A New Approach to the Dynamics of Roller Draft

Part 1: The Basic Scheme and an Application to a Model System

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Abstract: A theory of the dynamics of a sliver or roving under roller draft is presented in a general form based on the scheme of the fluid mechanics. The motion of a fiber bundle being drafted is described by three differential equations; one relates the local elongation of the bundle to the velocity distribution along the bundle, and the other two govern the conservation of mass and momentum. The momentum flux is composed of three terms due to mass flow, tension, and inter-fiber slip. The last term, the momentum transfer by interaction among fibers, has no trivial analogue in any of traditional theories proposed earlier, though indirectly it might connect with the force for pulling out fibers of the bundle. A hypothetical system simulated by a viscoelastic fluid flow is introduced as a standard model and its stationary motion is examined for illustrating how the basic scheme works, how the velocity and tension are formed along the bundle, how the drafting force depends on the system parameters, etc.

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1. Introduction

Roller draft is one of the vital elements that compose the whole process of yarn spinning, together with doubling, twisting, and winding. Because draft process compared with others affects most crucially the quality of spun yarns as final products, a great number of workers have wrestled with the technical problems of roller draft. Many theoretical approaches also have been tried in the analysis of drafting wave and irregularity [1] and in the mechanics of drafting [2-5].

Viewing from a pure-theoretical standpoint, none of the earlier authors seems to have presented a maximal scheme on the basis of which one could formulate any mechanical problem related to roller draft by following the underlying principles of the Newtonian mechanics of continua such as fluid, elastic body, etc. Some authors have made use of the law of mass conservation (*i.e.*, equation of continuity) when analyzing kinematically how fibers behave as the total in the drafting zone [3]. However, the law of motion (*i.e.*, equation of motion) seems to have been considered only in a restricted sense; in fact, any of the earlier workers has treated the behaviors of fibers in the drafting zone not as a dynamical problem but only as a static-mechanical one [3-5], and besides not given an explicit formalism that enables potentially to treat the dynamic relationship among the deformation of a fiber bundle (sliver or roving) under draft and the local physical quantities such as velocity and tension formed along the bundle.

In this series of articles, we try to formulate and cast a new light upon the roller draft dynamics by having recourse fully to the fundamental principles in the mechanics of continuous media. We assign this Part 1 to presenting a general framework of the theory as well as illustrating by use of an idealized model how it works for stationary drafting, and the second Part 2 to investigating the problems of nonstationary motion and instability also with the same model. In a subsequent article, we plan to develop a constitutive theory that can describe the deformational property of a fiber bundle, being necessary when applying the basic scheme to practical cases.

2. Fundamental scheme

2.1 Local variables

A fiber bundle to be under draft process is usually termed differently corresponding to at what stage it is from a card web to a spun yarn; the bundle is called a *sliver* at earlier stages but later a *roving* after given weak twist. We call however both sliver and roving commonly sliver in this article.

Suppose a cross-section S located at a point x on a sliver under draft between the back rollers B and the front rollers F, where x is the distance from the nip point of B (Fig. 1). Let $N(x, t)$ be the number of the fibers that cross with S at an instant t and called the *local number of fibers*, and $V(x, t)$ be the average of the instantaneous speeds which the fiber cross-sections on S have at time t and called the *local velocity of sliver*.

Fig.1 Geometry of drafting unit.

Let $M(x, t)$ be the local apparent linear density, *i.e.*, the mass per unit length at each position, of the sliver; M depends on x and t reflecting what irregularity the pre-draft (delivery) sliver has along length as well as how the sliver is deformed at (x, t) when drafted. We now introduce the *apparent linear density of fiber*, $m(x, t)$, by

$$m \equiv M/N \quad \text{or} \quad M \equiv mN. \quad (1)$$

Note here that m also depends in general on (x, t) , and coincides with the linear density of fiber only when the fiber is straight in the sliver.

