STATISTICAL ANALYSIS OF FLASHOVER

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ABSTRACT

Many successful fire tests were carried out so far and are now being accumulated as the data base which can be analysed statistically by an electric computer. In this paper, by statistically analysing such accumulated fire test data, the effects of the various factors on the flashover time and the presence of flashover in fire spread process are estimated, besides, the domain where flashover occurs is shown on the two-dimensional Euclidian space defined by the linear sum of the values given to the respective levels of the factors as the result of the analysis.

1. INTRODUCTION

Flashover is a very critical phenomenon in fire spread process, the occurrence of which gives a large effects especially on life safety in buildings. In this paper, the flashover time and the presence of flashover in fire spread process are statistically analysed by using fire test data accumulated so far.

When analysing fire test data, statistical techniques should be examined well since fire tests generally deal with the qualitative factors such as types of interior linings besides the quantitative ones such as water contents of materials. Similar does the subject to be analysed, that is, the flashover time is considered as a quantitative variable and the presence of flashover is considered as a qualitative one. "Theory of Quantification" developed by HAYASHI¹⁾ is very useful for analysing such phenomenon where qualitative variables and quantitative ones are thus mixed.

After statistical multi-variate analysis techniques including "Theory of Quantification" are introduced at the following paragraph, the result of the analysis of flashover in fire tests are reported.

2. STATISTICAL TECHNIQUES

In statistical terminology, the subject to be analysed or estimated, that is flashover in this study, is called dependent variable, while, the factors are called independent variables. These variables are either qualitative or quantitative. Qualitative variables have levels given with nominal scale and quantitative variables have values given with interval scale. What kind of multi-variate analysis techniques is available for some subject depends upon the types of dependent variable and independent variables. Table 1 shows the available techniques corresponding to the types of variables. All of these techniques premise linear sum model of factors, and the effect of each factor is estimated by way of minimizing whether the correlation coefficient (a-c) or the correlation ratio (d-f). Technique (c) covers (a) and (b), and technique (f) covers (d) and (e), so only techniques (c) and (f) are explained below. The other techniques can be led easily as the special cases of these techniques.

Table 1 Statistical Multi-variate Analysis Techniques

Dependent Var. Independent Var.	Interval Scale	Nominal Scale
Interval Scale	(a) Multiple Correlation Analysis	(d) Multiple correlation Ratio Analysis
Nominal Scale	(b) Theory of Quantification type I	(e) Theory of Quantification type II
Interval Scale & Nominal Scale are mixed	(c) Theory of Quantification type I'	(f) Theory of Quantification type II'

(b), (c), (e), (f) were developed by C. Hayashi ¹⁾

Either of the techniques premises the following linear sum model of factors.

$$\alpha_i = \sum_{j=1}^R \sum_{k=1}^{K_j} \delta_i(jk) X_{jk} + \sum_{l=1}^r W_l Z_{li} + C \quad (1)$$

where

α_i : Component variable on Sample i (sample score)

R : Number of factors expressed with nominal scale

K_j : Number of categories of factor j

$\delta_i(jk) = \begin{cases} 1 : \text{if sample } i \text{ is classified into category } k \text{ by factor } j \\ 0 : \text{otherwise} \end{cases}$

X_{jk} : Category Value which is given to category k of factor j . This value is estimated by the method explained later.

r : Number of factors expressed in interval scale

W_l : Weight given to factor l (interval scale)

Z_{li} : Value of factor l of sample i

C : Constant

The method to estimate X_{jk} , and W_l and C depends upon the type of the dependent variable.

(i) The case that dependent variable is interval scale. (c)

X_{jk} and W_l are estimated so that Pearson's product moment correlation coefficient between sample score α and dependent variable y may be maximum. The correlation coefficient $\rho_{y\alpha}$ is expressed as follows.

$$\rho_{y\alpha} = \frac{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})(\alpha_i - \bar{\alpha})}{\sigma_y \cdot \sigma_\alpha} \quad (2)$$

where, n is the total number of samples. $\bar{y}, \bar{\alpha}$ are the averages of y, α and σ_y, σ_α are the standard deviations of y, α respectively.

