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COLOR ORDER AND WEAVE ON A GIVEN COLOR-AND-WEAVE EFFECT

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Abstract

An efficient algorithm is developed for determining the color ordering of the warp and weft threads which yields a given pattern of color-and-weave effect. The conditions for the interlacement matrix determined by the color order of the threads are also given. All of the weaves satisfy the conditions which can be briefly represented by a design of "conditional weave".

The pattern of a two colored color-and-weave effect can be classified into three types by an adaptation of the Newton's⁸⁾ weave classification scheme. It was found that only type 3 (that classified as a multilayer weave without stitching) is a pattern suitable for color-and-weave effect.

Introduction

The color-and-weave effect is an effect that can be generated by a special arrangement of colors of warp and weft in woven fabrics, and is of practical importance for textile designers. There are many common textile fabrics in which the effect can be brought about, and a lot of literature¹⁾⁻⁴⁾ are dealing with such effects for a variety of fabric weaves and color orders of threads. In Fig. 1 and Fig. 2, some familiar color-and-weave effects and the corresponding weave and color order are diagrammatically shown. The color order of weft and warp is shown on the left hand side and at the bottom of the weave diagram, respectively. These color-and-weave effect designs

were obtained empirically through sample weaving or a computer aided simulation of a given pattern of either weave or color order of warp and weft threads.

At present there is no systematic method to obtain the color order and the weave which yield the desired design of color-and-weave effect.

The color order of warp and weft threads in a color-and-weave effect

The color-and-weave effect can be represented by a geometrical pattern which is a regular tiling of a plane by unit squares, each of which is colored. When the design uses n colors, the design is said to be an n -colored color-and-weave effect. For example, in Fig. 2, (a-E), (b-E) and (c-E) are

Fig. 1 Hound's tooth check. (a), (b): Appearance. (c): Weave (2/2 twill) and order of coloring in threads. (d): Design for this color-and-weave effect.

Fig. 2 Examples of color-and-weave effect. (a-D), (b-D), (c-D) and (d-D): weave and order of coloring in threads. (a-E): Two and two check. (b-E): Zig zag. (c-E): Star check. (d-E): A 3-colored color-and-weave effect.

designs with a 2-colored color-and-weave effect, and (d-E) is a 3-colored color-and-weave effect. In the following discussion, by “color-and-weave effect” we mean only the geometrical pattern as above (see, for example Fig. 1 (d)). We refer to each square by the notation (x, y) -square where x and y are the coordinates of its center with integers x and y . Each color in a n -colored color-and-weave effect is indicated by an integer (called a color number) from 1 to n . So the design can be described with a matrix of color numbers $C = (c_{xy})$, where c_{xy} is the x - y element of C and corresponds to the color of the (x, y) -square of the color-and-weave effect.

When

$$c_{ky} = c_{iy}, \quad k \equiv i \pmod{R_W}$$

$$\text{and } c_{xk} = c_{xi}, \quad k \equiv i \pmod{R_F}$$

the color-and-weave effect C will be called periodic, and R_W and R_F are called the periods of C . Although a fundamental $R_W \times R_F$ block determines the design of the color-and-weave effect uniquely, sometimes we show a larger part of the design for convenience, and indicate a fundamental block using gray and white squares instead of black and white ones as shown in Fig. 1 (d).

In real fabric, x and y correspond to warp number and weft number respectively, and c_{xy} to the color appearing at the intersection of x -th warp and y -th weft. Let w_x be the color number of x -th warp, and f_y that of y -th weft, and let $W = (w_x)$ and $F = (f_y)$ be the array of the color order of the warp and that of the weft respectively.