Denoting now the local elongational strain of the sliver by γ , we can derive the following relation (see APPENDIX 1):

$$D\gamma/Dt = \partial V/\partial x, \quad (2)$$

where D/Dt is a differential operator defined by $D/Dt \equiv \partial/\partial t + V\partial/\partial x$.

In a stationary case, the term $\partial\gamma/\partial t$ vanishes and Equation 2 reduces to

$$Vd\gamma/dx = dV/dx. \quad (2')$$

Integrating this leads to

$$\gamma = \ln(V/V_c), \quad (3)$$

where V_c is an arbitrary positive constant which we have to determine when solving the equations of motion developed below.

2.2 Equations of motion

The mass flux f_{ms} at x , *i.e.*, the mass transferred via cross-section S per unit time, is expressed by

$$f_{ms} = MV = mNV. \quad (4)$$

The law of mass conservation can be formulated in general as $\partial M/\partial t = -\partial f_{ms}/\partial x$, so that we have the continuity equation [7]

$$\frac{\partial(mN)}{\partial t} = -\frac{\partial(mNV)}{\partial x}. \quad (5)$$

On the other hand, the momentum flux f_{mm} at x , *i.e.*, the momentum transferred via S per unit time, is composed of three terms as

$$f_{mm} = -T + mNV^2 - S, \quad (6)$$

where $-T$ is the momentum transfer due to the tension (tensile force) T acting on the cross-section S , mNV^2 is the momentum brought by the mass flow, and $-S$ is the momentum which is transferred by inter-fiber slip; we name the quantity S the *slip stress* (see APPENDIX 2 and also refer to [7]). The momentum density, *i.e.*, the momentum contained per unit length of the sliver, is MV or mNV . Using the general formalism of momentum conservation, that is, $\partial(MV)/\partial t = -\partial f_{mm}/\partial x$, we have the equation of motion

$$\frac{\partial(mNV)}{\partial t} = -\frac{\partial(mNV^2)}{\partial x} + \frac{\partial T}{\partial x} + \frac{\partial S}{\partial x}. \quad (7)$$

2.3 Boundary conditions

Let the surface speed of the back rollers B be denoted by V_b and also, that of the front rollers F by V_f . We call V_b the *delivery speed* and V_f the *spinning speed*. The velocity has to satisfy the following conditions.

$$V(0, t) = V_b \quad \text{and} \quad V(L, t) = V_f \quad (8)$$

for any instant t , where L is the distance between B and F , *i.e.*, the *roller gauge*.

When a delivery sliver has a linear density $M_b(t)$ at the back nip point B just when it passes over the point, the following should be satisfied:

$$m(0, t)N(0, t) = M_b(t). \quad (9)$$

Notice that M_b has to be treated as a given function of time t when the delivery sliver is not uniform along length.

2.4 Stationary case

When a delivery sliver is uniform and the condition of draft is maintained constant, we can expect a stationary processing. In such a case, m , N , and V are all independent of time t and depend only on x , so that Equations 5 and 7 reduce to

$$\frac{d(mNV)}{dx} = 0 \quad \text{and} \quad -\frac{d(mNV^2)}{dx} + \frac{dT}{dx} + \frac{dS}{dx} = 0.$$

Therefore, we have

$$mNV = \text{const.} \quad \text{and} \quad T + S - mNV^2 = \text{const.} \quad (10)$$

2.5 Constitutive equations

Three Equations 2, 5, and 7 accompanied by the boundary conditions of Equations 8 and 9 form a fundamental set of differential equations which govern the along-sliver distributions of $V(x, t)$ and $N(x, t)$. To

solve this boundary-value problem, however, we need in general additional three constitutive relations which express the sliver tension T , the slip stress S , and the apparent linear density of fiber, m , as functions of variables N and V as well as other quantities which are derived from them, such as their gradients, higher derivatives, and/or possibly, integrations.

