A Calculation Method for Predicting Heat and Smoke Detector's Response

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ABSTRACT

Utilizing a zone model and a ceiling-jet model, a simple calculation method for predicting fire detector's response time is presented. The parameters are the floor area and the ceiling height of the room, the radial distance of the detector from the fire axis, time history of the heat and smoke release rates of the fire, and the response characteristics of the detector. For the heat detector, application of RTI-C model (a modified Response Time Index model) is discussed and a method utilizing two RTI's, one for the sensor element and the other for the detector body, is introduced. The method using two RTI's shows better results than the original RTI-C model, which was derived for predicting the response of sprinkler heads. In addition, a calculation method for predicting the response of rate-of-rise heat detector is presented. For the response of photoelectric smoke detector, a method utilizing a response threshold in the optical smoke concentration and a response threshold in the air velocity is introduced. It is shown that different response thresholds in the smoke concentration are needed for the smoke released from flaming sources and for the smoke released from smoldering sources respectively.

Keywords: Fire detection, Detector response, Heat detector, Smoke detector

1. INTRODUCTION

Prediction of fire detector's response for a given fire in a building is important in designing a fire safety system for the building. For example, the detection time determines the initiation of escape by building occupants. In addition, the detection signal may be used for activating a smoke control system, which keeps the escape route safe during evacuation of the building. The objective of this study is to provide simple engineering tools for estimating the response time of heat and smoke detectors installed on the ceiling of enclosed rooms.

A fire initiated in an enclosed room forms a buoyant upward plume above it. The plume carries heat and smoke released from the fire towards the ceiling. The flow rate of the plume increases with height as it entrains surrounding air, while the temperature and the smoke concentration decrease as they are diluted with the entrained air. When the plume impinges on the ceiling, the flow changes its course, and a radial outward

flow, the ceiling-jet, is formed beneath the ceiling. After the smoke front of the ceiling-jet reaches the wall, then a hot stratified layer, which contains smoke, is formed in the upper portion of enclosure. After this smoke layer is formed, environment of the plume and ceiling-jet changes, and heat and smoke in the upper layer are re-entrained to the plume and ceiling-jet, thus the local temperature and smoke concentration near the ceiling increase with time as the thickness of the upper layer increases. Plume, ceiling-jet, as well as the smoke layer play important roles in determining the response time of a fire detector installed on the ceiling of an enclosed room (Figure 1).

Figure 1 Fire detector in a room fire environment

Alpert [1] made an extensive experimental study of unconfined ceiling-jets made by steady-state fires, and provided a set of useful correlations for estimating the radial distributions of temperature and velocity near the ceiling, which can be used for estimating the response of heat sensitive detectors. Alpert [2] also developed a theoretical model which explains very well the radial temperature and velocity distributions of the ceiling-jet.

Heskestad and Delichatsios [3] made a thorough experimental study on the ceiling environment made by the fire and response to this environment by various types of detectors. They found that temperature and velocity distributions near a flat, unconfined ceiling can be predicted with reduced correlations of temperature and velocity as functions of time and ceiling height, where unsteady power-law growth rate of the fire is accounted for [4,5]. As for the radial distribution of smoke concentration, they found that there exists an approximately constant ratio between the optical density and the local temperature rise, where the value of the ratio depending on the burning material and time [3]. For evaluating the characteristics of smoke entry to the detection chamber, a concept of characteristic length is introduced (Heskestad [6]).

Heskestad and Smith [7] developed an RTI (Response Time Index) concept for evaluating the response of the fusible link of sprinkler heads. Given the environmental condition of the link, temperature and velocity, as functions of time, temporal variation of the link temperature can be calculated by integrating the heat absorbed by the link.

The RTI model is later upgraded to RTI-C model (Heskestad and Bill [8]) and being used as a standard model for evaluating the response of sprinkler heads. This RTI-C model may also be used for calculating the response of heat detectors.

Smoke detector's response to actually burning materials was experimentally studied by Bukowski and Bright [9], in which photoelectric detector's response was studied in comparison with the ionization detector. The materials used were shredded paper, polystyrene, gasoline, wood brand, cotton and polyurethane. Their findings include that the ionization detector has a superior performance over the photoelectric detector in detecting fires where open flame and little visible smoke predominated, while the photoelectric detector showed a superior response over the ionization detector to truly smoldering fires. They also pointed out that photoelectric detectors tended to show poor smoke entry characteristics at low air velocities. Lee and Mulholland [10], and Mulholland and Liu [11] later studied the particulate properties of smokes and showed that the different response characteristics of ionization and photoelectric detectors can be correlated to the smoke particle size and its distribution. They pointed out that mode of generation (flaming or smoldering), pyrolysis temperature, air velocity at smoke emitting site, and aging have large effect on the particle size distribution.

Because of the complex nature, precise modeling of smoke detector's response has been avoided for engineering purposes, and simple methodologies are in use for estimating the response time of smoke detectors – one is the temperature rise method and the other is the optical density method [12,13,14]. In the temperature rise method, the detection time is determined by the extent of temperature rise (of the gas at detector location) in representative of the smoke concentration. Although different threshold values should be used to take into account the difference of fuels and detector types [3], even a method using threshold values in the narrow range of 10 to 15 deg C is used for all flaming fires and detectors [12]. Beyler and DiNenno [15] discussed the validity of this temperature method and criticized that it has little scientific basis.

In the optical density method, the smoke concentration at the detector is directly calculated and compared with a threshold to determine the response time [12,13,14]. To take into account the smoke entry resistance, the smoke concentration inside the detection chamber is calculated with an equation using a characteristic length of the detector [6,16]. Setting an appropriate threshold value (in smoke concentration) for the detector in concern is another difficult problem, because the detector's output generally depends on the kind of fuel and combustion mode (flaming or smoldering) as well as the detection principle (ionization or photoelectric) [10,11]. Avoiding complex consideration of relating parameters, a method simply to use a nominal sensitivity value of the detector is even used as a compromising method [17].

