Prediction of Krill Target Strength by Liquid Prolate Spheroid Model

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A theoretical prolate spheroid liquid model is used to predict the target strength (TS) of krill. Scattering patterns are shown to demonstrate orientation dependence of krill TS. Length-to-wavelength ratio (L/λ) dependencies of reduced (normalized by square of the body length) target strength are shown for some orientation distributions. The results can explain the well-organized experimental results. The variability of the TS of krill is large when L/λ is larger than unity, therefore, a frequency around 70 kHz is superior to 120 kHz which is ordinarily used for krill surveys.

Key words: target strength, krill, spheroid, scattering pattern, frequency

Fisheries acoustics technology has been extensively used for surveys of small organisms such as zooplankton.1-3 These organisms, however, have a variety of body shapes and physical properties and so their acoustic scattering characteristics are sometimes very complicated at the acoustic frequencies ordinarily used. Further, their target strength (TS) is small and difficult to measure.4-6 These restrictions mean that the prediction of TS or the quantitative observation of small organisms must depend heavily on theoretical models.4-6

Theoretical models for scattering problems of small organisms are also being used more frequently for the following reasons: (1) The multi-frequency sonar method which depends on the scattering model is now in practical use7-9; (2) Acoustic surveys of Antarctic krill Euphausia superba, which are important to fisheries and therefore require proper management, have been conducted internationally.10 A new TS measurement of krill11 revealed that the previously recommended expression10) gave values of TS too high by more than an order of magnitude. Studies on krill scattering are thus very relevant.12

The target organism of this paper is Antarctic krill. This is one species of krill Euphausiids which is abundant and industrially and ecologically important in the world's oceans. The organism is similar in shape and body composition to many zooplankton in acoustic terms. Therefore, we may consider the Antarctic krill as a representative acoustic scatterer of small organisms in the ocean and the results of this paper may be applied to many other organisms with similar shape and body material. For convenience's sake, Antarctic krill will be referred to hereafter as krill.

Greenlaw13) cited Yeh14) and suggested that a prolate spheroid could be an effective scattering model for krill-like organisms. Furusawa4) took up this suggestion and using prolate spheroid models, derived the general fish size and keel characterizations of ordinary bladder fish and bladderless organisms such as krill. The aim of the study, however, was to derive the general scattering characteristics, so general parameters were used for the calculation. The need for a theoretical model and Foote's timely report15) on the speed of sound in krill led the authors to apply the liquid prolate spheroid model (LPSM) to krill scattering.

In this paper, we apply parameters specific to krill to LPSM, and analyze the krill scattering. We examine the aforementioned new TS value, compare our model with other models, and propose proper frequencies for krill surveys.

Our definition of TS (symbol for linear variable is Tq) is the ratio of the backscattered intensity transformed at the reference distance (r0 = 1 m) from a target to the incident intensity, giving the relation between the ordinary definition of scattering cross-section σ(9) and our TS as Tq = σ/(4πr02).

We use a decibel variable and its linear alternative interchangeably, but we distinguish the variables by using two capital letters for the decibel variable, for example TS[dB] = 10log Tq. We sometimes use practical units in which case we append the unit explicitly as in L(cm).

Methods

Liquid Prolate Spheroid Model

Since LPSM is described in detail in ref. 4, only an outline is given here. (The errata of ref. 4 are shown in the last part of the Discussion section.) Krill-like organisms can be modeled by a liquid prolate spheroid whose density and sound speed are slightly different from those of the surrounding medium. Figure 1 shows a side view of krill and its LPSM. The variables used to describe LPSM are: ξ is the spheroidal coordinates where ξ = ξ, =const. defines the spheroid surface, q is semi-focal length, a is major radius, b is minor radius, and ka or h = kq is used as the normalized frequency where k is the wavenumber. TS is normalized by the square of the length, L, and is expressed by the following function which includes complicated spheroidal wave functions:

\[ T_{\text{norm}} = T_q/[L/cm]^2 = T_{\text{norm}}(L/\lambda, \beta, b/a, \rho_1, \rho_2, c_1) \]  

(1)

where L/λ is body length normalized by wavelength, (L/λ), = kq/ξ, kq is wavelength of surrounding water, \( \beta \) is direction of backscattering (90° for dorsal), \( \epsilon = b/a \) is aspect ratio, \( \rho_1 = \rho_1/\rho_2 \) is density ratio (contrast) of animal body and medium, and \( c_1 = c_1/c_0 \) is sound speed ratio (contrast) of body and surrounding medium. This function shows a similarity law that "the normalized TS depends on the length measured by the wavelength and on the shape, and does not depend on the length itself."