To obtain these constitutive expressions, we have to describe the deformation mechanics of a sliver in terms of the structural parameters, such as fiber length, fiber crimp, apparent density of sliver, twist, fiber parallelity, inter-fiber contact, etc. We plan to present a micromechanical theory to this problem in Part 3 of our serial studies. In the present part, we confine ourselves only to illustrating and understanding how our underlying scheme works, by using a hypothetical model as below.

3. Model analysis

3.1 Standard model

Let an idealized system named the *standard model* be defined by the following assumptions:

1) The apparent linear density of fiber, m , is constant throughout draft process.

2) The tension of sliver, T , is given by

$$T = GN\gamma \quad \text{or} \quad T = GN \ln(V/V_c), \quad (11)$$

where γ is the elongational strain of sliver (see Equation 2); constant coefficient G may be assumed as the elongational rigidity averaged per fiber.

3) The slip stress S is given by (see Equation 2)

$$S = \eta ND\gamma/Dt \quad \text{or} \quad S = \eta N\partial V/\partial x, \quad (12)$$

where constant η may be interpreted as the viscosity coefficient averaged per fiber.

The first assumption above means neglect of the change in m due to decrimping deformation which fibers having crimps should undergo when the sliver is drafted. The second assumption will be reasonable if the fibers stretched show Hookean elasticity. The third corresponds to simulating a sliver as a viscous fluid [7], since the viscous stress is proportional to the strain rate $D\gamma/Dt$ for Newtonian flow. Note here that the strain rate, *i.e.*, the time derivative of the strain, at a point on a running sliver is given not by $\partial\gamma/\partial t$ but by $D\gamma/Dt$ (see APPENDIX 1).

Although the standard model thus defined cannot bear any of the structural factors characterizing the sliver as a fiber mass, it will be useful for clarifying essential features proper to the dynamics of roller draft.

In the rest of this article, we analyze the stationary motion of the standard model to get fundamental aspects on the draft dynamics.

3.2 Stationary equations

We can rewrite Equation 10 for stationary cases, as

$$NV = N_b V_b = N_f V_f, \quad (13)$$

$$(T - T_b) + (S - S_b) - m(NV^2 - N_b V_b^2) = 0, \quad (14)$$

and especially,

$$(T_f - T_b) + (S_f - S_b) - m(N_f V_f^2 - N_b V_b^2) = 0, \quad (15)$$

where V_b, N_b, T_b , and S_b are the values the corresponding variables assume at the back rollers ($x = 0$), and V_f, N_f, T_f , and S_f are those at the front rollers ($x = L$). The boundary conditions of Equation 8 reduce to

$$V(0) = V_b \quad \text{and} \quad V(L) = V_f. \quad (16)$$

Using Equations 11 and 12, we obtain the following differential equation from Equation 14:

$$G \left(\frac{V_b}{V} \ln \frac{V}{V_c} - \ln \frac{V_b}{V_c} \right) + \eta \left(\frac{V_b}{V} \frac{dV}{dx} - V'(0) \right) - mV_b(V - V_b) = 0, \quad (17)$$

where we eliminated variables N and N_b with noting the relation $N = N_b V_b/V$ (Equation 13); and $V'(0)$ is the gradient of $V(x)$ at $x = 0$. Notice that undetermined constants V_c and $V'(0)$ in Equation 17 have to be so chosen that $V(x)$ satisfies Equation 15 as well as Equation 16.