For a purpose of defining an appropriate threshold smoke optical density outside of a detector at which a smoke detector would be expected to alarm, Geiman and Gottuk [18] reviewed a number of full-scale fire test results in which the optical smoke density is recorded as well as the alarm time. Variables evaluated were detector type, fire type

and nominal detector sensitivity. They provided smoke optical density thresholds, each represents a level at which 20, 50 and 80 % of the detectors alarmed respectively in full-scale fire tests. However, they failed to precisely correlate the threshold values with the conditional variables that may exist, and concluded that determining a precise alarm time with the optical density (outside of a detector) is difficult. In the current study, a series of full-scale fire tests have been conducted with similar purposes. The detector kind was limited to photoelectric, but its analog output was monitored as well as the optical smoke concentration. In order to exclude the effect of smoke entry resistance in smoldering fires, small fans were used to increase the air velocity in the vicinity of the detector, so as to measure the true relationship between the optical smoke concentration and the detector output. Results of this experiment is shown later in section 2.5.

In conventional ionization detector response theories, such as Hosemann [19], Scheidweiler [20], and Helsper et al. [21], the change in the ionization current is correlated to the particle number concentration and the characteristic particle size of smoke aerosol. The particle number concentration rapidly varies after the smoke is released from a source and accumulates in a fire room. Yamauchi [22] showed that by tracking the number concentration in the smoke layer with an additional algorithm to a zone model, simulation of output variation of a measuring ionization chamber (MIC [23], which has an aspiration mechanism) is possible. Newman [24] later studied the effect of electrically charged particles in the smoke aerosol released from fire sources. He found that ionization detector's response is strongly dependent on the smoke particle charges influenced by both the combustion mode and the fuel kind. He showed that the ionization detector response theory can be modified with defining a detector/material sensitivity factor that can be predicted from the fuel soot yield. No additional work is done in the current study concerning the prediction of ionization detector's response.

Alpert's correlation [1] and Heskestad and Delichatsios's correlation [5] both are useful for estimating the temperature and velocity of the unconfined ceiling-jet, however, they are not directly applicable to the response prediction of detectors installed in closed rooms. In this case, effect of the smoke layer is indispensable for accurate predictions, provided that a conservative estimation is not the actual question. Cooper [25] and Evans [26], provided separately calculation methods which take into account the effect of the smoke layer to plume and ceiling-jet temperatures. Yamauchi [22] extended these methods to the prediction of smoke concentration near the ceiling. Yamauchi's method needs to solve differential equations of the plume and ceiling-jet at each calculation time step, which may be too complex for general use. In this paper, a more simplified calculation method is introduced.

2. MODEL DESCRIPTION

The calculation procedure consists of four parts, smoke layer, plume, ceiling-jet and sensor's response. Each part is described as follows.

2.1 Prediction of Smoke Layer with a Zone Model

A zone model bears a part of the performance of this calculation method, because the calculated layer temperature and smoke concentration as well as the layer height are very important in the ceiling-jet calculation. A choice from widely recognized codes, such as BRI2002 [27] and CFAST [28] is recommended, however, other simpler models may be used for analysis or to obtain more general results. A condition is that it must have a feature to calculate the convective heat loss to the ceiling and walls from the upper layer. Inclusion of radiative heat transfer is not a must, because the source strength is relatively small during the initial stage of the fire and the temperature variation (concerning the detector response) is generally less than 100 deg C at detector location, so the effect of radiation (other than the flame zone) may be neglected. With the same reason, the flame height is assumed to be small relative to the interface height.

Following is a set of equations used in this study. Detailed description of the zone model can be find elsewhere [27,28,29], therefore only the equations are listed below. A schematic is shown in Figure 2 and notations are listed in the last part of this paper.

Figure 2 A schematic of zone model

Mass balance:

$$\rho_0 S \frac{dz_i}{dt} = m_l - m_p, \quad (1)$$

$$S \frac{d}{dt} [\rho_u (H - z_i)] = m_p - m_u. \quad (2)$$

Energy balance:

$$\frac{dP}{dt} = \frac{R}{C_p S H} (Q_p - q_c + q_l - q_u) = 0. \quad (3)$$

Smoke mass:

$$S \frac{d}{dt} [C_{su} (H - z_i)] = Q_s - C_{su} \frac{m_u}{\rho_u}, \quad (4)$$

where

$$Q_p = Q_f - Q_R, \quad (5)$$

$$Q_c = h_w (T_u - T_w) A, \quad (6)$$

$$q_l = m_l C_p T_0, \quad q_u = m_u C_p T_u. \quad (7)$$

Equation of state:

$$P = \rho \cdot R \cdot T. \quad (8)$$

Heat conduction in the wall (one dimensional):

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}, \quad (9)$$

where boundary conditions are

$$-\lambda_c \left. \frac{\partial T}{\partial x} \right|_{x=0} = h_w (T_u - T_w), \quad -\lambda_c \left. \frac{\partial T}{\partial x} \right|_{x=l} = 0. \quad (10)$$

2.2 Plume in a two-layer environment

Morton, Taylor and Turner [30] provided a theoretical model for the steady-state buoyant plume. Their simple solution for the axial temperature and velocity distributions for the plume in a uniform environment is widely recognized among fire researchers. In the same paper, they extended their analysis into the behavior of the plume in a stratified environment.

Radial distributions of the plume temperature and velocity are assumed to be axisymmetric in Gaussian forms,

$$u = u_m \exp(-r^2/b^2), \quad (11)$$

$$\Delta T = \Delta T_m \exp(-r^2/(\lambda b)^2). \quad (12)$$

Then, the integrated mass, momentum and energy conservation equations respectively can be written as,

$$\frac{d}{dz} (\pi \cdot b^2 u_m) = 2\pi \cdot b \alpha \cdot u_m, \quad (13)$$

$$\frac{d}{dz} \left(\frac{1}{2} \pi \cdot b^2 u_m^2 \right) = \pi \cdot \lambda^2 b^2 g \frac{\Delta T_m}{T_0}, \quad (14)$$

$$\frac{d}{dz} \left(\frac{1}{(1+1/\lambda^2)} \pi \cdot b^2 u_m \rho_0 C_p \Delta T_m \right) = -\pi \cdot b^2 u_m \rho_0 C_p \frac{dT_a}{dz}. \quad (15)$$

