A Hybrid Method to Improve Forecasting Accuracy Utilizing Genetic Algorithm and Its Application to Stock Market Price Data

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Abstract. Focusing that the equation of the exponential smoothing method (ESM) is equivalent to \((1,1)\) order ARMA model equation, a new method of estimation of the smoothing constant in the exponential smoothing method was proposed before by us which satisfied the minimum variance of forecasting error. In this paper, we utilize the above-stated theoretical solution. Firstly, we estimated the ARMA model parameter and then estimate the smoothing constants. Thus the theoretical solution is derived in a simple way and it may be utilized in various fields. This new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

Keywords: minimum variance, exponential smoothing method, forecasting, trend, genetic algorithm

1. INTRODUCTION

In this paper, utilizing above-stated method, revised forecasting method is proposed. In making forecast such as stock market price data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the stock market price data of consumer electronics industry. Genetic Algorithm is utilized to search optimal weights for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Measuring method of forecasting accuracy is exhibited in 5. GA model to search optimal weights for the weighting parameters of linear and non-linear function is introduced and forecasting is executed in section 6, and estimation accuracy is examined.

2. DESCRIPTION OF ESM USING ARMA MODEL\textsuperscript{[1]}

In ESM, forecasting at time \(t + 1\) is stated in the following equation.

\[
\hat{x}_{t+1} = \hat{x}_t + \alpha (x_t - \hat{x}_t) \quad (1)
\]

Here,

\[
\hat{x}_{t+1} : \text{forecasting at } t + 1 \\
x_t : \text{realized value at } t \\
\alpha : \text{smoothing constant } \ (0 < \alpha < 1)
\]

(2) is re-stated as:

\[
\hat{x}_{t+1} = \sum_{j=0}^{\infty} \alpha (1 - \alpha)^j x_{t-j} \quad (3)
\]

By the way, we consider the following \((1,1)\) order ARMA model.

\[
x_t - x_{t-1} = e_t - b e_{t-1} \quad (4)
\]

Generally, \((p,q)\) order ARMA model is stated as:

\[
x_t + \sum_{j=1}^{p} a_j x_{t-j} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \quad (5)
\]

Here,

\[
\{x_t\}: \text{Sample process of Stationary Ergodic Gaussian Process } x(t) \\
\{e_t\}: \text{Gaussian White Noise with 0 mean } \sigma^2 \text{ variance}
\]

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that:

\[
E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0
\]
we get the following equation from (4).
\[ \hat{x}_{t+1} = x_{t+1} - \beta \hat{e}_{t+1} \]  (6)
Operating this scheme on \( t + 1 \), we finally get:
\[ \hat{x}_{t+1} = \hat{x}_t + (1 - \beta) \hat{e}_t \]
\[ = \hat{x}_t + (1 - \beta)(x_t - \hat{x}_t) \]  (7)
If we set \( 1 - \beta = \alpha \), the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model.
Comparing (4) with (5) and using (1) and (7), we get:
\[
\begin{align*}
    a_1 &= -1 \\
    b_1 &= -\beta
\end{align*}
\]  (8)
From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model.
But, generally MA part of ARMA model become non-linear equations which are described below.
Let (5) be:
\[ \tilde{x}_i = x_i + a_j x_{i-j} \]  (9)
\[ \tilde{x}_i = e_i + b_j e_{i-j} \]  (10)
We express the autocorrelation function of \( \tilde{x}_i \) as \( \tilde{r}_k \) and from (9), (10), we get the following non-linear equations which are well known\(^2\)
\[
\begin{cases}
    \tilde{r}_k = \sigma_e^2 \sum_{j=0}^{q} b_j b_{k-j} & (k \leq q) \\
    0 & (k \geq q + 1)
\end{cases}
\]  (11)
For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only \( b_1 \), so it can be solved in the following way.
From (4) (5) (8) (11), we get:
\[
\begin{align*}
    q &= 1 \\
    a_1 &= -1 \\
    b_1 &= -\beta = \alpha - 1 \\
    \tilde{r}_0 &= \sigma_e^2 \\
    \tilde{r}_1 &= b_1 \sigma_e^2
\end{align*}
\]  (12)
If we set:
\[
\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0}
\]  (13)
the following equation is derived.
\[
\rho_1 = \frac{b_1}{1 + b_1^2}
\]  (14)
We can get \( b_1 \) as follows.
\[
b_1 = \frac{1 \pm \sqrt{1 - 4 \rho_1^2}}{2 \rho_1}
\]  (15)
In order to have real roots, \( \rho_1 \) must satisfy
\[
|\rho_1| \leq \frac{1}{2}
\]  (16)
As \( \alpha = b_1 + 1 \)
\( b_1 \) is within the range of
\[-1 < b_1 < 0\]
Finally we get
\[
b_1 = \frac{1 - \sqrt{1 - 4 \rho_1^2}}{2 \rho_1}
\]
\[
\alpha = 1 + 2 \rho_1 - \sqrt{1 - 4 \rho_1^2}
\]  (17)
which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

3. TREND REMOVAL METHOD
As ESM is a one of a linear model, forecasting accuracy for the time series with non-linear trend is not necessarily good. How to remove trend for the time series with non-linear trend is a big issue in improving forecasting accuracy. In this paper, we devise to remove this non-linear trend by utilizing non-linear function.
As trend removal method, we describe linear and non-linear function, and the combination of these.