3.3 Dimensionless expression

Before solving Equation 17, let this be transformed into a dimensionless expression by introducing the following notations:

$$\xi \equiv x/L, \quad \phi \equiv V/V_b, \quad \phi_c \equiv V_c/V_b,$$

$$\alpha \equiv LV'(0)/V_b, \quad \tau \equiv \eta V_b/GL, \quad \mu \equiv mV_b^2/G. \quad (18)$$

This normalization casts Equation 17 into

$$\frac{d\phi}{d\xi} = \alpha\phi + \frac{\mu}{\tau}\phi(\phi - 1) - \frac{1}{\tau} \left[\ln \frac{\phi}{\phi_c} + (\ln \phi_c)\phi \right]. \quad (17')$$

The boundary conditions of Equation 16 are normalized as

$$\phi(0) = 1 \quad \text{and} \quad \phi(1) = \delta, \quad (16')$$

where δ is the *draft ratio*, that is,

$$\delta \equiv V_f/V_b. \quad (19)$$

Correspondingly, we introduce the dimensionless variables for T and S as

$$\widehat{T}(\xi) \equiv T(x)/G \quad \text{and} \quad \widehat{S}(\xi) \equiv S(x)/G. \quad (20)$$

Equations 11, 12, and 15 reduce to

$$\widehat{T} = N_b \frac{1}{\phi} \ln \frac{\phi}{\phi_c}, \quad \widehat{S} = N_b \tau \frac{1}{\phi} \frac{d\phi}{d\xi}, \quad (11', 12')$$

and

$$[\widehat{T}(1) - \widehat{T}(0)] + [\widehat{S}(1) - \widehat{S}(0)] - N_b \mu (\delta - 1) = 0. \quad (15')$$

When solving Equation 17' as a differential equation, we have to notice the following two points. Undetermined constants ϕ_c and α in Equation 17' have to be so chosen that velocity $\phi(\xi)$ satisfies Equations 15' and 16'. Furthermore, the gradient $d\phi/d\xi$ cannot take any negative value because we have no physical reason for expecting that the local velocity ϕ may decrease somewhere within the draft zone. This implies that a solution having physical meaning can exist only in a restricted range of system-parameter combination (δ, τ, μ) .

3.4 Characteristic parameters

As is seen from Equation 17', our dynamical system is characterized by three dimensionless constants δ , μ , and τ . The first, the draft ratio δ , is given as a parameter defining the condition of drafting. The second, μ , may be assumed as a measure which represents the relative weight of the kinetic energy of fiber motion to the elastic energy of fiber extension, or the relative effect of fiber inertia to fiber elasticity. The third parameter τ can be related to the ratio of the rates at which two typical motions proceed, as follows:

We can define two characteristic time constants related to our system. One is

$$\tau_d \equiv L/V_b. \quad (21)$$

This constant is a rough measure of the time which a fiber takes when passing through the draft zone; the exact pass time should be of course less than this value. We call τ_d the *draft time constant*. Another time constant is defined by

$$\tau_r \equiv \eta/G. \quad (22)$$

Let this be named the *relaxation time*, which we may assume as a measure of the time that elapses before the tension in a stretched fiber decays appreciably.

The dimensionless parameter τ in Equation 17' is related to these two characteristic times by

$$\tau = \tau_r/\tau_d. \quad (23)$$

Thus, τ is the relaxation time relative to the draft time constant τ_d . As is seen later, τ together with δ plays a key role for the stability of solution.

3.5 Practical ranges of parameters

This article focuses only on presenting the basic framework and illustrating how it works with the standard model for various test cases. We do not touch upon any actual drafting process, since the standard models for T and S seem distant far from those that can successfully simulate reality. However, referring to experimental information available is useful for restricting the test ranges of the model parameters near to practical ones. So, we here survey the typical orders of magnitude of the relevant parameters.

A typical order of mN_b , the apparent linear density of a delivery sliver is around 10^{-1} kg/m at stages of sliver and 10^{-2} kg/m at stages of roving ([6, 8]).

The rigidity GN_b of a sliver may be estimated from the initial slope of the force-elongation relation. From the experimental curves for polyester slivers [6], GN_b is estimated typically to be about 10 N when $mN_b \approx 0.4$ kg/m. We may roughly assume $G/m \approx 200$ N·m/kg for sliver. As to roving, we failed in finding such information about the rigidity.

