Short Paper

Mathematical Approach to Virtual Population Analysis*

Tatsuro Akamine

National Research Institute of Fisheries Science, Fukuura, Kanazawa, Yokohama 236, Japan

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Virtual Population Analysis (VPA) is a method of obtaining fishing mortality coefficients and stock sizes by solving the fishing equations.¹⁾ The solution of VPA must be obtained by iteration because VPA is a non-linear model. In this paper, we consider a new method using mortality rates instead of coefficients, and obtain a simple algorithm to solve the equations and to evaluate the error of the solution easily.

VPA requires the solution of the following simultaneous equations:

$$C_i = N_i \frac{F_i}{F_i + M} \{1 - \exp(-F_i - M)\} \quad (1)$$

$$N_{i+1} = N_i \exp(-F_i - M) \quad (2)$$

where C and N are catch and stock sizes, F and M are fishing and natural mortality coefficients, and i is age. Forward calculation obtains $F_2 \sim F_n$ and $N_1 \sim N_n$ using $C_1 \sim C_n$, M and F_1 . In the backward calculation, we use F_n (terminal F) instead of F_1 , and obtain $F_1 \sim F_{n-1}$ in the same way.

Equation (1) gives the following definition of fishing and natural mortality rates:

$$E = \frac{F}{F + M} \{1 - \exp(-F - M)\} \quad (3)$$

$$D = \frac{M}{F + M} \{1 - \exp(-F - M)\} \quad (4)$$

These equations give the survival rate:

$$S = \exp(-F - M) = 1 - E - D \quad (5)$$

and the following inverse transformations:²⁾

$$F = -\frac{E}{E + D} \ln(1 - E - D) \quad (6)$$

$$M = -\frac{D}{E + D} \ln(1 - E - D) \quad (7)$$

We can rewrite Eqs. (1) and (2) using Eqs. (3) and (4) as follows:

$$C_1 = N_1 E_1, \quad C_2 = N_2 E_2 \quad (8)$$

$$N_2 = N_1(1 - E_1 - D_1) \quad (9)$$

These equations give the following solutions:

$$E_2 = \frac{C_2 E_1}{C_1(1 - E_1 - D_1)} \quad (10)$$

$$E_1 = \frac{C_1 E_2(1 - D_1)}{C_1 E_2 + C_2} \quad (11)$$

Equation (10) gives the forward calculation, and Eq. (11) gives the backward calculation. The general case is given by changing the subscripts from $1 \rightarrow i$, and $2 \rightarrow (i + 1)$.

From these equations, we obtain a simple method of solving Eqs. (1) and (2) provided that D_i is known. However, this new algorithm gives a different solution for VPA if M is constant. It is not natural to use F and M , which are defined for the continuous model originally, for the discrete model (1) and (2). The condition of constant M is less realistic than the condition of "known M ". Therefore, this new method is more natural and realistic than VPA.

Next, we consider the error evaluation. The differential of Eq. (8) gives

$$\frac{dC}{C} = \frac{dN}{N} + \frac{dE}{E} = 0 \quad (12)$$

This gives the following approximation:

$$\frac{\Delta N}{N} = -\frac{\Delta E}{E} \quad (13)$$

On the other hand, the differential of Eq. (9) gives

$$\frac{dN_2}{N_2} = \frac{dN_1}{N_1} - \frac{dE_1}{1 - E_1 - D_1} \quad (14)$$

Because Eq. (12) shows that dN has the opposite sign to dE , Eq. (14) gives the following approximation:

$$\left| \frac{\Delta N_2}{N_2} \right| > \left| \frac{\Delta N_1}{N_1} \right| \quad (15)$$

Backward calculation is considered to be better than forward calculation, because Eqs. (13) and (15) show that the relative error of N_i increases in the forward calculation. This is a general theory of numerical analysis. However, Eqs. (1) and (2) are only equations and not a stochastic model. Therefore, Eqs. (13) and (15) do not suggest the mathematical superiority of the backward calculation.

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