Given a color-and-weave effect C , c_{xy} must be either w_x or f_y , so the condition of the warp and weft coloring in (x, y) -square is

$$(w_x = c_{xy}) \vee (f_y = c_{xy}) = T, \quad (1)$$

where T means true and $\vee$ is the logical sum operator. For all squares of the color-and-weave effect C , the condition of the warp and weft coloring order is $\Psi(C) = T$ where

$$\Psi(C) = \prod_{x=1}^{R_W} \bigwedge_{y=1}^{R_F} \{ (w_x = c_{xy}) \vee (f_y = c_{xy}) \}, \quad (2)$$

where, $\bigwedge$ denotes the logical product and $\prod$ accumulation in $\bigwedge$.

By expanding and simplifying the formula (2), we obtain the condition for coloring order of threads and define the coloring order.

As a simple example, suppose

$$C = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix},$$

then,

$$\begin{aligned} \Psi(C) &= (w_1=1 \vee f_1=1) \wedge (w_1=1 \vee f_2=1) \\ &\quad \wedge (w_2=2 \vee f_1=2) \wedge (w_2=2 \vee f_2=2). \end{aligned}$$

Here, we introduce the following symbols: $a_1 = (w_1=1)$, $a_2 = (w_2=2)$, $b_1 = (f_1=1)$, $b_2 = (f_2=1)$, $b_3 = (f_1=2)$ and $b_4 = (f_2=2)$, and omit the operator $\wedge$, then

$$\begin{aligned} \Psi(C) &= (a_1 \vee b_1)(a_1 \vee b_2)(a_2 \vee b_3)(a_2 \vee b_4) \\ &= a_1 a_1 a_2 a_2 \vee a_1 b_2 a_2 a_2 \vee b_1 a_1 a_2 a_2 \\ &\quad \vee b_1 b_2 a_2 a_2 \vee a_1 a_1 b_3 a_2 \vee a_1 b_2 b_3 a_2 \\ &\quad \vee b_1 a_1 b_3 a_2 \vee b_1 b_2 b_3 a_2 \vee a_1 a_1 a_2 b_4 \\ &\quad \vee a_1 b_2 a_2 b_4 \vee b_1 a_1 a_2 b_4 \vee b_1 b_2 b_2 b_4 \\ &\quad \vee a_1 a_1 b_3 b_4 \vee a_1 b_2 b_3 b_4 \vee b_1 a_1 b_3 b_4 \\ &\quad \vee b_1 b_2 b_3 b_4 \\ &= a_1 a_2 \vee a_2 b_1 b_2 \vee a_1 b_3 b_4 \\ &= (w_1=1)(w_2=2) \vee (w_2=2)(f_1=1)(f_2=1) \\ &\quad \vee (w_1=1)(f_1=2)(f_2=2) \end{aligned}$$

Thus we obtain the sequence of warp coloring $W = (w_1, w_2)$ and weft coloring $F = (f_1, f_2)$, which generates the color-and-weave effect C . Those from the first term are: $(w_1, w_2)(f_1, f_2) = (1, 2)(1, 2)$, $(1, 2)(2, 1)$, $(1, 2)(1, 1)$ and $(1, 2)(2, 2)$; from the second term: $(1, 2)(1, 2)$, $(2, 2)(1, 2)$ and from the last term: $(1, 1)(2, 2)$, $(1, 2)(2, 2)$, so that, apart from common ones, there are 6 types.

Algorithm for computation

Expansion and simplification of formula (2) by the method described above will become more difficult with increasing periods. The following is a simpler method of determining the color order by means of depth-first search: For given C , R_W , R_F and n , assume $R_W = R_F = R$ for simplicity. Let $S = (s_i)$ be the arrangement of the colored threads, where $s_{2i-1} = w_i$ and $s_{2i} = f_i$ ($i = 1, 2, 3, \dots, R$). The status where the color order from s_i to s_{i-1} has been determined and s_i under consideration is called i -stage.

Algorithm:

(i) Initial stage

Call the 1-stage. If returned from the 1-stage, then the search completes.

(ii) Procedures for the i -stage

(a) If $i > 2R$ then output S as one of the solution, and return to the $(i-1)$ -stage.

(b) Set $s_i = 1$

(c) If all evaluations of Formula (1) on s_i with $(s_{i-1}, s_{i-2}, \dots, s_j (j > 0))$ are true, then call the $(i+1)$ -stage.