If the radial distribution of the smoke concentration and the temperature are assumed to be similar with each other, $\Delta C_s / \Delta C_{sm} = \Delta T / \Delta T_m$, and the smoke is passively transported without affecting the flow, then the conservation equation of the smoke mass can be written as,

$$\frac{d}{dz} \left(\frac{1}{(1+1/\lambda^2)} \pi \cdot b^2 u_m \Delta C_{sm} \right) = -\pi \cdot b^2 u_m \frac{dC_{sa}}{dz}. \quad (16)$$

Equations 13-16 is applicable to the two layer environment generally observed in a room fire. If each of the lower and the upper layer is assumed uniform, then the right hand terms of Equations 15 and 16 are zeros at all heights (in each layer) but non-zero at the interface. The connecting conditions at the interface are illustrated in Figure 3 and discussed later in this section.

Figure 3 Connecting conditions of the conserved quantities of plume at the interface of upper and lower layers

Within the lower layer, an analytical solution of Equations 13-16 can be obtained assuming a virtual point source at an appropriate elevation. This solution provides the flow rate and the momentum flux at the interface height, where the enthalpy flux and the smoke mass flux relative to the environment abruptly change. Same equations (Equations 13-16) can be applied to the upper-part of the plume with these as new upstream boundary conditions.

The upper-layer plume flow is a buoyant plume whose initial mass and momentum fluxes are non-zero at the starting point, or a forced plume according to Morton's definition [31]. Morton [31] provided theoretical analysis of this forced plume, and Evans [26] applied this theory to solve the upper part of the plume in a two-layer environment. Evans [26] explored several methods to obtain the centerline temperature distribution of the plume, and pointed out that a sharp decrease in the centerline temperature across the interface is a problem in the exact solution of Equations 13-15, when the same radial distribution profile is assumed in the upper- and lower-layer plumes (Equation 12). Yamauchi [22] re-adjusted the plume parameters (such as entrainment constant α

and Gaussian width ratio λ) and showed that the exact solution of *Equations 13-15* gives a best fit to Evans's experimental data [26] and the sharp decrease can be avoided by extrapolating the lower-layer plume until the temperature value coincides with that of the upper-layer plume.

A drawback in Yamauchi's method [22] is that the plume equations (*Equations 13-16*) have to be solved numerically at each time step, because the layer height, upper-layer temperature and smoke concentration as well as the heat and smoke release rates vary with time through the course of calculation. This procedure may be inappropriate for engineering purposes. The calculation procedure introduced below is a simpler version of Yamauchi's method [22].

According to Morton's theory [31], the parametric ratio of buoyancy and momentum of a forced plume, which may be unbalanced at the source, quickly varies near the source and self-adjusts to a constant value of the natural plume sufficiently away from the source. In this sense, the fire plume in the upper layer is a forced plume whose buoyancy suddenly reduced (unbalanced) at the interface, which may quickly adjust to the balance of natural plume with increased entrainment near the interface. Neglecting this gradual variation of the buoyancy-momentum balance, approximation for the upper-layer plume is possible assuming a virtual source which gives the same flow rate and enthalpy flux (at the interface height) in a uniform upper-layer environment. This is equivalent to assume that the plume flow self-adjusts instantly to the buoyancy-momentum balance of natural plume at the interface. The position of this virtual source should be set somewhat below the actual source to give the same flow rate with a reduced enthalpy flux (*Figure 4*). Relations for the equivalent source parameters Q_{pe} , Q_{se} and the adjusted ceiling height H_e can be written as:

Figure 4 A plume in a two-layer environment and equivalent plume in a uniform upper-layer environment

$$Q_{pe} = Q_p - W_p \cdot \rho_0 C_p (T_u - T_l), \quad (17)$$

$$Q_{se} = Q_s - W_p \cdot C_{su}, \quad (18)$$

$$H_e = H + \left(\left(Q_p / Q_{pe} \right)^{1/5} - 1 \right) \cdot z_i, \quad (19)$$

where

$$W_p = \frac{6}{5} \pi \alpha \left(\frac{9}{5} \frac{\pi}{c_1} Q_p \alpha \right)^{1/3} z_i^{5/3}, \quad c_1 \equiv \frac{g(\lambda^2 + 1)}{\rho_0 C_p T_0}. \quad (20)$$

Equation 19 can be derived from the relation

$$W_p = \frac{6}{5} \pi \alpha \left(\frac{9}{5} \frac{\pi}{c_1} Q_p \alpha \right)^{1/3} z_i^{5/3} = \frac{6}{5} \pi \alpha \left(\frac{9}{5} \frac{\pi}{c_1} Q_{pe} \alpha \right)^{1/3} z_i'^{5/3}, \quad (21)$$

where $z_i' = z_i + (H_e - H)$.

Using these equivalent parameters, the ceiling-jet correlations such as Heskestad and Delichatsios [5] for the unconfined ceiling can be used for the estimation in the two-layer environment.

Figure 5 shows calculation results for the plume centerline temperature distribution compared with Evans's bench-scale experimental data [26]. In Figure 5, exact solution of Equations 13-15 obtained with numerical integration is referred to as 'numerical method', and approximate solution obtained with equivalent source parameters (Equations 17-19) is referred to as 'approximate method'. Both the exact solution and the approximate solution show reasonable agreement with the experimental data, although there is a sharp decrease at the interface as pointed out by Evans [26]. This abrupt change can be avoided by extrapolating the lower-layer plume (as shown by the dotted line in Figure 5). This approximate method allows a discontinuity in the axial velocity across the interface as shown in Figure 6, which may not affect directly the calculation result of detector's response, because it quickly comes close and merge with the exact solution.