For cotton spinning, the draft ratio δ is chosen within a range up to about 2 and the roller gauge L is set between 30 mm and 50 mm [8]. Okamura's work [6] for polyester slivers are concerned with δ up to 10 and L of around 100 mm. The order of V_b , the delivery speed is about 1 m/s for cotton spinning [8].

If a representative value of V_b is assumed as 1 m/s, the draft time constant τ_d ($\equiv L/V_b$) is nearly 50 or 100 times 10^{-3} s for L being 50 or 100 mm, and the parameter μ ($\equiv mV_b^2/G$) is about 5×10^{-3} .

As to the viscosity coefficient η per fiber or ηN_b per sliver, we cannot find any experimental information in earlier works. The viscosity η means the rate at which a fiber moving faster accelerates neighboring slower fibers and gives them additional momenta by friction and/or collision. It may possibly have some relationship to the forces necessary for pulling out fibers from the sliver. However, we have no logic to estimate η or ηN_b from the earlier authors' data (refer also to APPENDIX 2).

Remember that as was pointed out earlier, parameter τ represents how long it takes, relatively to the draft time constant τ_d , for a fiber tension to decay appreciably. This suggests that as the reference value of τ , unity is of theoretical interest; if τ is far above (under) unity, the local tension formed in the sliver and the draft force as well will be great (small). In the numerical case study, we give τ a test value in various orders of magnitude and examine the behavior of the standard model system.

3.6 Numerical case studies

3.6.1 Calculation procedure

The differential Equation 17' seems unsolvable analytically, so that we have recourse to numerical method using the following algorithm of calculation:

1. Give a set of values to (δ, τ, μ) .
2. Give a tentative value to ϕ_c .
3. Give a tentative value to α .
4. Let $\phi(0) = 1$ and solve Equation 17' to get function $\phi(\xi)$ up to $\xi = 1$.
5. If $\phi(1) = \delta$ is satisfied then advance, else modify the value of α and return to procedure 4.
6. If Equation 15' is satisfied then end, else modify the value of ϕ_c and return to procedure 4.

3.6.2 Dependency on δ

Fig. 2 shows the distributions of V , T , S , and mV^2 for various draft ratios δ with $\tau = 4$ and $\mu = 0.005$ being fixed. The figure illustrates the followings:

For any δ value (up to 10), the tension takes a maximum at a point which shifts toward the back rollers with increasing δ (Fig. 2b). When δ is near unity, the tension is almost uniform along the sliver like in an ordinary static test in which a sliver sample of definite length is extended with its ends being gripped. As δ is increased, the difference from the static behavior becomes remarkable; in the right-hand side to the maximum, T decays monotonously but in the opposite side, once it goes down but then up.

The slip stress S has a minimum that corresponds to the maximum in tension and contrary to T , goes up monotonously with increasing δ (Fig. 2c).

The momentum flux brought by mass flow, mV^2 , increases monotonously both with δ and with x as shown in Fig. 2d.

(2a) Velocity.

(2b) Tension.

(2c) Slip stress.

(2d) Momentum flux due to mass flow.

Fig. 2 Distributions of local variables along the sliver; velocity (a), tension (b), slip stress (c), and momentum flux due to mass flow (d).

3.6.3 Dependency on τ

Fig. 3 depicts how the distributions of V , T , and S depend on τ the relaxation time relative to the draft time constant, for $\delta = 3$ and $\mu = 0.005$ being fixed.

The τ -dependency of tension distribution shows a notable behavior (Fig. 3b); the curve of T goes up with increasing τ in a range of smaller values up to around 0.6, but then jumps down discontinuously, and at near 0.7, turns back toward increase. Analyzing in detail a wider ranged τ -dependency of T can verify that such jumps break out at other different values of τ , though here we do not present them explicitly. Such discontinuous behaviors suggest occurrence of the *bifurcation of solution*; in general, bifurcation may occur in a dynamical system containing parameters [9], *i.e.*, even if a parameter characterizing the system be varied continuously, a solution being stable within a range may become unstable at a specific value of the parameter

(3a) Velocity.