(d) Set $s_i = s_i + 1$

(e) If $s_i > n$ then return to the $(i-1)$ -stage, else go to (c).

All distinct color orders can be obtained by the above method. For example, from the design of Fig. 3(E), we obtain 8 distinct color orders by this method, and they are shown in Figs. (C-1), (C-2),

Fig. 3 Example of color orders and conditional weaves. (E): A given design. (C-1), (C-2), (C-3), (C-4), (C-5), (C-6), (C-7) and (C-8): Possible color orders in threads and conditional weaves for (E). (W): An example of weaves obtained from (C-8).

(C-3), (C-4), (C-5), (C-6), (C-7) and (C-8). For other examples, from each of the designs in Fig. 4(E) and Figs. 5(a-E), (b-E), (c-E), (d-E), (e-E), we obtain other than two trivials only one color order, each of which is also shown in Fig. 4(C) and Figs. 5(a-C), (b-C), (d-C) and (e-C). The trivial cases will be described later. For the design in Fig. 5(f-E), 22 distinct color orders are obtained. In Fig. 5(f-C), we show one of them as an example.

Color order and weave condition

From the color orders satisfying formula (2) for a given color-and-weave effect, d_{xy} which is the interlacement condition on (x, y) is found to be one of the following values:

$$\left. \begin{aligned} (w_x = c_{xy}) \wedge (f_y \neq c_{xy}) &\Rightarrow d_{xy} = DW \\ (w_x \neq c_{xy}) \wedge (f_y = c_{xy}) &\Rightarrow d_{xy} = DF \\ (w_x = c_{xy}) \wedge (f_y = c_{xy}) &\Rightarrow d_{xy} = DL \\ (w_x \neq c_{xy}) \wedge (f_y \neq c_{xy}) &\Rightarrow d_{xy} = DU \end{aligned} \right\} \quad (3)$$

where DW means that the interlacement must be a warp float, DF a weft float, DL a warp float or weft float and DU means that the interlacement cannot develop the color-and-weave effect. Since DU can be rejected by formula (2), we obtain a conditional interlacement array $D_C = (d_{xy})$ consisting of DW , DF and DL . We represent the conditional interlacement array by means of a design of "conditional weave". The design has three types of squares. A white square means DF , a black square means DW and a dotted square means DL . Figs. 3(C-1), (C-2), (C-3), (C-4), (C-5), (C-6), (C-7) and (C-8) show the conditional weaves corresponding to the color order of the warp and weft shown on the bottom side and left hand side.

Determination of weave

Interpreting DL as the warp float or weft float, we can obtain ordinary interlacement array D consisting of the warp float and weft float. There exist a great number of possibilities, but not all of these weaves corresponding to D are suitable for color-and-weave effects. Some conditional weaves can be eliminated immediately. If there are any columns (or rows) consisting of all warp (or weft) floats, the conditional weave as shown Figs. 3(C-1) and (C-2), can be ignored since that can not be a fabric weave.

Newton and Sarkar⁸⁾ developed a method for determining how a given weave would cause fabrics to form layered structures, and outlined a weave classification scheme. A subsequent paper by Gu Ping and Newton⁹⁾ supplemented a classification of interchange weaves, and gave a method for identifying and analyzing interchanging double weaves.

The weave classifications are as follows:

i) Single-layer weave: All threads interlaced into one coherent layer.

ii) Multilayer weave without stitching: Threads separate into two or more unconnected fabrics.

iii) multilayer weave with stitching

a) Single-direction multilayer weave with stitching (warp-backed or weft-backed weave): Ends or picks form two or more layers but threads in the other direction remain in a single layer.

b) Two-direction multilayer weave with stitching: Threads form two or more fabric layers connected by stitching points, or by stitching threads.

iv) Multilayer with interchanging (Interchange weave): Threads form two or more layers connected along interchange lines.

a) Interchanging in the warp or weft.

b) Interchanging in the warp and weft.