McCaffrey's correlations [32] were used to obtain the calculation results in Figures 5 and 6,

Figure 5 Calculation results for the plume centerline temperature distribution in a two-layer environment compared with Evans's bench-scale experimental data [26]

Figure 6 Calculation results for the plume centerline velocity distribution in a two-layer environment

$$\Delta T_m / T_0 = 21.6 / T_0 \cdot z^{-5/3} Q_p^{2/3}, \quad (z / Q_p^{2/5} > 0.2), \quad (22)$$

$$u_m = 1.12 \cdot z^{-1/3} Q_p^{1/3}, \quad (z / Q_p^{2/5} > 0.2), \quad (23)$$

where equivalent values $z_i' = z_i + (H_c - H)$ and Q_{pe} were used for z and Q_p respectively for the calculation in the upper layer. Equations 22 and 23 are consistent with the analytical solution of Equations 13-15 with the entrainment constant set as $\alpha = 0.118$ and the Gaussian width ratio set as $\lambda^2 = 1.157$. Again, the flame height ($z \approx 0.2 \cdot Q_p^{2/5}$) is assumed to be small relative to the interface height z_i .

As the smoke concentration and the temperature rise in the plume are assumed similar with each other, the ratio of the local smoke concentration to the temperature rise can be correlated to the ratio of smoke and heat release rates at the source, as follows,

$$\frac{\Delta C_s}{\rho_0 C_p \Delta T} = \frac{Q_s}{Q_p} = \frac{Q_s}{k_p \cdot Q_f}, \quad (24)$$

where $k_p \equiv Q_p / Q_f$. Equation 24 confirms Heskestad and Delichatsios's observation [3] that the ratio of the optical smoke density to the temperature rise depends mainly on the burning material. In theory, this relation should be conserved in the plume region, but gradually changes in the ceiling-jet region, as the enthalpy flux in the ceiling-jet is gradually lost to the ceiling by convection, while the smoke mass flux is conserved.

2.3 Ceiling-jet

Alpert [2] provided a theoretical model for the steady-state ceiling-jet formed under flat horizontal ceilings. Figures 7 and 8 show schematics of plume and ceiling-jet flows and the coordinate system. Following Alpert's theory, conservation of mass, momentum and energy in the radial direction can be written as follows,

Figure 7 A schematic of plume and ceiling-jet flows and the coordinate

Figure 8 Mass, momentum and energy balance in the ceiling-jet

$$\frac{1}{r} \frac{d}{dr} (r \cdot \bar{v} \cdot h) = E \cdot \bar{v}, \quad (25)$$

$$\frac{1}{r} \frac{d}{dr} (r \cdot \bar{v}^2 \cdot h) = -\frac{1}{2} f_w \bar{v}^2 - \frac{1}{2} \frac{d}{dr} \left(S \cdot g \cdot h^2 \frac{\bar{\Delta T}}{T_0} \right), \quad (26)$$

$$\frac{1}{r} \frac{d}{dr} \left(r \cdot \bar{v} \cdot h \frac{\bar{\Delta T}}{T_0} \right) = -\frac{h_w}{\rho_0 C_p T_0} (\bar{T} - T_w), \quad (27)$$

where

$$E = E_0 \exp[\beta(Ri_0 - Ri)], \quad Ri = \frac{g \cdot h}{\bar{v}^2} \cdot \frac{\bar{\Delta T}}{T_0}. \quad (28)$$

In Equations 25-28, the horizontal bars over the variables, such as $\bar{v}$ and $\bar{\Delta T}$, means that they are the averaged representative values (at a certain radial position) in the ceiling-jet. They can be converted to maximum values according to the assumed vertical

profile of each. In the current study, they are assumed as half-Gaussians. For solving Equation 27, the temperature of the wall surface T_w has to be known. For simplicity, T_w can be regarded as constant ($\bar{T} - T_w = \overline{\Delta T}$) for isothermal ceilings or equalized to $\bar{T}$ ($\bar{T} - T_w = 0$) for adiabatic ceilings.

According to Alpert [2], solutions of Equations 25-27 is weakly dimensional by the effect of viscous sub-layer which affects the value of f_w , the friction factor. Alpert [2] himself pointed out that f_w has little effect on the flow properties. In other experimental studies on fire induced ceiling-jet [33,34], values of f_w from 0.02 to 0.04 have been found to match the ceiling heat transfer data. You [35] pointed out that if the stagnation point heat flux is correlated, then $f_w=0.025$ gives a best fit to their experimental data. If f_w is assumed to be constant, and the Reynolds' analogy between heat transfer and fluid friction is applied, then the equations can easily be converted to non-dimensional form,

$$\frac{1}{r^*} \frac{d}{dr^*} (r^* \cdot \bar{v}^* \cdot h^*) = E \cdot \bar{v}^*, \quad (29)$$

$$\frac{1}{r^*} \frac{d}{dr^*} (r^* \cdot \bar{v}^{*2} \cdot h^*) = -\frac{1}{2} f_w \bar{v}^{*2} - \frac{1}{2} \frac{d}{dr^*} (S \cdot h^{*2} \overline{\Delta T^*}), \quad (30)$$

$$\frac{1}{r^*} \frac{d}{dr^*} (r^* \cdot \bar{v}^* \cdot h^* \overline{\Delta T^*}) = -k_w \cdot St \cdot \bar{v}^* \cdot \overline{\Delta T^*}, \quad (31)$$

where

$$r^* = r/H, \quad h^* = h/H, \quad (32)$$

$$\bar{v}^* = \frac{\bar{v}}{\sqrt{gH}} (Q^*)^{-1/3}, \quad (33)$$

$$\Delta T^* = \frac{\Delta T}{T_a} (Q^*)^{-2/3}, \quad (34)$$

$$Q^* = Q / (\rho_0 C_p T_0 \sqrt{gH} H^2), \quad (35)$$

$$St \equiv \frac{h_w}{\rho_0 C_p \bar{v}} = \frac{f_w}{2} \cdot \frac{1}{Pr^{2/3}}, \quad (36)$$

$$k_w = \frac{\bar{T} - T_w}{\overline{\Delta T}}. \quad (37)$$

If the smoke is assumed to be transported passively by the flow, with no loss to the ceiling, then the conservation of smoke mass in non-dimensional form can be written as

$$\frac{1}{r^*} \frac{d}{dr^*} (r^* \cdot \bar{v}^* \cdot h^* \overline{\Delta C_s^*}) = 0, \quad (38)$$

where

$$\Delta C_s^* = \frac{\Delta C_s}{Q_s} \frac{Q}{\rho_0 C_p T_0} (Q^*)^{-2/3}. \quad (39)$$