(3b) Tension.

(3c) Slip stress.

Fig. 3 Dependencies of variables on relaxation time; velocity (a), tension (b), and slip stress (c).

and there leaps suddenly to a different solution being more stable.

In marked contrast to T , the distribution curve of S does not show such discontinuity and goes up monotonously with τ (Fig. 3b).

3.6.4 Dependency on μ

Fig. 4 shows the effects of μ the mass of the sliver relative to the rigidity, on the distributions of T and S , for $\delta = 3$ and $\tau = 4$. Increasing μ lowers the tension T over the whole range of x and shifts the point of maximum toward the front (Fig. 4a); inversely, the momentum transfer S increases and shifts toward the back in an intermediate region but behaves oppositely in others near the back and front rollers (Fig. 4b).

Fig. 4a Dependency of tension on relative mass.

Fig. 4b Dependency of slip stress on relative mass.

3.6.5 Drafting force

Let the force that the rollers drag the sliver under draft be called the *back drafting force* (BDF) at the back and the *front drafting force* (FDF) at the front, with denoting them by F_b and F_f , respectively. These are related to the boundary values of tension T as

$$F_b = T(0) \quad \text{and} \quad F_f = T(L).$$

Fig. 5 illustrates how BDF and FDF depend on the draft ratio for a few values of τ . From this we see that BDF differs greatly from FDF in magnitude except for a narrow range of small δ ; that both forces have cusp-like peaks at δ values near unity (Fig. 5a); and that with δ increasing over the peaks, FDF decays monotonously but BDF once goes down and then up via a minimum (Fig. 5b). Furthermore, it is noticeable

(5a) $1 < \delta \leq 3$.

Fig. 5 Dependency of drafting force on draft ratio; for $1 < \delta \leq 3$ (a) and for $3 \leq \delta \leq 10$ (b).

that the δ -dependency curves show leaps at several values of δ . These jumps may be attributed to the bifurcation of solution occurring at specific δ values.

Fig. 6 depicts the τ -dependency of drafting force separately for $1.2 \leq \delta \leq 2.0$ (a); $2.0 \leq \delta \leq 10.0$ and $1.3 \leq \tau \leq 2.4$ (b); and $2.0 \leq \delta \leq 10.0$ and $2.4 \leq \tau \leq 10.0$ (c). The τ -dependency curves also leap at several values of τ . Jump occurring at a smaller τ is the greater in height. The value of τ at which the first leap appears seems to have a systematic dependency on δ , though we need further detailed investigation for identifying it.

As to the μ -dependency of drafting force, as is seen from Fig. 4a, both BDF and FDF decay monotonously with μ ; there is seen no notable feature, so we omit plotting a figure.

4. Concluding remarks

Following after the general scheme of the fluid mechanics, we formulated the dynamics of a fiber bundle under roller draft in terms of three differential equations. The first equation relates the extensional deformation of the bundle as a continuum to the local macroscopic velocity which is defined as the average speed of the fibers passing through a cross-section of the bundle; the second represents the law of mass conservation; and the third is the equation of motion which is derived naturally from the law of momentum conservation.

In the expression of the total momentum flux, besides the fluxes due to mass flow and tensile impulse,

(6a) $1.2 \leq \delta \leq 2.0$.

(6c) $2.0 \leq \delta \leq 10.0$ and $2.4 \leq \tau \leq 10.0$.

(6b) $2.0 \leq \delta \leq 10.0$ and $1.3 \leq \tau \leq 2.4$.

