Original Article

A method for analyzing the static response of submerged rope systems based on a finite element method

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ABSTRACT: A method is presented for determining the equilibrium configuration and tensions of submerged rope systems in a uniform current based on a non-linear finite element formulation. A standard straight rope element is adopted to model rope segments of the system. Rope stretch is included in the formulation, and a hydrodynamic model is introduced for the rope. The structural and hydrodynamic models are coupled in order to account for the interaction between the structural response and hydrodynamic load acting on the rope itself. The Newton–Raphson method is employed in the formulation, ensuring the global convergence of the iteration to the correct response (and hence to the correct equilibrium configuration and tension of the rope system) from any set of initially estimated responses. Good agreement between this numerical simulation and experimental behavior was revealed in an application of this method to a long-line model.

KEY WORDS: finite element method, flume testing, numerical analysis, static response, submerged rope system.

INTRODUCTION

In commercial fishery, almost all fishing gears consist partly or entirely of supple ropes and netting. The fishing efficiency of fishing gear depends heavily on its working shape during fishing. For this reason, it is desirable to predict the behavior of fishing gear loaded by external forces, before the equipment is manufactured. However, this problem is complex because the flexibility and elasticity of the net materials allow the working shape of the fishing gears to change under external forces. It is noted that large relative displacements of line elements introduce geometric non-linearities when the line is in motion. The stress–strain relationship for the material under such conditions is highly non-linear.

Early studies on supple rope analysis in a uniform current took the form of solving the differential equations for the transformed continuous curve line. As the basic equations are highly non-linear, solutions can be obtained only under particular conditions. An approximation method for calculating tension and configuration for towing or mooring rope has been proposed, assuming a uniform current and catenary curves.1–5 Hu and Matuda studied the static characteristics of a mid-water trawl system by integrating the basic difference equations of supple rope.6 However, recent advances in the application of discrete energy techniques to highly non-linear problems have provided sufficient grounds for the development of efficient and reliable non-linear analyses.7,8 In particular, the rapid advances in numerical computation methods and computer performance have allowed the simulation of fishing gear behavior and ocean installations. In the past 10 years, theoretical analyses have been applied as a more straightforward and practical tool for seeking solutions to the configuration and tension of fishing gear and other ocean installations and moorings under various external forces.9–12

The present paper describes an analytical method for evaluating the static response of a submerged rope system in which a finite element method (FEM),13 which was originally applied to

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suspension structures in the field of architecture, is adopted whereby nodal displacements and element tensions are taken as unknown variables. In order to establish the validity of the calculation method, the equilibrium configuration and tension of a simplified tuna long-line model was calculated, and the results were compared with observation by flume testing.

MATERIALS AND METHODS

Numerical model

In order to develop a simulation method for the static analysis of a submerged rope system, the following assumptions for the numerical model need to be established:

1. There is only tension in the direction of the axis of the rope and the tension is constant across the entire cross-section of the rope.
2. The relative displacements of all points on the cross-section of the rope are equal.
3. The cross-sectional area of the rope remains constant during deformation.
4. The rope is completely flexible and is easily bent without resistance.
5. The rope material is elastic and isotropic and, hence, the relationship between tension and strain follows Hooke’s law.

Based on these assumptions, the rope is modeled as a series of straight rope elements, an example of which is shown in Fig. 1. Each element of the system incurs significant relative displacement during movement, from the initial state to the deformed state. In the figure, \( u, v \) and \( w \) denote the nodal displacements of the element in the direction of the \( X, Y \) and \( Z \) axes, respectively, and the subscripts \( i \) and \( j \) indicate the ends of the element. Taking into account the elastic elongation of elements due to the acting tensile forces or finite displacement of elements during large deformation, the 3-D position and tensile force of the elements are regarded as unknown variables in the derivation of the basic equations for this problem. Assuming that two adjacent elements are connected in a hinge-like manner, the rope system can be modeled as a supple structure composed of hinged truss members. These hypotheses are valid when the size of the elements are sufficiently small to realize the desired accuracy.