Parameters

Table 1 lists the parameters used to calculate LPSM, which are assumed
Table 1. Physical parameters used in calculating scattering models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbols</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspect ratio</td>
<td>$\varepsilon = b/a$</td>
<td>0.117</td>
<td>See the text</td>
</tr>
<tr>
<td>(Spheroid surface)</td>
<td>$\varepsilon_s$</td>
<td>1.0069</td>
<td></td>
</tr>
<tr>
<td>Density contrast</td>
<td>$\rho_b = \rho_c/\rho_0$</td>
<td>1.0357</td>
<td>Foote$^{15}$</td>
</tr>
<tr>
<td>Sound speed contrast</td>
<td>$c_s = c_c/c_0$</td>
<td>1.0279</td>
<td>Foote$^{15}$</td>
</tr>
</tbody>
</table>

Fig. 1. Antarctic krill and its spheroid model with definition of variables.

In order to compare this model with LPSM, some modifications are performed using the relationships of to get to be constant.

The aspect ratio is given by the following formulas, cited from Foote et al.$^{13}$:

$W[g] = 9.60 \times 10^{-8}L[mm]^{2.94}$ (Morris et al., 1988)

$V[cm^3] = 0.939W[g] - 0.003$ (Kils, 1979)

$V = (4\pi/3)a_h^3$, $a_h[cm] = L[mm]/20$

where $W$ is body mass and $V$ is volume. Proceeding from $L(a)\rightarrow W \rightarrow V \rightarrow b$, we obtain a relationship between $L$ and $b/a$. Since $b/a$ is nearly the same for $L=20-50$mm, we use its average value in this length range.

Density$^{11}$ and sound speed contrasts$^{15}$ were measured for Antarctic krill by the density bottle method and the time-of-flight method, respectively.

**High-pass Sphere Model**

The TS of krill-like organisms is usually modeled by Johnson’s high-pass sphere model$^5$ (HPSM) which was deduced from Anderson’s liquid sphere model$^{17}$ as follows.

$$T_s = \frac{2(k_i a_h)^2}{2 + 3(k_i a_h)^2}d^2$$

$$d = \frac{1 - \rho_c c_s}{\rho_b c_s}, \frac{1 - \rho_c}{3\rho_b c_s}, \frac{1 + 2\rho_b}{1 + 2\rho_c},$$

where $a_h$ is equivalent sphere radius. (Note the typographical errors in Johnson$^{18}$ and Greenlaw$^{19}$ as pointed out by Greenlaw.$^{11}$)

In order to compare this model with LPSM, some modifications are performed using the relationships of $a_h = a^{1/3}$, $k_i a_h = a^{2/3}L/\lambda$

to get

$$T_{s_{cm}} = \frac{6\pi(aL/\lambda)^4}{4 + 66.57(aL/\lambda)^4}d^2.$$ (3)

**Results**

Figure 2 shows the backscattering patterns, that is $T_{s_{cm}} (=10\log T_{s_{cm}})$ as a function of $\theta$ for parameters $L/\lambda = 1, 2, 3$. Dashed curves are the experimental data by Foote et al. (1988) and the recommendation of the post-FIBEX report ($\times$). The parameter pair in parentheses are $(\theta_i, s)$ where $\theta_i$ is the (average) tilt angle (0° for horizontal, and positive for head up) and $s$ is the standard deviation of the tilt angle. Constant tilt angle is expressed by $s=0$. The aspect ratio $\varepsilon = b/a$ is 0.117.

Figure 3 shows the normalized TS as a function of $L/\lambda$ for constant orientation, and Fig. 4 is the same for the averaged TS. The averaging method and its indication was established by Foote$^{18}$ and the present authors have followed these.$^{4,9}$ The parameter pair in parentheses are $(\theta_i, s)$ where $\theta_i$ is the (average) tilt angle (0° for horizontal, and positive for head up) and $s$ is the standard deviation of the tilt angle.
Table 2. TS data measured by Foote et al.\textsuperscript{11)} and TS value computed according to Post-FIBEX report\textsuperscript{10)}

<table>
<thead>
<tr>
<th>( f ) [kHz]</th>
<th>( L ) [mm]</th>
<th>( L/\lambda )</th>
<th>( TS ) [dB]</th>
<th>( TS_{cm} ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foote et al.</td>
<td>38</td>
<td>33.7</td>
<td>0.85</td>
<td>-85.1</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>33.7</td>
<td>2.70</td>
<td>-76.1</td>
</tr>
<tr>
<td>Post-FIBEX</td>
<td>50</td>
<td>-</td>
<td>1.12</td>
<td>-60.1</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>-</td>
<td>2.70</td>
<td>-63.3</td>
</tr>
</tbody>
</table>

Figure 3 also shows the result by HPSM.

