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ABSTRACTS FROM THE TRANSACTIONS

HYDRATION OF THE FIBER III.

Fluctuation of Fiber Length by the Change of Atmospheric Humidity and its Time Lag.

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Theoretical Part

In this paper, the author investigated a sorption of water by fiber, assuming that the shape of the fiber is macroscopic homogenous cylinder. The micellar structure of fiber is not considered in this case.

Assume that the change of the atmospheric humidity and the density of moisture on the surface of fiber is represented with a simple harmonic motion as follows,

$$\rho = \rho_0 \sin \omega t \quad \dots\dots\dots(1)$$

where ρ = density of moisture, $\rho_0 = \frac{1}{2}$ amplitude of the change,
 t = time, $\omega = 2\pi/T$ (T = Period),

then this change diffuses into the fiber and the fluctuation in length of fiber is caused by the water content and its distribution in the fiber.

The general equation of diffusion as to the cylinder co-ordinate is as follows (ρ is independent to θ and z)

$$\partial\rho/\partial t = K(\partial^2\rho/\partial r^2 + 1/r \cdot \partial\rho/\partial r) \quad \dots\dots\dots(2)$$

r, θ, z = cylinder co-ordinates, t = time,
 ρ = water content in the unit volume of fiber,
 K = coefficient of diffusion.

Assuming that the radius of fiber is R and ρ is the function of t and $(R-r)$, we have

$$\rho = Ae^{\alpha t + \beta(R-r)} \quad \dots\dots\dots(3)$$

$$\alpha = K(\beta^2 - \beta/r)$$

$$\beta = \frac{1}{2r} (1 \pm \sqrt{1 + 4r^2\alpha/K})$$

put $\alpha \equiv \pm i\gamma$

Substituting (4) & (5) in (3)

$$\rho = Ae^{\pm i\gamma t + \frac{(R-r)}{2r}(1 \pm \sqrt{1 + 4r^2\alpha/K})}$$

put $M = \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4r^4\gamma^2}{K^2}}}$

$$\therefore 2M\sqrt{M^2+1} = \frac{4r^2\gamma}{K}$$

$$\therefore \sqrt{1 \pm 4r^2\gamma i/K} = \sqrt{M^2+1} \pm Mi$$

$$\begin{aligned} \rho &= Ae^{\pm i\gamma t + \frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1} \pm Mi)} \\ &= Ae^{\frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1})} e^{\pm i\left(\gamma t + \frac{(R-r)}{2r}M\right)} \\ &= Ae^{\frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1})} \left[e^{i\left(\gamma t + \frac{(R-r)}{2r}M\right)} \pm e^{-i\left(\gamma t + \frac{(R-r)}{2r}M\right)} \right] \\ \rho &= Be^{\frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1})} \sin\left(\gamma t + \frac{(R-r)}{2r}M\right) \dots\dots\dots(10) \end{aligned}$$

$$\rho = Ce^{\frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1})} \cos\left(\gamma t + \frac{(R-r)}{2r}M\right) \dots\dots\dots(11)$$

Boundary condition;

at $r = R$
 $\rho = \rho_0 \sin \omega t$
 put $r = R$ in (11) $\rho = C \cos \gamma t$
 put $r = R$ in (10) $\rho = B \sin \gamma t$
 $\therefore \rho_0 = B \quad \gamma = \omega$

$$\therefore \rho = \rho_0 e^{\frac{(R-r)}{2r}(1 \pm \sqrt{M^2+1})} \sin\left(\omega t + \frac{(R-r)}{2r}M\right) \dots\dots\dots(12)$$

Substituting (7), (8) and $\gamma = \omega$ in (12)

$$\rho = \rho_0 e^{\frac{(R-r)}{2r}\left(1 \pm \sqrt{\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4r^4\omega^2}{K^2}}}\right)} \sin\left(\omega t + \frac{(R-r)}{2r}\sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} \pm \frac{4r^4\omega^2}{K^2}}}\right)$$

The amplitude of this curve at r is represented with

$$\rho_0 e^{\frac{(R-r)}{2r}\left(1 \pm \sqrt{\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4r^4\omega^2}{K^2}}}\right)}$$

If it is assumed that ρ is maximum when n is even in $\omega t = (2n+1)\pi/2$, and that ρ is minimum when n is odd in $\omega t = (2n+1)\pi/2$, then ρ will be minimum on the surface at the time when $\omega t = 3\pi/2$. The time t , when ρ is minimum at the point r is given as follows,

$$t = \frac{3\pi}{2\omega} - \frac{(R-r)}{2r\omega} \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4r^4\omega^2}{K^2}}}$$

$$\omega = 2\pi/T$$

$$\therefore \text{ Time lag } L = - \frac{(R-r)T}{4\pi r} \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4\pi^2 r^4}{K^2 T^2}}}$$

$$\text{Velocity } V = (R-r)/L = - \frac{4\pi r}{T \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4\pi^2 r^4}{K^2 T^2}}}}$$

$$\text{Wave length } \lambda = VT = - \frac{4\pi r}{\sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4\pi^2 r^4}{K^2 T^2}}}}$$

at $r = R/2$

$$L = - \frac{T}{4\pi} \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4r^4\omega^2}{K^2}}}$$

If it is assumed that the time lag at $R/2$ represents the time lag of change in fiber length by the periodic change of humidity, the following results are obtained.

For the same time lag in the two fibers having each coefficients of diffusion K_1, K_2, R_1 and R_2 must be related as follows

$$\pi^2 R_1^4 / K_1^2 = \pi^2 R_2^4 / K_2^2$$

$\pi R_1^2 = \text{sectional area } A_1$, and $\pi R_2^2 = A_2$

$$\therefore A_1 : A_2 = K_1 : K_2$$

It is clear that the fiber thread having a large coefficient of diffusion, for example viscose silk thread, has a small time lag for the same denier of thread.

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