Only single-layer weave is suitable for a color-and-weave effect. In every multilayer weave, some threads cover up the floating points of other threads, so they can not develop a given design. If no pairs of adjacent rows or columns have DL , we can see beforehand by Newton and Sarkar's method⁸⁾ whether the conditional weave becomes only multilayer weaves. We can see Fig. 3(C-3) will become only multilayer weaves by their method. Thus there are five suitable conditional weaves left in Fig. 3.

A great number of weaves can be obtained from each of the conditional weaves. But it is difficult to investigate all of them. If there are long warp (or weft) floats in a weave such as a satin weave, the warp (weft) floats will tend to spread, and thus cover up the floating points of other threads. Hence the weaves which have a long float are not suitable for a color-and-weave effect. In Fig. 3, it may be possible to make the conditional weave (C-8) into a weave which has the shortest maximum float length, because the maximum length

of the sequence of D_W or D_F in (C-8) is the shortest of all. So let's select (C-8) for consideration.

Searching for a well-known weave is a very simple method for making a weave from a conditional weave. In Fig. 4, weaves (W-1), (W-2), (W-3) and (W-4) are examples obtained from (C). All of them are well-known weaves. In this way, sometimes different well-known weaves can

Fig. 4 Examples of weaves obtained from a conditional weave. (E): A given design. (C): Color order of threads and the conditional weave. (W-1), (W-2), (W-3) and (W-4): Examples of weaves obtained from (C).

Fig. 5 Examples of the construction of a color-and-weave effect. The given design (denoted by E), color order and the conditional weave (C), and an example of weaves obtained from the conditional weave (W).

develop the same color-and-weave effect with the same coloring order. Regularity in textile weaves has been described by Go et al.¹⁰⁾ If there exists any regularity in conditional weaves, we can use the structures to determine the weaves. In Fig. 5, weaves (a-W), (b-W), (c-W), (d-W) and (e-W) are obtained by this method. But in Fig. 3(C-8) we could not make any well-known weaves, and not find any regularity. In general, we determine the weave considering the following two points.

- i) The weave must be a single layer.
- ii) The length of each float must be as short as possible.

Although there is no effective method to find the weave which satisfies the above, from these points of view, in Fig. 3 we get (W) from (C-8). Another example is shown in Fig. 5(f-W).

Classification of 2-colored color-and-weave effects

In this section we shall only discuss the 2-colored color-and-weave effect. There exist two trivial solutions of the warp and weft coloring order W, F for any given color-and-weave effect C . These are $W=(1), F=(2)$ and $W=(2), F=(1)$ (for

example, see Figs. 3(C-1) and (C-2)). In these two cases, from Eq. (3), D_C does not contain D_L , so D is equal to D_C and is equivalent to C . By saying that D is equivalent to C , we mean that C is equal to D when one element of C is regarded as the warp float and the other element of C as the weft float or vice versa.

If two distinct rows i, j and two distinct columns k, l , of a color-and-weave effect C satisfy the relation

$$\begin{bmatrix} c_{ik} & c_{il} \\ c_{jk} & c_{jl} \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \quad (a \neq b), \quad (5)$$

then it may be concluded that rows i, j and columns k, l are directly connected to each other in the color-and-weave effect (Some authors say "have a circuit" in a weave diagram. See Newton and Sarkar⁸⁾).

If the submatrix

$$C' = \begin{bmatrix} c_{ik} & c_{il} \\ c_{jk} & c_{jl} \end{bmatrix}$$

satisfies Eq. (5) then

$$\begin{aligned} \Psi(C') &= (w_k = w_l = a \wedge f_i = f_j = b) \\ &\quad \vee (w_k = w_l = b \wedge f_i = f_j = a) \end{aligned}$$

and we obtain the conditions for the coloring of i, j -th weft and k, l -th warp, which are

$$w_k = w_l, \quad f_i = f_j \quad (w_k \neq f_i) \quad (6)$$

If rows i, j and columns i', j' are connected to each other and at the same time rows i, j and columns j'', k' are connected, then it also may be said that these columns and rows are connected indirectly with each other. In the above case the conditions for coloring i, j, k -th warps and j'', i', j', k' -th wefts are

$$w_i = w_j = w_k, \quad f_{i'} = f_{j'} = f_{j''} = f_{k'} \quad (w_i \neq f_{i'})$$