Fig. 6 Dependency of drafting force on relaxation time; for $1.2 \leq \delta \leq 2.0$ (a); $2.0 \leq \delta \leq 10.0$ and $1.3 \leq \tau \leq 2.4$ (b); and $2.0 \leq \delta \leq 10.0$ and $2.4 \leq \tau \leq 21.5$ (c).

we took into account the flux which is transferred by frictional interaction among fibers. This term corresponding to the viscous momentum flux in the fluid dynamics has no trivial analogue in any of earlier theories of roller draft. Indirectly it might connect with the force for pulling out fibers of the bundle, to which the earlier authors' attentions were devoted in their kinetic analyses of draft process, but no theory to realize this has been explored so far. We plan to develop a micromechanics to make up the slip stress, by treating the elemental motion and deformation in a fiber bundle being roller-drafted.

To illustrate how our basic scheme should be handled, we assumed an idealized system simulated by a viscous fluid having Hookean elasticity and tested its stationary motion by numerical calculation. The results suggested notable information which has not been identified evidently by earlier workers, as follows:

The distribution of local tension along the sliver being drafted has a maximum point in the draft zone. This has been pointed out by Okamura in his doc-

toral thesis [6], where the tension distribution is made up of the forces for pulling-out fibers which are estimated sectionally for short elemental portions of the sliver under draft. However, his procedure of making up the tension is rather unsophisticated from a theoretical viewpoint and the result he obtained seems appreciable only in a qualitative sense.

The back drafting force and the front one are not necessarily the same. This seems not to have been clearly recognized by earlier authors; presuming from the fact that drafting force is usually detected only at the front rollers, most workers might have been convinced that both forces would coincide. This mistake may be possibly caused by confusion between the static elongation of a definite sliver specimen with its terminals being fixed and the dynamic extension without fixation at the terminals but under incessant renewal of a portion to be stretched. Thus, we can conclude that simple static-mechanical consideration may fail in describing the dynamics of roller draft.

The solution of the standard model bifurcates when some of the system parameters are varied continuously. This means that stable steady motion can be realized only within restricted ranges of parameters and the same mechanical variable may assume discontinuously different values in various ranges. Such catastrophic phenomena will be of theoretical interest as typical problems in the nonlinear dynamics, though the model may be far from real draft systems. Further detailed test calculations also revealed that Equation 17' cannot be solved for τ values less than about 0.2, though here we do not present the systematic results about it. We expect that these problems may have essential connections with the stable draftability and draft irregularity.

The model analysis suggests that drafting force shows a steep rise in a range of low draft before falling down with increasing draft. Similar behavior is seen in some of the actual data that Ogawa and Ishihara [10] reported for cotton slivers. No author seems to have theoretically elucidated why such steep increase occurs in drafting resistance, though the subsequent decrease can be understood reasonably, for example, by Hannah's idea [5]. Our theory may have a potential to clarify this problem.

APPENDIX 1

In relation to the strain rate, *i.e.*, the time derivative of the strain γ , two expressions can be derived as follows:

1) Suppose any point P located at x on the sliver

at an instant t . Let $\gamma(x, t)$ be the local sliver strain around P. After a short time lapse of Δt , point P running with velocity V arrives at point P' cited at $x + V\Delta t$. The local sliver strain around P' at $t + \Delta t$ is expressed by $\gamma(x + V\Delta t, t + \Delta t)$. The rate of strain change per unit time can be evaluated by

$$\lim_{\Delta t \rightarrow 0} \frac{\gamma(x + V\Delta t, t + \Delta t) - \gamma(x, t)}{\Delta t}.$$

Calculating this limit by an ordinary mathematical technique and denoting it by $D\gamma/Dt$, we have

$$D\gamma/Dt = \partial\gamma/\partial t + V\partial\gamma/\partial x. \quad (A1)$$