The condition that $\rho_{y\alpha}$ is maximum leads to the condition that

$$\frac{\partial \rho_{y\alpha}}{\partial X_{uv}} = 0 \quad (u = 1, 2, \dots, R; v = 1, 2, \dots, K_u), \quad \frac{\partial \rho_{y\alpha}}{\partial W_l} = 0 \quad (l = 1, 2, \dots, r)$$

are satisfied simultaneously.

These expressions lead to the simultaneous linear equation concerning X_{uv} and W_L . This simultaneous linear equation is an arbitrary one as it is, by the fact that whether some sample is classified into arbitrary one of the categories of each factor or not depends upon the state of classification into other categories. Consequently, this equation is solved generally by adding the following conditions, the number of which is R .

$$\sum_{k=1}^{K_j} X_{jk} n_{jk} = 0 \quad (j = 1, 2, \dots, R) \quad (3)$$

where, n_{jk} is the number of samples which are classified into category k by factor j .

Further, the correlation coefficient is constant even if α is transformed linearly, so constant value C and the scale of the solution can be determined arbitrarily. If they are determined as $\bar{y} = \bar{\alpha} = C$, and $\sigma_\alpha = \rho_{Y\alpha} \cdot \sigma_Y$, the result of the analysis can be interpreted easier since the solution thus obtained is equal to that by least square method, namely, the solution solved under the condition that the square sum of error term, which is given as the following expression, is minimum.

$$S = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \alpha_i)^2 \quad (4)$$

(ii) The case that dependent variable is nominal scale : (f)

This method is a kind of discriminant analysis using the correlation ratio as the criterion of preciseness of discrimination. X_{jk} and W_L are estimated so that the correlation ratio may be maximum. The correlation ratio η^2 is the ratio of "between class variance" σ_b^2 to the "total variance" σ^2 , which are expressed as follows.

$$\eta^2 = \frac{\sigma_b^2}{\sigma^2} \quad (5)$$

where

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (\alpha_i - \bar{\alpha})^2$$

$$\sigma_b^2 = \sum_{t=1}^{t_m} \frac{n_t}{n} (\bar{\alpha}_t - \bar{\alpha})^2$$

n : Total number of sample

n_t : The number of the samples which is classified into category by dependent variable.

$\bar{\alpha}_t$: The average of sample score of the samples belonging to category

t_m : The number of categories of dependent variable

The condition that η^2 is maximum leads to the condition that

$$\frac{\partial \eta^2}{\partial X_{uv}} = 0 \quad (u = 1, 2, \dots, R; v = 1, 2, \dots, k_u), \quad \frac{\partial \eta^2}{\partial W_l} = 0$$

($l = 1, 2, \dots, r$) are satisfied simultaneously.

Substituting the expression (5),

$$\begin{aligned} \frac{\partial \sigma_b^2}{\partial X_{uv}} &= \eta^2 \frac{\partial \sigma^2}{\partial X_{uv}} \quad (u = 1, 2, \dots, R; v = 1, 2, \dots, k_u) \\ \frac{\partial \sigma_b^2}{\partial W_l} &= \eta^2 \frac{\partial \sigma^2}{\partial W_l} \quad (l = 1, 2, \dots, r) \end{aligned} \quad (6)$$

These simultaneous equations lead to the eigenvalue problem after all, where η^2 is the eigenvalue and $X^t = [X_{11} \ X_{12} \ \dots \ X_{1k_1} \ X_{21} \ \dots \ X_{jk} \ \dots \ X_{Rk_R} \ W_1 \ \dots \ W_l \ \dots \ W_r]$ is the eigen vector.

In order to solve this problem uniquely, besides the condition (3) is necessary to be considered, σ^2 and C are also necessary to be determined appropriately since the correlation ratio is constant even if α is transformed linearly. Here, the conditions that $\sigma^2 = \eta^2$ and $C = 0$ are added.