The solutions of the plume equations (*Equations 13-16*) can be used as upstream boundary conditions for the ceiling-jet at the exit of the turning region, as described by Alpert [2], which can also be set in non-dimensional form,

$$z_{j0}^* = (1 + \sqrt{6}/5 \cdot \alpha)^{-1}, \quad (40)$$

$$r_0^* = \sqrt{3} \cdot b^* (z_{j0}^*), \quad (41)$$

$$h_0^* = h^*(r_0^*) = b^* (z_{j0}^*) / \sqrt{6}, \quad (42)$$

$$\overline{v_0^*} = \overline{v^*(r_0^*)} = u_m^* (z_{j0}^*) / \sqrt{2}, \quad (43)$$

$$\overline{\Delta T_0^*} = \overline{\Delta T^*(r_0^*)} = \frac{\lambda^2}{\lambda^2 + 1} \Delta T_m^* (z_{j0}^*), \quad (44)$$

$$\overline{\Delta C_{s0}^*} = \overline{\Delta C_s^*(r_0^*)} = \frac{\lambda^2}{\lambda^2 + 1} \Delta C_{sm}^* (z_{j0}^*). \quad (45)$$

Definitions of the reduced variables used in *Equations 40-45* for the plume part are

$$b^* = b/H, \quad (46)$$

$$u_m^* = \frac{u_m}{\sqrt{gH}} (Q^*)^{-1/3}, \quad (47)$$

$$\Delta T_m^* = \frac{\Delta T_m}{T_0} (Q^*)^{-2/3}, \quad (48)$$

$$\Delta C_{sm}^* = \frac{\Delta C_{sm}}{Q_s} \frac{Q}{\rho_0 C_p T_0} (Q^*)^{-2/3}. \quad (49)$$

As all parameters could be set in non-dimensional form, then just one calculation is enough in obtaining a solution for the ceiling-jet flow for all ceiling heights and heat release rates. *Figures 9-11* show a result of calculation in radial temperature, velocity and smoke concentration respectively for isothermal ceiling-jet, where boundary conditions and parameters are set same as the values used in Yamauchi [22]. Exact value of each parameter can be found in the nomenclature listing and detailed explanation can be found in Yamauchi [22]. The subscript m in ΔT_m , v_m and ΔC_{sm} means that they are the maximum values in the vertical distribution of respective quantities at a certain radial distance. In *Figures 9 and 10*, Heskestad's correlation [4] and Heskestad and Delichatsios's correlations [5] are plotted on the same scale for comparison. As it is seen

in *Figures 9 and 10*, Heskestad and Delichatsios's correlations [5] may be used instead of the numerical solution of *Equations 29-31* for ΔT_m^* and v_m^* . Also as shown in *Figure 11*, instead of using the numerical solution for ΔC_{sm}^* , the correlation expressed with *Equation 50* may be used for estimating the radial distribution of smoke concentration, which makes the calculation procedure quite simple. It should be noted here, that detailed measurement of the smoke concentration in the ceiling-jet is not available in the literature. The experimental plot in *Figure 11* is taken from [36], in which the r/H range was less than 1.5 and the heat release rate was not constant. Therefore, *Equation 50* ought to be modified when a detailed measurement is done.

$$\Delta C_{sm}^* = 1 / (0.035 + 0.15 \cdot r/H)^{2/3}, \quad r/H \geq 0.2. \quad (50)$$

Figure 9 A result of calculation in the ceiling-jet temperature compared with other ceiling-jet correlations

Figure 10 A result of calculation in the ceiling-jet velocity compared with other ceiling-jet correlations

Figure 11 A result of calculation in the ceiling-jet smoke concentration; the experimental plot is taken from reference [36]

2.4 Heat detector's response

According to Heskestad and Bill [8], variation of the temperature of a heat sensitive device placed in a fire environment can be described by the equation,

$$\frac{dT_s}{dt} = \frac{v^{1/2}}{RTI_s} (T_g - T_s) - C'(T_s - T_b), \quad (51)$$

where T_b stands for the temperature of the piping which the sprinkler head is connected. T_b is generally assumed constant in the sprinkler calculation. However, if T_b is interpreted as the temperature of the detector body, it may change gradually when exposed to a fire induced ceiling-jet. Applying the RTI model to the detector body, then T_b can be calculated with the following equation,

$$\frac{dT_b}{dt} = \frac{v^{1/2}}{RTI_b} (T_g - T_b). \quad (52)$$

In the actual calculation, it is more convenient to use the following expressions,

$$\frac{d\Delta T_s}{dt} = \frac{v^{1/2}}{RTI_s} \left[(\Delta T_g + C/v^{1/2} \Delta T_b) - (1 + C/v^{1/2}) \Delta T_s \right], \quad (53)$$

$$\frac{d\Delta T_b}{dt} = \frac{v^{1/2}}{RTI_b} [\Delta T_g - \Delta T_b], \quad (54)$$

where variables are converted by the relations: $T_g = T_0 + \Delta T_g$, $T_s = T_0 + \Delta T_s$, $T_b = T_0 + \Delta T_b$ and $C' = C/RTI_s$.

Figure 12 shows a comparison of experimental data with calculation in the sensor temperature. The experimental data was taken in a wind tunnel [37] in which the gas temperature (T_g) was control to rise at a constant rate (3 deg C/min) while the velocity was kept constant (1.0 m/s). The detector is a fixed-temperature type with a thermistor as a sensor. The response characteristics of the detector are shown in Table 2 in section 3.2. After placing the detector in a wind tunnel for sometime to become equilibrium, the gas temperature was set to rise. As clearly seen in Figure 12, the simulated temperature

with 2-RTI model almost exactly traces the variation of the sensor temperature. However, when the body temperature is assumed to be constant (in 1-RTI model), the simulated sensor temperature stays below the measured value. The difference becomes greater with time.