In general, if in a two-colored color-and-weave effect C , all columns and all rows are connected with each other, then there exist only two trivial solutions of warp and weft coloring order, and in this case the interlacement matrix D is equivalent to C .

The two-colored color-and-weave effect can be classified into the following three types according to the connections.

Type 1: All adjacent pairs of rows are connected

directly and all adjacent pairs of columns are also connected directly.

Type 2: All rows are connected and all columns are connected directly or indirectly, but not Type 1.

Type 3: There exist disconnected rows or columns.

Newton and Sarkar's weave classification scheme according to the circuit⁸⁾ is as follows:

Single-layer (provided that it contains inter-change weaves⁹⁾): Circuits between every pair of adjacent rows and columns.

Multilayer without stitching: Columns and rows divided into sets that exhibit circuits with in but not between each other.

Multilayer with stitching: Some pairs of adjacent columns or (and) rows do not form circuits.

Following their scheme where connections correspond to circuits, we regard the color-and-weave effect as a weave diagram. Then, Type 1 becomes to single-layer weave, Type 2 a multilayer weave with stitching (It contains a fabrics which are obtained by doubling like a matt weave) and the Type 3 a weave without stitching (Some authors say that "a fabric that does not hang together". See Grünbaum and Shephard⁵⁾). Fig. 6 shows color-and-weave effect designs belonging to the three types classified by the method.

Only Type 3 is a design of interest as a color-and-weave effect, since in Type 1 and Type 2 designs, all columns and all rows are connected to each other, and they have only two trivial solutions of the color order, that is all warp threads are the same in color, all weft threads are the same in color and they are distinct. In these case the interlacement matrix are equivalent to the color-and-weave effect. Procedures to determine whether or not a design represents a fabric that hangs together, are given by Clapham⁶⁾, Grünbaum and Shephard⁷⁾ and by Enns¹¹⁾. These procedures can be applied to determine whether or not a design falls into Type 3 classification.

Conclusions

An effective method has been introduced for determining the color ordering of the threads and weave which yield a given pattern of color-and-weave effect.

Fig. 6 Examples of the three types of designs. (a), (b): Type 1.
(c), (d): Type 2. (e), (f): Type 3.

All of the suitable color order of threads are obtained immediately by the use of a computer with the search algorithm.

The conditional weave contains the conditions for all of the possible weaves corresponding to the color order of threads. Since the weave must be a single-layer weave, searching for a well-known or a regular single-layer weave is a simple method for determining the weave from the conditional weave. Another general method is making the length of thread floats as short as possible.

In 2-colored color-and-weave effect, only the pattern of Type 3 (multilayer weave without stitching) is suitable for a color-and-weave effect. So another patterns can be rejected for a consideration of a new pattern of the color-and-weave effect.

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色糸効果における色糸配列と組織の決定

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従来試行錯誤により行われてきた色糸効果の設計に関し織物設計者を支援する系統的手法を検討した。色糸効果を数論的にとらえ, 与えられた色糸効果の意匠を織り出すことができる色糸配列の条件式を作成し, それを評価する効率的なアルゴリズムを開発した。

ある目的とする意匠を織り出す場合, 色糸配列を与えると, その意匠を満足する複数個の組織が得られる。こ

の複数個の組織を1つの条件組織図と呼ばれる図で簡潔に表わすことができた。

Newtonら⁸⁾の織物分類法を発展させることから, 2色の色糸効果の意匠を3種類に分類することができた。そのうちの第3種の接結のない多重組織に分類される意匠のみが色糸効果に適することが分った。