In the two states of the \( g \)-th element of the rope illustrated in Fig. 1, the total potential energy \( \Pi \) for this system can be expressed as:

\[
\Pi = -\sum_{i=1}^{f} F_i D_i + \sum_{g=1}^{m} \left[ T_g (L_g(D_i) - L_g(0)) \right. \\
\left. - \frac{L_g(0) T_g}{2EA_g} \right],
\]

whereby \( F_i \) is the equivalent nodal loading on the \( i \)-th node; \( D_i \) is the nodal displacement; \( T_g \) is the axial force of the \( g \)-th element; \( L_g(0) \) is the initial length of the \( g \)-th element; \( L_g \) is the length of the \( g \)-th element after deformation; \( A_g \) is the cross-section area of the \( g \)-th element; \( E \) is Young’s modulus of the material; \( f \) is the nodal degree of freedom; and \( m \) is the number of the element. It should be noted that the term \( (L_g(D_i) - L_g(0)) \) in equation 1 represents the elongation of elements, and can be expressed explicitly as:
\[ L_g(D_g) - L_{g0} = \sqrt{X(u)^2 + Y(v)^2 + Z(\omega)^2} \]

whereby

\[ X_0 = X_j - X_i, \quad Y_0 = Y_j - Y_i, \quad Z_0 = Z_j - Z_i, \]

and \( X, Y, \) and \( Z \) are the nodal coordinates of the element in the direction of the \( X, Y, \) and \( Z \) axes, and \( u, v, \) and \( \omega \) denote the nodal displacements of the element in the direction of the \( X, Y, \) and \( Z \) axes, and the subscripts \( i \) and \( j \) indicate the ends of the element. Equations 2, 3 and 4 describe the relationship between strain and displacement of the element.

From the stationary condition of \( \Phi \) with respect to \( D_i \) and \( T_g \); that is, \( \partial \Phi / \partial D_i = 0 \) and \( \partial \Phi / \partial T_g = 0 \), basic equations for this system can be obtained, as shown in the following equations 5 and 6:

\[
\begin{bmatrix}
\frac{\partial L_1}{\partial D_i} & \cdots & \frac{\partial L_m}{\partial D_i} \\
\vdots & \ddots & \vdots \\
\frac{\partial L_1}{\partial D_f} & \cdots & \frac{\partial L_m}{\partial D_f}
\end{bmatrix}
\begin{bmatrix}
T_i \\
\vdots \\
T_m
\end{bmatrix}
= \begin{bmatrix}
F_i \\
\vdots \\
F_m
\end{bmatrix}
\]

whereby the elements in the matrix are the direction cosines of the \( g \)-th element axis after deformation with respect to the \( X, Y, \) and \( Z \) axes.

\[ \{L_g(D_g) - L_{g0}\} - \frac{L_{g0}}{EA_g} T_g = 0, \quad g = 1, 2, \ldots m. \] (6)

It is clear that equation 6 gives the relationship between element tension and nodal displacement.

Equations 5 and 6 constitute the basic simultaneous equations of \((f + m)\) degrees of freedom with unknowns of nodal displacement \( D_g \) and tension \( T_g \) for the rope system.

Configuration-dependent external forces

In equation 5, the external force \( F \) acting on elements of the submerged rope system includes hydrodynamic forces \((R_d, R_b)\), which are dependent on rope configuration, the weight of the rope in water, and other concentrated loads. Unfortunately, it is still very difficult to determine the hydrodynamic forces theoretically, including the interaction between rope and fluid flow. In the present paper, the hydrodynamic forces are estimated using the following equations:\(^{18}\)

\[ R_d = \frac{1}{2} \rho C_d d V^2, \]

\[ R_b = \frac{1}{2} \rho C_b d V^2, \]

whereby \( R_d \) and \( R_b \) are the hydrodynamic drag and lift per unit length of element, respectively; \( d \) is the rope diameter; \( V \) is the relative velocity of the flow; and \( \rho \) is the fluid density. The hydrodynamic lift and drag coefficients \((C_d, C_b)\) for the ropes are determined by the following Miyazaki model,\(^{18}\) respectively:

\[ C_d = C_{d0} (1 - \sin^2 \theta \cos^2 \delta), \]

\[ C_b = C_{b0} \sin \theta \cos \theta \cos^2 \delta, \]

in which \( C_{d0} \) is the coefficient of drag when the axis of the element is vertical to the flow, \( \delta \) is the deflected angle, and \( \theta \) is the inclined angle of element.