At this eigen value problem, the spectral factorization of the matrix concerned give the solutions, the number of which is equal to the rank of the matrix, where the eigen vectors are independent with each other. When discriminating the categories of dependent variable, namely, when predicting the phenomenon, better discrimination can be achieved generally by taking up not only the solution which gives the maximum correlation ratio (eigen value) but some other solutions together.

After X_{jk} , W_l and C were determined, the effect of the factors can be com-

pared with the values of partial correlations between dependent variable and each factor (When calculating the partial correlation, $\bar{a}_t$ is substituted for the value of dependent variable, if it is nominal scale). The comparison of the effects only among the factors of nominal scale ²⁾ can be achieved also by the range of category value X_{jk} within each factor.

By the way, Theory of Quantification type I' (c) is mathematically equal to the multiple correlation analysis using 1-0 dummy variable for the independent variable of nominal scale. However, it would be a remarkable point that the comparison of the effects of qualitative factors with the partial correlations or the ranges is generated by the conception to quantify the qualitative variables, not merely to calculate the partial regression coefficient of 1-0 dummy variable.

3. ANALYSIS OF FLASHOVER BY FIRE TEST DATA

Occurrence of flashover can be analysed by applying Theory of Quantification type I' and type II' introduced at the previous paragraph.

The Fire Test Data analysed here are those done by Dr. F. SAITO, the main purpose of which is to examine the effect of the thickness of plywood on smoke generation. Flashover time is also observed in this test. Such results of the tests are reported at literature 3).

The model box used was 95 cm width x 46 cm high x 43 cm depth in size, and plywood was set on the ceiling and the back and side walls. Cribs were arranged in the corner of the box. Small piece of soft fiber board (1cm x 1cm) dipped with alcohol was used for ignition. The figure of the model box is shown at appendix I.

The tests were designed to examine the effects of the following factors.

1. Rate of Opening Area to Front Area ---- 2 levels

(1) 1/4 (2) 1/2

2. Size of Fuel ---- 3 levels

(1) 8 Cribs, the length of which are 6 cm (4 layers)

(2) 16 Cribs, the length of which are 14 cm (4 layers)

(3) 25 Cribs, the length of which are 17 cm (5 layers)

Each Crib has the section 2cm x 2 cm.

3. Thickness of plywood ---- 5 levels

(1) 3.0 mm (2) 5.5 mm (3) 7.5 mm (4) 9.0 mm (5) 12.0 mm

Rate of water content of Crib and Plywood were also measured, and they were also taken up as the factors of flashover together with the above three factors.

(i) Analysis of Flashover Time

Theory of Quantification type I' can be applied in this case since flashover time, which is dependent variable, is given with interval scale, and the factors, which are independent variables, are given with both nominal scale and interval scale.

The result of the analysis of flashover time is shown at table 2.

Table 2 Result of the Analysis of Flashovertime by Theory of Quantification Type

	No. of Sample	Category Value	Range	Partial Corr.
Openings 1/4	30	0.089	0.168	0.063
1/8	34	-0.079		
Crib 6 cm	7	3.897	5.549	0.761
Length 14	36	0.206		
17	21	-1.652		
Thickness 3.3 mm	17	-1.457	2.788	0.573
of 5.5	16	0.292		
Plywood 7.5	11	1.331		
9.0	11	0.155		
12.0	9	0.417		
Rate of Water Content of Crib*	64	0.079 (Weight)***		0.052
Rate of Water Content of Plywood**	64	-0.035 (Weight)****		-0.024

* AV = 9.4%, SD = 1.7 % ** AV = 7.9 %, SD = 1.6 %

*** Weight is calculated using standerdised value of Rate of Water Content $Z_{Li} = (X_{Li} - AV_L)/SD_L$

Comparing the values of partial correlation, the effects of Size of Fuel is the largest among 5 factors, and the effect of Thickness of Plywood is next to it. On the other hand, Rate of Opening Area and Rate of Water Content have little effect. Of course such results are based on the levels of factors taken up.