[1] Linear function
We set:
\[
y = a_1 x + b_1
\]  (18)
as a linear function, where \( x \) is a variable, for example, time and \( y \) is a variable, for example, stock market price, \( a_1 \) and \( b_1 \) are parameters which are estimated by using least square method.

[2] Non-linear function
We set:
\[
y = a_2 x^2 + b_2 x + c_2
\]  (19)
\[
y = a_3 x^3 + b_3 x^2 + c_3 x + d_3
\]  (20)
as a 2nd and a 3rd order non-linear function. \( a_2, b_2, c_2 \) and \( a_3, b_3, c_3, d_3 \) are also parameters for a 2nd and a 3rd
order non-linear functions which are estimated by using least square method.

[3] The combination of a linear and a non-linear function

We set:

\[ y = \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) + \alpha_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \]  
(21)

\[ 0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq \alpha_3 \leq 1 \]

\[ \alpha_1 + \alpha_2 + \alpha_3 = 1 \]  
(22)

as the combination linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the original data by (21). The optimal weight parameter \( \alpha_1, \alpha_2, \alpha_3 \) are determined by utilizing GA. GA method is precisely described in 6.

4. MONTHLY RATIO

For example, if there is the monthly data of L years as stated bellow:

\( \{X_{ij}\} \ (i = 1, \ldots, L) \ (j = 1, \ldots, 12) \)

where \( X_{ij} \in \mathbb{R} \) in which \( j \) means month and \( i \) means year and \( X_{ij} \) is a shipping data of \( i \)-th year, \( j \)-th month. Then, monthly ratio \( \bar{X}_j \ (j = 1, \ldots, 12) \) is calculated as follows.

\[ \bar{X}_j = \frac{1}{L} \sum_{i=1}^{L} \frac{X_{ij}}{L} \]

(23)

Monthly trend is removed by dividing the data by (23). Numerical examples both of monthly trend removal case and non-removal case are discussed in 7.

5. FORECASTING ACCURACY

Forecasting accuracy is measured by calculating the variance of the forecasting error. Variance of forecasting error is calculated by:

\[ \sigma^2_e = \frac{1}{N-1} \sum_{i=1}^{N} (e_i - \bar{e})^2 \]  
(24)

Where, forecasting error is expressed as:

\[ e_i = \hat{X}_i - X_i \]  
(25)

\[ \bar{e} = \frac{1}{N} \sum_{i=1}^{N} e_i \]  
(26)

6. NUMERICAL EXAMPLE

6.1 Application to stock market price data

Following five typical stocks of consumer electronics industry are selected.

- Sharp Corporation: "SHARP"
- Panasonic Corporation: "Panasonic"
- Sony Corporation: "SONY"
- Hitachi, Ltd.:"HITACHI"
- TOSHIBA CORPORATION:"TOSHIBA"

The above mentioned 5 companies for 2 cases from April 2011 to March 2014 are analyzed. Furthermore, GA results are compared with the calculation results of all considerable cases in order to confirm the effectiveness of GA approach. First of all, graphical charts of these time series data are exhibited in Figure 6-1 ～ 6-5.
6.2 Execution Results

GA execution condition is exhibited in Table 6-1.

We made 10 times repetition and the minimum of the variance of forecasting error and the average of convergence generation are exhibited in Table 6-2 and 6-3.

In the case monthly ratio is not used, the combination of linear and 3rd order non-linear function model is best in SHARP and Panasonic. On the other hand, the combination of 2nd+3rd order non-linear function model is best in SONY, HITACHI and TOSHIBA.

In the case monthly ratio is not used, the combination of linear and 3rd order non-linear function model is best in SHARP and TOSHIBA. On the other hand, the combination of linear and 2nd+3rd order non-linear function model is best in SONY and Panasonic. And the combination of linear function model is best in HITACHI.

Parameter estimation results for the trend of equation (21) using least square method are exhibited in Table 6-6 for the case of 1st to 24th data.

### Table 6-1: GA Execution Condition

<table>
<thead>
<tr>
<th>GA Execution Condition</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>100</td>
</tr>
<tr>
<td>Maximum Generation</td>
<td>50</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation ratio</td>
<td>0.05</td>
</tr>
<tr>
<td>Scaling window size</td>
<td>10</td>
</tr>
<tr>
<td>The number of elites to retain</td>
<td>2</td>
</tr>
<tr>
<td>Tournament size</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 6-2: GA execution results (Monthly ratio is used)

<table>
<thead>
<tr>
<th>Case</th>
<th>Minimum the variance of forecasting error</th>
<th>Average of convergence generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>4,145</td>
<td>9.6</td>
</tr>
<tr>
<td>SONY</td>
<td>71,020</td>
<td>8.5</td>
</tr>
<tr>
<td>Panasonic</td>
<td>12,299</td>
<td>8.4</td>
</tr>
<tr>
<td>HITACHI</td>
<td>5,163</td>
<td>6.9</td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>2,385</td>
<td>13.3</td>
</tr>
</tbody>
</table>