Solution

It is clear that the basic equations for the aforementioned system are highly non-linear. In order to solve these non-linear simultaneous equations, the Newton–Raphson method was employed.\(^{19-21}\)

Let \((D_i', T_g')\) denote the solution obtained in the \( r \)-th step of the iteration, followed by the solution \((D_i' + \Delta D_i', T_g' + \Delta T_g')\); hence, the linear equations for the correction increments \((\Delta D_i', \Delta T_g')\) take the following forms:

\[
\begin{cases}
\sum_{i=1}^{f} \frac{\partial f_1}{\partial D_i} \Delta D_i' + \sum_{g=1}^{m} \frac{\partial f_1}{\partial T_g} \Delta T_g' + f_1(D_i', T_g') = 0 \\
\sum_{i=1}^{f} \frac{\partial f_2}{\partial D_i} \Delta D_i' + \sum_{g=1}^{m} \frac{\partial f_2}{\partial T_g} \Delta T_g' + f_2(D_i', T_g') = 0 \\
\vdots \\
\sum_{i=1}^{f} \frac{\partial f_{f+m}}{\partial D_i} \Delta D_i' + \sum_{g=1}^{m} \frac{\partial f_{f+m}}{\partial T_g} \Delta T_g' + f_{f+m}(D_i', T_g') = 0
\end{cases}
\]

which can be transformed into the following matrix form:

\[
\begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta D_i' \\
\Delta T_g'
\end{bmatrix} = \begin{bmatrix}
F - \sum_{g=1}^{m} \frac{\partial L_{g0}}{\partial D_i} T_g \\
\sum_{g=1}^{m} \frac{\partial L_{g0}}{EA_g} T_g + \{L_{g0} - L_g(D_i')\}
\end{bmatrix}
\]

whereby the submatrix \([N_{11}, (D_i', T_g')]\) corresponds to the geometrical stiffness matrix; \([N_{12}]\) is the equi-
librium matrix of the system after deformation, 
\[ [N_{22}] = \text{diag}(-L_g/EA_g) \]; and \([N_{21}]\) is the compatibility matrix related to \([N_{12}]\) by\(^{15}\)

\[ [N_{21}] = [N_{12}]^T. \]  \(13\)

The total stiffness coefficient matrix is a function of the direction cosines of the element axis and nodal coordinates of elements in the global system after deformation. Superscript \(T\) on the right side of equation 13 denotes the transposition of the matrix.

**Experimental and calculating condition**

In order to verify the validity of the numerical model as stated above, the geometry of a simplified tuna long-line model was calculated, as an example of a typical commercial fishery task.\(^{17}\) The computed results were compared with experimental results obtained by flume testing in the circulating water tank at Tokyo University of Fisheries. The simplified model was set at a given inclined angle (\(\alpha\)) relative to the water current, as shown in Fig. 2 in a global coordinate system. As there is a limit to the depth of the circulating water tank, we let the shortening rate of the mainline equal to 0.95 in the experiment. The equilibrium configurations of the simplified model were measured and analyzed by means of a digital image analysis system.

The ropes of the simplified flume test model were made of polypropylene, and the length of mainline and branch-line were 2.4 m and 0.6 m, respectively. In order to aid observation of the working shape of the system, a line-shaped lead was wrapped inside the ropes. The ropes were \(\varnothing 6\) mm in diameter and weighed 0.075 kg/m in water, and the weight of each sinker attached to the end of each branch-line in water was 0.027 kg.