2) Besides P at x , imagine another point Q at $x + \delta x$ on the sliver at the same instant t . After a short time lapse of Δt , point P running with velocity $V(x, t)$ arrives at point P' located at $x + V(x, t)\Delta t$. On the other hand, point Q with velocity $V(x + \delta x, t)$ arrives at point Q' cited at $x + \delta x + V(x + \delta x, t)\Delta t$. The portion PQ of length δx is transformed to P'Q' of length $\delta x + [V(x + \delta x, t) - V(x, t)]\Delta t$, so that the increment in length equals $[V(x + \delta x, t) - V(x, t)]\Delta t$. The incremental strain $\Delta\gamma$ in the time lapse Δt is evaluated by

$$\Delta\gamma = \lim_{\delta x \rightarrow 0} \frac{[V(x + \delta x, t) - V(x, t)]\Delta t}{\delta x} = \frac{\partial V}{\partial x} \Delta t,$$

and hence the rate of strain per unit time, $D\gamma/Dt \equiv \lim_{\Delta t \rightarrow 0} \Delta\gamma/\Delta t$, can be expressed as

$$D\gamma/Dt = \partial V/\partial x. \quad (A2)$$

APPENDIX 2

Suppose a short portion of a sliver, SS', which is between two cross-sections S at x and S' at $x + \delta x$. The momentum contained within SS' equals around $MV\delta x$, since the portion brings mass of $M\delta x$ with a velocity near to V . If no interaction exists among fibers, the rate of change in this local momentum per unit time, *i.e.*, $\partial(MV\delta x)/\partial t$, is composed of two terms as follows:

One originates from the mass flow. In a unit time lapse, mass flows in (with velocity $V(x, t)$) via S by the amount of $M(x, t)V(x, t)$ and flows out (with velocity $V(x + \delta x, t)$) via S' by the amount of $M(x + \delta x, t)V(x + \delta x, t)$, so that momentum flows in by $M(x, t)[V(x, t)]^2$ and flows out by $M(x + \delta x, t)[V(x + \delta x, t)]^2$. The net increment in momentum per unit time is hence evaluated as

$$\begin{aligned} & M(x, t)[V(x, t)]^2 - M(x + \delta x, t)[V(x + \delta x, t)]^2 \\ & \approx -\frac{\partial(MV^2)}{\partial x} \delta x. \end{aligned} \quad (A3)$$

Another comes from the tension generated by elongation in the mass flow. The cross-section S is dragged backward by tension $T(x, t)$ and S' forward by $T(x + \delta x, t)$, so that the net force acting on the portion SS' is equal to

$$T(x + \delta x, t) - T(x, t) \approx \frac{\partial T}{\partial x} \delta x. \quad (\text{A4})$$

This force means just the impulse that acts on the portion per unit time. This impulse also increases the momentum within the portion.

Thus, the conservation of momentum is formulated as

$$\frac{\partial(MV\delta x)}{\partial t} = -\frac{\partial(MV^2)}{\partial x}\delta x + \frac{\partial T}{\partial x}\delta x,$$

and δx being eliminated from this leads to

$$\frac{\partial(MV)}{\partial t} = -\frac{\partial}{\partial x}(MV^2 - T). \quad (\text{A5})$$

The expression $MV^2 - T$ that appears under gradient operator $\partial/\partial x$ is called in general the *momentum flux*, which means what amount of momentum is transferred per unit time via a cross-section in the flow [7].

To take into account interaction among fibers, we have to add to the momentum flux above, the third term $-S$ which expresses the momentum transferred per unit time by frictional interaction between fibers. (This term due to inter-fiber friction is an analogue of the flux term due to viscous stress in the theory of the fluid dynamics [7].) Thus, the total momentum flux is represented as $MV^2 - T - S$ (Equation 6).

To avoid erroneous interpretation which may be taken about the momentum flux due to fiber slip, we call readers' attention to the followings:

The quantities S and T appear as parallels with the same dimension in the momentum-flux expression. Although any of T and S implies a *stress* (being not normalized per unit area), the stress T acts normal to a cross-section of a sliver but the stress S does not so. Therefore, differing from T , S has no direct connection with the drafting force. The stress S is related to the local tension T only in an implicit manner through the

dynamics described by Equations 2, 5, and 7. In addition, S has no explicit relation to the force necessary to pull out fibers from a sliver.

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