Figure 12 Temperature variation of the sensing element of heat detector compared with simulated temperatures by 2-RTI and 1-RTI models respectively

In the case of fixed-temperature heat detector, the detection time can be set as the time the temperature of the sensor exceeds a threshold. In the case of rate-of-rise detectors, a more sophisticated calculation is needed. A representative response characteristic of the rate-of-rise detector may be derived from the conventional pneumatic type detector, which has an air chamber with a small orifice to cancel the effect of slow temperature variation (Figure 13).

Figure 13 A schematic of the pneumatic type rate-of-rise detector

In a fire environment, the pressure in the chamber rises quickly by the heat absorbed by the chamber while the air inside the chamber leaks slowly. When the pressure difference exceeds a predetermined level, an electric contact closes sending an alarm signal. The energy balance in the air chamber can be expressed by the following equation,

$$\left(\frac{C_V V}{R} \right) \frac{d\Delta P}{dt} + m_e C_p T_e = q(t), \quad (55)$$

where ΔP is the pressure rise in the chamber and $q(t)$ is the instantaneous rate of heat absorbed by the chamber air, m_e is the mass flow rate at the orifice, and T_e is the exiting gas temperature. Defining equivalent temperatures by $\Delta T_p = \Delta P \cdot (C_V / (\rho \cdot R))$ and

$d\Delta T_s/dt \equiv q(t)/(\rho \cdot C_V V)$, and assuming that m_e is proportional to ΔT_p , then Equation 55 can be rewritten as,

$$\frac{d(\Delta T_s - \Delta T_p)}{dt} = \frac{1}{\tau} \Delta T_p. \quad (56)$$

In Equation 56, ΔT_s stands for the real temperature rise of the chamber air and ΔT_p for the equivalent temperature rise representing the change in the chamber pressure. τ is the specific time constant of the detector, determined mainly by the chamber volume and the orifice resistance. The heat transfer process from the environment to the chamber air is not included in Equation 56, which must be calculated separately by solving the heat transfer equations such as Equations 53 and 54.

Equation 56 can also be applied to the response of the electric type rate-of-rise detector comprising a temperature sensor connected to a differential circuit as shown in Figure 14. In Figure 14, the capacitor (C) acts as an air chamber and the resistance (R) acts as an orifice. The response characteristic of this electric-type detector is determined by the time constant τ of the differential circuit and the response threshold expressed in temperature rise ΔT_{th} . Therefore, Equation 56 can be applied to either the conventional pneumatic type detector or the electric type comprising a temperature sensor and a differential circuit. In the actual calculation, it may be convenient to use the following expression,

$$\frac{d(\Delta T_{dif})}{dt} = \frac{1}{\tau} (\Delta T_s - \Delta T_{dif}), \quad (57)$$

where $\Delta T_{dif} \equiv (\Delta T_s - \Delta T_p)$.

Figure 14 A schematic of the electric type rate-of-rise heat detector comprising a temperature sensor connected to a differential circuit

2.5 Photoelectric smoke detector's response

The photoelectric smoke detector detects smoke by the light scattering principle. The nominal sensitivity of the photoelectric detector is generally expressed with extinction coefficient. However, it is known that the magnitude of the scattered light is strongly dependent on the particulate properties of smoke [11]. Thus the photoelectric smoke detector's response is also dependent on the smoke properties.

Concerning the delay due to smoke entry resistance, Heskestad's characteristic length model [6] can be written as,

$$K_r = K_{th} + L \cdot [dK/dt] \cdot v^{-1}, \quad (58)$$

where K_r is the optical smoke concentration (extinction coefficient) outside of the detector at response, and K_{th} is the threshold smoke concentration specific to the detector and the smoke property. L is the characteristic length of the detector, which is dependent on the geometrical design of the detector including the labyrinth and the mesh size of bug screen. $[dK/dt]$ is the smoke concentration build-up rate, and v is the air velocity at the detector location. If the smoke concentration build-up rate $[dK/dt]$ is assumed constant, then Equation 58 can be rewritten as,

$$t_{delay} = \frac{K_r - K_{th}}{[dK/dt]} = L \cdot v^{-1}. \quad (59)$$

Equation 59 simply tells that the delay time due to entry resistance is proportional to v^{-1} with the factor of L , but independent of the smoke concentration build-up rate.

Figure 15 shows a result of experiment in which the delay time of a photoelectric detector is measured in a plunge test with different air velocities. It is noted here that the detector's analog output is smoothed with a processor in the detector, so that the delay time shorter than 3 seconds could not be measured. Because of this limitation, it is difficult to draw a functional relationship from the data in Figure 15, but it is apparent that the delay time does not follow a simple linear relation to v^{-1} . Bjoerkman et al. [16] reported that the smoke entry time delay of six commonly used detectors ranged in 10 – 50 s with a flow rate of 0.2 m/s, therefore the experimental result in Figure 15 may not be a special case. Also seen from Figure 15, the delay time is less than 10 s with velocities greater than 0.1 m/s. The velocity in the ceiling-jet produced by a flaming fire in an ordinary room configuration may generally be greater than 0.1 m/s, therefore the effect of entry resistance may be little in flaming fires. On the other hand, Figure 15 shows a sudden increase in the delay time with the velocity of 0.05 m/s, suggesting that there is a critical value. In authors' experience, it is often observed that a photoelectric detector not responding for several tens of minutes to a truly smoldering fire after the smoke concentration (outside of the detector) exceeded the threshold value, but then suddenly responds to a transition to flaming or even to a door opening which produce sudden air movements. Precise prediction of the response time in such cases may be difficult. However, for engineering purposes, it is convenient to use a threshold in the air velocity to obtain an approximate time of response in a smoldering fire.