The measurements by Foote et al.\textsuperscript{11)} and the values calculated for the same \( L/\lambda \) by the expressions in the Post-FIBEX report\textsuperscript{10)} are shown in Table 2 and plotted in Figs. 3 and 4.

**Discussion**

**Backscattering Pattern**

As can be seen from Fig. 2, the backscattering pattern varies largely with changes in \( L/\lambda \). Such patterns for the liquid spheroid have much sharper lobes and more lobes compared with the soft spheroid model of the swimbladder (Fig. 4 of Ref. 4). Traditional sphere models such as HPSM do not provide such directivity and can not predict exact scattering characteristics for larger \( L/\lambda \).

**Comparison with High-pass Sphere Model**

The LPSM and HPSM agree well at \( L/\lambda \) values of less than 0.5 and agree somewhat to about 1. At larger values of \( L/\lambda \), however, there is a big difference in accordance with the tilt angle change in the case of the constant orientation (Fig. 3). Comparing this with the average TS (Fig. 4), the result of HPSM reaches its asymptotic value earlier than LPSM, causing a large difference for large \( L/\lambda \). The HPSM is, however, effective for small values of \( L/\lambda \) and also useful for large \( L/\lambda \) as a first approximation because of its simple form.

**Comparison with Measurements**

The data of Foote et al.\textsuperscript{11)} shown in Fig. 3 approximately agree with the LPSM curve of (10, 0), but for other tilt angles we see great differences. On the other hand, if we compare the data with the averaged LPSM curves (Fig. 4), we see fairly good agreement in the range of the present calculation. Foote et al. did not successfully observe the orientation distribution, but since their results must be somewhat averaged with respect to the orientation distribution, then a fairly good agreement exists between the measurements and LPSM.

The Post-FIBEX recommendation data are too high, although the 120 kHz point agrees with the maximum curve of (0, 0) (see Fig. 3).

**Effectiveness of Prolate Spheroid Model**

As shown above, since LPSM can reasonably explain the well-organized new measurement\textsuperscript{11)} and can reasonably evaluate other models, then LPSM appears to be an effective model.

The normalized TS (\( T_{scm} \)) of Eq. (1) is greatly simplified through effective normalization. That is, if the shape \((b/a)\) and the physical properties of the body \((\rho_s, c_s)\) are fixed, then the relationship of \( L/\lambda - T_{scm} \) is determined exclusively by the orientation \((\theta)\). Therefore, the graphs shown in Figs. 3 and 4 satisfy almost all cases we encounter. For details of the effect of changes in shape and physical properties, refer to Figs. 4 and 5 of ref. 4. Since the final simple expression for LPSM is obtained by reducing the complexity of the animal body (Fig. 1), further examination of LPSM is desirable.

The main problem with LPSM is that the calculation is complex and the computation time is long, but advanced computer technology has made it possible to perform the computation on some work-station class computers in a reasonable time. Also, FORTRAN programs are available\textsuperscript{19)} for calculating complicated spheroidal wave functions that are the key functions in LPSM.

**Effect of Orientation**

Figures 3 and 4 suggest that there should be a large variation in TS according to variations in the orientation distribution of krill. Sameoto\textsuperscript{3)} and Endo\textsuperscript{20)} showed that the orientation of krill is very variable compared with fish and tends to be positive (head up). This may come from planktonic characteristics; they may have an upward posture because of the lack of swimbladder.

The fact that the orientation variation is large and that the scattering pattern is sharp doubly makes the variation in TS large. Fortunately, however, as seen in Fig. 4 if orientation has a proper distribution, then the range of TS variations becomes considerably small.

There have been few studies on the orientation distribution of krill, yet such studies are crucial for designing methods of estimating krill abundance. On the other hand, since the observed value of TS is governed by the orientation of krill, we may be able to develop some method to detect krill orientation by combining some acoustic observation methods.