The category values of Size of Fuel show that the larger is the Size of Fuel, the smaller is the category value. This means that the larger is the fuel, the shorter is flashover time.

Seeing the category value of thickness of Plywood, the value for 3.0 mm is particularly small, therefore you can see that flashover time is particularly short in this case. The relation between Thickness of Plywood and category value is not monotonous, the value is maximum in the case of 7.5 mm.

Figure 1 show the relation between observed flashover time and estimated one. The correlation coefficient which indicates the preciseness of the analysis is 0.785.

Fig. 1
Relation between Flashover
Time and Estimated Value
(Correlation Coef. = 0.785)

(ii) Analysis including the case that flashover did not occur

Theory of Quantification type II' can be applied in this case by arranging the dependent variables as follows. Namely, flashover time is classified into the following five categories. (1) 199 sec. below (2) 200 - 299 sec. (3) 300 - 399 sec. (4) 400 - 499 sec. (5) 500 sec. above. Further, the category (6) no flashover is added.

As the dependent variable thus arranged is nominal scale, the analysis is

done so that above categories may be well discriminated. Table 3 shows the two solutions which give the largest two correlation ratios (eigen values). These two solutions, namely axis I and axis II are independent with each other since they are obtained by spectral factorization of the matrix concerned. The correlation ratios, 0.770 of axis I and 0.357 of axis II are fairly good.

Table 3 Result of Multiple Correlation Ratio Analysis (Quality & Quantity Mixed)

AXIS		I			II		
Correlation Ratio		0.770			0.357		
Nominal Scale	No. of Sample	Category Value	Range	Partial Corr.	Category Value	Range	Partial Corr.
Openings WxHx1/4	33	-0.022	0.042	0.043	-0.138	0.261	0.157
WxHx1/8	37	0.020			0.123		
Crib Length							
6 cm	13	-1.700	2.390	0.863	-0.382	0.852	0.454
14	36	0.211			0.401		
17	21	0.690			-0.451		
Thickness of Plywood							
3.0 mm	17	0.535	1.036	0.573	-0.543	0.949	0.381
5.5	17	0.002			0.134		
7.5	13	-0.501			-0.020		
9.0	12	-0.064			0.229		
12.0	11	-0.169			0.406		
Water Content of Crib*	70	-0.076 (weight)***		-0.136	-0.038 (weight)***		-0.041
Water Content of Plywood**	70	0.058 (weight)***		0.102	-0.216 (weight)***		-0.231

* AV = 9.4 %, SD = 1.7 % ** AV = 7.8 %, SD = 1.6 %

*** Weight is calculated using standardized value of Rate of Water Content

$$; Z_{Li} = (X_{Li} - AV_L) / SD_L$$

Either the partial correlations and the category values of axis I can be interpreted almost similarly with the results of the analysis of flshover time.

Figure 2 shows the distribution of sample score . The distribution on the direction of axis I shows that the shorter is the flashover time, the larger is the sample score, and the category (6) no flashover separates remarkably. Axis II shows that the longer is the flashover time, the larger is the sample score except two categories of (5) 500 sec. above and (6) no flashover.

The ellipses in figure 2 show 90 % confidence bounds on two dimensional distribution of sample scores based on the assumption of normal distribution. As being expected from this figure, the prediction of flashover time or presence of flashover can be achieved by determining the domains appropriately on the multi-dimensional Euclidian space where sample scores distribute.

Figure 2 Distribution of Sample Score

Note: Ellipses show 90 % confidence bounds of two dimensional distribution of sample scores based on the assumption of normal distribution

5. CONCLUDING REMARKS

It was made out that the multi-variate analysis techniques introduced in

this paper is useful for the factor analysis and the estimation of flashover, the results of which can be easily interpreted in relation to the phenomenon by showing them with a diagram.

The development of such analysis of Fire Test Data which were accumulated so far will give more valuable and direct informations to the estimation of flashover.

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