As shown in Fig. 2, there were 17 elements and 18 nodes in this numerical analysis example. The mainline was divided equally into eight elements of 30 cm, and each branch-line was divided into three elements of 20 cm. Because the ends of the mainline are fixed, their displacements are considered to be zero during movement of the system. As the convergence of the numerical model is not seriously dependent on initial values, we started our calculations from a simple shape; that is, assuming that the mainline is an isosceles triangle and branch-lines are straight lines.

**RESULTS AND DISCUSSIONS**

The calculated and measured shapes of the simple model, parallel to the current (\(\alpha=0^\circ\)) and at a flow velocity of 70 cm/s, are shown in Fig. 3. It is apparent that the theoretical and experimental values are in close agreement, although the calculated displacements of the middle and the downstreammost branch-line are somewhat inaccurate. The equilibrium configuration for this rope system is skewed downstream because of hydrodynamic forces, and the mainline does not conform to the catenary. The tension simulation results suggest that tensile forces on the rope decrease gradually in the downstream direction.

In commercial fishery, long-line operation is not always parallel to the current. The validity of the proposed method was then examined by comparing experimental data with the calculated static response of the rope system aligned at an angle (\(\alpha\)) across the current flow. In contrast to when the rope is parallel to the current, the rope system has a 3-D configuration in this situation. Side and top views of the system for \(V=50\) cm/s and \(\alpha=10^\circ\) are shown in Figs 4 and 5, respectively. In Fig. 4, the calculated result for the geometry of the system is very close to that of the experiment, and the downstream decrease in rope tension is also observed in this state.

As shown in Fig. 5, the model demonstrates that the mainline curves slightly downstream, whereas the branch-lines are reflected straight and parallel to the current because of hydrodynamic forces.

Comparing Figs 3 and 4, there is a better agreement between the computed values and measured values for the geometry of the system set across the current at an angle (\(\alpha=10^\circ\)). The reason may be that the flow disturbance from upstream lines is less in this case. The influence of upstream lines on the flow is responsible for a remarkable reduction
in the velocity of the downstream flow and, hence, in the hydrodynamic lift when the system is placed parallel to the water flow. As this effect is not considered in this numerical method, the position of the downstream-most branch-line becomes a little inaccurate.

A calculation procedure for determining the equilibrium configuration and tension of sub-
merged rope systems in a uniform current, based on a non-linear finite element method, has been described in the present report. The supple rope system can be validly divided into finite elements and the rope can be modeled as a series of straight rope elements. In comparison with conventional solutions, rope extension can be accounted for easily, and the elongation–displacement relationship is strictly and exactly maintained in the formulation. Second, Tronstad et al. have neglected terms of higher than second order in strain–displacement relationships in order to linearize non-linear basic equations. It is said that the approximation procedure may greatly affect the convergence and accuracy of calculations for a large displacement problem. Hence, in the present paper, a mixed method with respect to tensions of element and nodal displacements were used so that it was possible to maintain the strain–displacement relationship completely without any omission. Because the mixed formulation gives rise to increased unknowns by the auxiliary variables of element tension, the condensation procedure of these auxiliary unknowns can be considered to economize computing time for large-sized problems such as fishing nets. However, because there are not too many elements in the example used, good calculation accuracy was reached at the cost of computing time. Moreover, to represent the dependent relationship between hydrodynamic force and shape of the rope system, we introduced the Miyazaki model to predict the hydrodynamic forces acting on an element in each cycle of the Newton–Raphson iteration, iterating until the solution converged. It is also shown that the external forces on elements can be determined conveniently by the Miyazaki hydrodynamic model and that any referential transformations such as those done by Tronstad et al., need not be made.

In the prediction of the static response, not only of a long-line but also of various flexible rope systems including nets, the proposed method can be employed. With the support of the Newton–Raphson method, a solution of the desired accuracy is ensured even in situations involving large deformations.

REFERENCES