**Effect of Density and Sound Speed**

The density and sound speed contrasts we used in the present calculations (see Table 1) are nearly the same as those used by Furusawa in his previous work\textsuperscript{4)} (\( \rho_s = 1.04, \ c_s = 1.02 \)). Figure 5 of ref. 4 shows that this, like small differences in \( \rho_s \) and \( c_s \), cause no large difference in TS, so the general trend shown in ref. 4 does not need to be changed.

However, in the reference, in explaining the past measurements of krill TS, the result of LPSM was considered to be rather lower, and the possibility of ignoring some body components such as the carapace was suggested. This study does not necessitate such explanations because of the rather good agreement with the experimental data. Foote’s \( \rho \) and \( c \) include to some extent the contribution from all body components and, although of course in not exact sense, the present calculation includes the effect of the whole body structure.

**Frequency Appropriate for Krill Surveys**

Here we consider acoustic systems for estimating krill abundance.

Equation (3) shows that if \( L/\lambda \) is small (Rayleigh region), the normalized TS of krill is proportional to \((L/\lambda)^4\) or TS
itself is proportional to $L^6$. Therefore, a change in $L$ results in a large change in TS. This means that if we do not know the exact length distribution, the estimate of TS may contain large errors. Also, TS becomes very small for small $L/\lambda$ and this lowers the signal-to-noise ratio of the acoustic system. Thus, low frequencies are not good for krill surveys.

If $L/\lambda$ is large, variations caused by variations in orientation becomes large and again precise estimates of TS or abundance are difficult. Therefore, we should select $L/\lambda$ such that variations are small.

In view of the above, $L/\lambda=0.5-1$ may be a good band for krill surveys. Traditionally, a frequency of 120 kHz has been used for krill surveys. If the above condition is applied to this frequency, we have length range of $L=0.63-1.25$ cm, but in view of the larger size of adult krill, 120 kHz may be too high.\(^{21,22}\)

The authors thus propose a lower frequency such as 70 kHz as the standard frequency for krill surveys. We are developing a two-frequency system operating at 38 kHz and 70 kHz as a general purpose quantitative echo sounder which can be used for krill as well as ordinary fish.

**Multi-frequency Sonar**

A multi-frequency sonar is a new and promising acoustic remote sensing tool for plankton surveys,\(^{23}\) and can also be used for krill surveys.\(^{24}\) This method wholly relies upon the modeling of the TS of the object organism(s). Let us consider the method, referring to LPSM.

Greenlaw\(^{25}\) stated that "... the ratio of scattering strength is a function of both the sizes and the frequencies and it is this region which a multi-frequency echo-sounder can exploit." From Eq. (2) in the Rayleigh region we have

$$T_s \propto a_4^2(k_0a_4)^4$$

Then the ratio of TS at the two frequencies is proportional to $k_0^4$ and the $a_4$ term does not appear in the ratio, therefore size estimation is impossible. This means that we must utilize a shoulder part of the $L/\lambda - T_{\text{Rayleigh}}$ characteristics. As seen in Fig. 3 and 4, however, there are variations in the shoulder part caused by orientation changes and this makes an exact estimation difficult for krill-like organisms without introducing an orientation dependent TS model.

This difficulty is caused by the directional scattering characteristic of krill. For more omni-directional scatterers such as small zooplankton the above problem is less important. However, caution is required since for a certain size of organism, only a few frequencies in a narrow band contribute to determine the size distribution. Finding this narrow band is not always easy for many species without sophisticated work on scattering models.

**Future Tasks**

From the above discussion it is necessary to clarify, from the acoustical view points, the in situ orientation, swimming behavior, shape, body component of object organisms and how these change according to the surrounding conditions, growth, season, and time of day.

Acoustic systems must be designed to withstand such variations. A procedure for designing a quantitative echo sounder for fish has been proposed,\(^{21,22}\) and this procedure should be extended to smaller organisms.

Stanton's bent finite-length fluid cylinder model can also reasonably explain the experimental data.\(^{6}\) This model is similar to LPSM in expressing elongated shapes, curvature, and so on, but is different in how it expresses weight distribution along the body, the shape of the body ends, and so on. A comparative evaluation will give us a better understanding of the scattering characteristics of krill-like organisms.

There is little data on the scattering pattern measured for plankton. Such measurements are necessary to examine further theoretical models and to establish a more accurate theoretical-experimental hybrid model.

**References**