and volume of objects, they should have compared the shapes based on the lengths of shapes without calculating the areas and volumes.

As shown in Tables 8 and 9, the kindergartners and the first graders tended to use the lengths of a single side of a shape as a comparison cue. They rarely aligned sides of the same length of the shapes and compared lengths of the remaining sides. Further, as shown in the results for task SC2 (Table 10), when examining whether participants used the height of the shape or the height of the water surface as a cue, younger children tended to compare shapes using not the heights of the water surfaces but the heights of the containers as cues.

These results showed that the comparisons of younger children depended on their visual impressions of shapes. The differences among grades, and an interaction between grades and the types of shapes (flat shapes vs. solid shapes), were significant. The number of dimensions junior and middle kindergartners, and the second and third graders used in the flat shape comparison tasks were significantly larger than what was used in the solid shape comparison tasks. Because the number of dimensions that the second and third graders used in the flat shape comparison tasks was approximately two, and those in the solid shapes comparison tasks was approximately three, they most likely compared the shapes by integrating width, depth, and height of shapes. However, as both the number of elements the junior and middle kindergartners used in the flat and solid shape comparison tasks were about one, it implausible that they compared shapes based on the integration of lengths.

Sena and Smith (1990) demonstrated that the rate of children who use the height of shapes in area comparison tasks increases with age. In contrast, the results of this study indicated that kindergartners and first graders use multiple elements; and, that they do not integrate multiple elements simultaneously, but separately.

The results also indicated that the second or third graders integrated multiple elements using strategies that included the aligning shapes through rotation. When we consider these results, and the fact that the correct response rate of area comparison tasks decreased from the junior kindergartners to the middle kindergartners (while that of the volume comparison tasks did not), the interpretation of Bausano and Jeffrey (1975), that three-year-old children compare areas of rectangles based on the area but on the length of the longest side of the rectangles, is a plausible explanation for the phenomenon that area comparison ability develops in a u-shaped manner.

4.3. Instruction of area and volume calculation

The units of area and area calculation is taught in the fourth grade, and the units of volume and volume calculation is taught in the fifth grade.

Area calculation is taught through counting the number of units of one square centimeter, with which a flat shape is filled. Volume calculation is taught through counting the number of units of one cubic centimeter with which a container is filled. Thus, learning about area and volume calculation requires pupils to understand that area is determined by width and depth, and that volume is determined by multiple elements of width, depth, and height. However, the methods taught in schools focus on the learning and utilization of formulas, and assume that the understanding described above has already been developed.

As shown in this study, children have not developed an understanding that area and volume are determined by multiple elements, whereas the number of elements children use increases with age. For example, some of the third graders could not integrate multiple elements. Therefore, this understanding should be formed in the fourth and fifth grade.

5. CONCLUSION

We conducted an experiment to clarify age-related changes in area and volume comparison ability, by examining the elements of shapes used by kindergartners and pupils in an elementary school. As a result, the flat shape comparison ability developed in a u-shaped manner, though the age-related differences were not significant. In contrast, the solid shape comparison ability developed in a linear manner. The number of dimensions used in the comparison tasks increased largely in the first and second grade, as the first and second graders tended to integrate multiple elements, whereas the numbers of dimensions increased with age, irrespective of the types of shapes involved. These results are consistent with previous findings that three-year-olds outperformed four-
five-year-olds in the area comparison, because three-year-old children compare areas of shapes based not on the area, but on the length of the longest side of the shapes. In addition, we discussed the necessity of fostering children’s understanding of the idea that area and volume are determined by multiple elements, in the teaching methods of area and volume calculation.

ACKNOWLEDGEMENTS

We are very grateful to the principals and the teachers of the kindergarten and the elementary school who understood and supported our study.

REFERENCES