### Table 6-3: GA execution results (Monthly ratio is not used)

<table>
<thead>
<tr>
<th>Case</th>
<th>Minimum the variance of forecasting error</th>
<th>Average of convergence generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>4,269</td>
<td>7.7</td>
</tr>
</tbody>
</table>

### Table 6-4: Optimal weights (Monthly ratio is used)

<table>
<thead>
<tr>
<th>Case1</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>0.55</td>
<td>0.00</td>
<td>0.45</td>
</tr>
<tr>
<td>SONY</td>
<td>0.00</td>
<td>0.47</td>
<td>0.53</td>
</tr>
<tr>
<td>Panasonic</td>
<td>0.04</td>
<td>0.00</td>
<td>0.96</td>
</tr>
<tr>
<td>HITACHI</td>
<td>0.00</td>
<td>0.76</td>
<td>0.24</td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>0.00</td>
<td>0.67</td>
<td>0.33</td>
</tr>
</tbody>
</table>

### Table 6-5: Optimal weights (Monthly ratio is not used)

<table>
<thead>
<tr>
<th>Case2</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>0.86</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>SONY</td>
<td>0.62</td>
<td>0.06</td>
<td>0.32</td>
</tr>
<tr>
<td>Panasonic</td>
<td>0.96</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>HITACHI</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>0.97</td>
<td>0.00</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table 6-6: Parameter estimation results for the trend of equation (21)

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>-28.014</td>
<td>840.924</td>
<td>0.062</td>
<td>-29.558</td>
<td>846.614</td>
<td></td>
</tr>
<tr>
<td>SONY</td>
<td>-44.513</td>
<td>1982.830</td>
<td>4.651</td>
<td>-160.792</td>
<td>2486.707</td>
<td></td>
</tr>
<tr>
<td>Panasonic</td>
<td>-18.984</td>
<td>924.091</td>
<td>1.538</td>
<td>-57.425</td>
<td>1090.667</td>
<td></td>
</tr>
<tr>
<td>HITACHI</td>
<td>3.333</td>
<td>422.670</td>
<td>0.309</td>
<td>-4.402</td>
<td>456.190</td>
<td></td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>-1.796</td>
<td>367.826</td>
<td>1.082</td>
<td>-28.836</td>
<td>484.998</td>
<td></td>
</tr>
</tbody>
</table>

Trend curves are exhibited in Figure 6-6～6-10.

![Figure 6-6: Trend of SHARP](image1)

![Figure 6-7: Trend of SONY](image2)

![Figure 6-8: Trend of Panasonic](image3)

Figure 6-9: Trend of HITACHI

![Figure 6-10: Trend of TOSHIBA](image4)

Figure 6-11: Forecasting Result of SHARP

Table 6-7: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is used)

<table>
<thead>
<tr>
<th></th>
<th>$\rho_1$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>-0.4286</td>
<td>0.4341</td>
</tr>
<tr>
<td>SONY</td>
<td>-0.4346</td>
<td>0.4183</td>
</tr>
<tr>
<td>Panasonic</td>
<td>-0.0243</td>
<td>0.9756</td>
</tr>
<tr>
<td>HITACHI</td>
<td>-0.0983</td>
<td>0.9007</td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>-0.2681</td>
<td>0.7093</td>
</tr>
</tbody>
</table>

Table 6-8: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is not used)

<table>
<thead>
<tr>
<th></th>
<th>$\rho_1$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHARP</td>
<td>-0.3904</td>
<td>0.5194</td>
</tr>
<tr>
<td>SONY</td>
<td>-0.3829</td>
<td>0.5339</td>
</tr>
<tr>
<td>Panasonic</td>
<td>-0.0107</td>
<td>0.9893</td>
</tr>
<tr>
<td>HITACHI</td>
<td>-0.2943</td>
<td>0.6746</td>
</tr>
<tr>
<td>TOSHIBA</td>
<td>-0.3156</td>
<td>0.6445</td>
</tr>
</tbody>
</table>

Forecasting results are exhibited in Figure 6-11～6-15.
6.3 Remarks

In 80% cases, that monthly ratio was not used had a better forecasting accuracy (Table7-2,7-3). SHARP and TOSHIBA had a good result in 1st+3rd order, SONY and Panasonic had a good result in 1st+2nd+3rd order. And HITACHI had a good result in 1st order.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

7. CONCLUSION

Focusing on the idea that the equation of exponential smoothing method (ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of the smoothing constant in the exponential smoothing method was proposed before by us which satisfied the minimum variance of forecasting error. Generally, the smoothing constant was selected arbitrary. But in this paper, we utilized the above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus the theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to increase forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the stock market price data of consumer electronics industry. The combination of linear and non-linear function was also introduced in trend removing. Genetic Algorithm is utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

REFERENCES