Figure 15 A characteristic of the smoke entry delay time of photoelectric detector measured with different air velocities

As discussed above, one must be careful in obtaining a detector's response threshold in the optical smoke concentration, especially in smoldering fires. In order to obtain a true relationship between the optical smoke concentration and the detector's output, a series of fire test have been conducted. The experimental configuration is the same as the experiments described in section 3, and so are the burning methods and materials. The only difference is that six 9-cm-diameter fans were placed in parallel at a 1 m upstream position near the ceiling facing the detector, in order to increase the velocity at the detector location up to 0.5 - 0.7 m/s to avoid the effect of entry resistance. Figure 16 is a result of this experiment, showing a photoelectric detector's response characteristic to smokes from different sources (Table 1). As apparently seen in Figure 16, the detector output can be regarded as proportional to the optical smoke concentration, however, the factors are different with smoke sources. The proportionality factor of the detector output to the obscuration meter reading (extinction coefficient) is 1 for the smoke from smoldering sources, but it is about 1/5 for the smoke from flaming sources. There may be small differences by detector designs, e.g. wave length of the light or the scattering angle. However, it is convenient to assume a constant value each for all flaming fires and for all smoldering fires respectively, because detector designs are somewhat similar with each other.

Figure 16 Photoelectric detector's response to various smokes; TF-1 - TF-5 are the test fire names specified in ISO catalog for full scale test fires for smoke and fire detectors [38]

It is noted here that the output reading of the detector used in this experiment is calibrated with the smoke from a smoldering source. This may be a reason that the output proportionality factor for the smoldering source is 1. One must be careful in using the nominal sensitivity value of the detector, because the detector's output reading changes relatively with the calibration method.

2.6 Calculation Procedure

The calculation procedure can be summarized as follows:

Set dimensions of the room and detector location

- Ceiling-height: H [m]
- Floor area of the room: S [m]
- Detector's radial distance from the fire axis: r [m]

Specify detector's response characteristics

- Thermal characteristics of heat detector: RTI_s , C , and RTI_b ;
 T_{th} for fixed temperature detector; τ and ΔT_{th} for rate-of-rise detector
- Response characteristics of smoke detector: K_{th} and v_{th}

Specify burning characteristics of the fuel

- Mass loss rate as a function of time: $m_f(t)$ [g/s]
- Heat of combustion: H_c [kJ/g]
- Smoke yield: Y_s [g/g]

The heat release rate $Q_f(t)$ and the smoke generation rate $Q_s(t)$ can be calculated by the relation

$$Q_f(t) = H_c \cdot m_f(t), \quad Q_s(t) = Y_s \cdot m_f(t). \quad (60)$$

H_c and Y_s can be specified as functions of time if necessary.

Calculation by zone model

Instantaneous upper-layer temperature T_u , smoke concentration C_{su} , and interface height z_i are calculated as functions of time using a two-layer zone model.

Calculation of equivalent source strengths

With the following relations (same as *Equations 17-19*), the strength of the source placed in an idealized uniform upper-layer environment can be specified as functions of time.

$$Q_{pe} = Q_p - W_p \cdot \rho_0 C_p (T_u - T_l),$$

$$Q_{se} = Q_s - W_p \cdot C_{su},$$

$$H_e = H + \left((Q_p / Q_{pe})^{1/5} - 1 \right) \cdot Z_i.$$

Calculation of ceiling-jet variables

Instantaneous ceiling-jet velocity v , temperature T_g and smoke concentration C_s at the detector location are calculated using the equivalent source parameters and the ceiling-jet correlations. In the present study, Heskestad and Delichatsios's correlations [5] are used for estimating temperature and velocity (Equations 61 and 62), and Equation 63 for smoke concentration.

$$\Delta T_m^* = (0.188 + 0.313 \cdot r/H_e)^{-4/3}, \quad (61)$$

$$v_m^* = 0.59 \cdot (\Delta T_m^*)^{1/2} \cdot (r/H_e)^{-0.63}, \quad r/H_e \geq 0.3, \quad (62)$$

$$\Delta C_{sm}^* = 1 / (0.035 + 0.15 \cdot r/H_e)^{2/3}, \quad r/H_e \geq 0.2. \quad (63)$$

The actual values of temperature and smoke concentration can be obtained with the relations $T_g = T_u + \Delta T_m$ and $C_s = C_{su} + \Delta C_{sm}$. C_s [g/m³] is then converted to the extinction coefficient K [1/m] with the following relation,

$$K = k_s \cdot C_s, \quad (50)$$

where the value of k_s for the smoke from a flaming source is 7.6 m²/g and for the smoke from a smoldering source is 4.4 m²/g respectively (Seader and Einhorn [38]).

Calculation of detector's output

Variation of the sensor's output, such as the sensor temperature T_s and the extinction coefficient K , is calculated with the method described in section 2.4 and 2.5, according to detector types.

Check operating conditions

Sensor's output values are compared with the operating conditions of the detector. When the output exceeds the predetermined threshold, then the response time is determined.

- Fixed temperature heat detector: $T_{th} \leq T_s$
- Rate-of-rise heat detector: $\Delta T_{th} \leq \Delta T_p$
- Photoelectric smoke detector: $K_{th} \leq K$ and $v_{th} \leq v$

3. OVERALL PERFORMANCE OF THE PREDICTION METHOD

To test the performance of the prediction method as described above, a series of experiments were conducted in a rectangular room with the size specified in the ISO catalog for full scale test fires for smoke and fire detectors [39]. Detector placement was set exactly as specified in the ISO catalog, and so with the burning materials. Test results were compared with calculation.

3.1 Experimental setup

Figure 17 shows the experimental room and instruments set up. The size of the room is 10m x 6m with the ceiling height of 4 m. The fire source is placed on the floor at the center of the room. The detectors are placed on the ceiling 3 m from the fire axis. In order to measure the overall environment, thermocouples are placed in several places. For velocity measurement, a set of two thermocouples and a set of two smoke obscuration meters were placed 10 cm apart along the radial direction near the detector. With the time-wise correlation of these two sensors' output variation, the radial velocity was evaluated. For the flaming fire, the thermocouple data were used, and for the smoldering fire, the obscuration meter's data. As a fire source, one of the ISO test fires listed in Table 1 is chosen at a time.

Figure 17 Experimental room and instruments setup

Table 1 Brief description of ISO test fires [39]

Test fire types	Description
TF-1	Wood fire
TF-2	Rapid smoldering wood on hot plate
TF-3	Glowing (smoldering) cotton
TF-4	Polyurethane fire
TF-5	Liquid heptane fire
TF-6	Liquid ethanol fire

3.2 Heat detector's response

For the heat detector experiment, TF-6 (liquid ethanol fire) is chosen, in which pure ethanol is burnt in a 0.2 m² pan. Three types of detectors as specified in Table 2 are used for the experiment. The heat release rate of the fire is measured separately and used for the calculation. The thermal response characteristics are also measured separately as shown in Table 2. The experimental data and the result of calculation with the method described in section 2 are compared.

Table 2 Response characteristics of heat detectors used in the experiments

Detector type (sensor)	RTIs [m ^{1/2} s ^{1/2}]	C [m ^{1/2} s ^{-1/2}]	RTIb [m ^{1/2} s ^{1/2}]	Threshold [C]
Fixed temperature (thermistor)	15	0.2	1000	70 (T _{th})
Fixed temperature (bimetal)	15	0.2	1000	60 (T _{th})
Rate-of-rise (pneumatic)	15	0.2	1000	15 (ΔT _{th})

Figure 18 shows the observed variation of temperatures measured with a thermocouple tree placed 4 m from the fire axis, compared with the calculated upper-layer temperature using the zone model described in section 2.1. The calculated temperature seems to coincide with the measured temperature at $z = 2.5$ m, which may be appropriate as a representative temperature for the ceiling-jet environment. Figure 19 shows similarly the observed and calculated layer heights respectively for comparison. Observation is taken from the variation of vertical temperature distribution measured with the thermocouple tree. The agreement is quite good, thus the validity of the zone model is confirmed.

Figure 18 Observed variations of temperatures measured with a thermocouple tree placed 4 m from the fire axis, compared with the calculated upper layer temperature with a zone model

Figure 19 Observed variation of the layer height compared with calculation

Figure 20 shows the measured and the calculated ceiling-jet temperatures respectively at several radial positions. Figure 21 shows similarly the measured and calculated air velocities at a position near the detector ($r = 3$ m). A general agreement is seen in both cases, although there is some difference in the temperature values at a position near the fire axis ($r = 1$ m), probably due to the ceiling-jet model.

Figure 20 Comparison of the measured and the calculated ceiling-jet temperatures at several radial positions

Figure 21 Comparison of the measured and the calculated air velocities at a position near the detector ($r=3$ m)

Figure 22 shows a result of 2-RTI calculation in the sensor temperature, compared with observed sensor output variation. The agreement seems quite good as a base for the response estimation of various type heat detectors. In Table 3, the observed response time of the three detectors used in the experiment are compared with the calculated response time. The differences are small enough when the experimental dispersions are considered. Not only the fixed-temperature type, but the estimation in the response of rate-of-rise detector is also successful with the calculation method described in section 2.4.

Figure 22 A result of 2-RTI calculation in the sensor temperature, compared with observed sensor output variation

Table 3 Comparison of experimental response time with calculation
(heat detectors in TF-6: liquid ethanol fire)

Detector type (sensor)	Observed response time [s]	Calculated response time [s]	Difference [s]
Fixed temperature (thermistor)	252	260	+8
Fixed temperature (bimetal)	198	170	-18
Rate-of-rise (pneumatic)	48	55	+7

3.3 Photoelectric smoke detector's response

For testing the calculation method in estimating the photoelectric smoke detector's response, TF-1 (wood fire) and TF-2 (rapid smoldering wood) are chosen. The room, fire position and detector placement are the same as in the tests for the heat detector. The response characteristics of the detector are summarized in Table 4. The heat release rates and the smoke yields are measured separately and used for the calculation.

Table 4 Response characteristics of the photoelectric smoke detector used in the experiments

K_{th} [1/m]	0.05 for smoke from smoldering sources 0.25 for smoke from flaming sources
v_{th} [m/s]	0.05

In Figure 23, calculated results for the smoke concentration, detector output, and air velocity near the detector are compared with the measured values in TF-1 (wood fire). As for the smoke concentration, both the obscuration meter reading and the detector's output are precisely estimated. As for the velocity, only a few points were measured however, the estimation shows a consistent agreement. In this case, the velocity value is quite higher than the threshold, thus the response time is determined with the threshold in the extinction coefficient.

Figure 23 Calculated results for the smoke concentration, detector output, and air velocity neat the detector compared with the measured values in TF-1 (wood fire)

In Figure 24, calculated results for the smoke concentration, detector output, and air velocity near the detector are compared with the measured values in TF-2 (rapid smoldering wood). The estimation of the smoke concentration agrees well with the experimental data. However, observed detector output is somewhat delayed by about 100 s. This may be caused by the slow air velocity. The situation is that the smoke concentration near the detector exceeded the threshold, but the smoke is not yet entered the detection chamber. The smoke enters the chamber at about the time the velocity exceeded the threshold value of 0.05 m/s. The smoke detector's output quickly rises after that and becomes saturated. Through this experiment, it is understood that the air velocity plays an important role in determining the response time of smoke detector in a smoldering fire, as well as the smoke concentration.

Figure 24 Calculated results for the smoke concentration, detector output, and air velocity near the detector compared with the measured values in TF-2 (rapid smoldering wood)

In Table 5, the observed response time of the smoke detector are compared with the calculated response time. In the case of the flaming fire, the agreement is quite satisfactory. However, in the case of the smoldering fire, there is some difference between the measurement and estimation. Modification in the model may be needed to increase the accuracy. Presently, use of two thresholds in the smoke concentration and in the air velocity respectively may be practical for engineering purposes.

Table 5 Comparison of experimental response time with calculation (photoelectric smoke detector)

Fire type	Observed response time [s]	Calculated response time [s]	Difference [s]
TF-1 (flaming wood)	230	215	-15
TF-2 (smoldering wood)	730	800	+70

4. CONCLUSION

For the purpose of providing engineering tools for estimating of the response time of heat and smoke detectors installed on the ceiling of enclosed rooms, a simple calculation method is presented. The combination of a zone model and a ceiling-jet model showed a successful result in estimating the temperature, smoke concentration and the velocity in the ceiling-jet formed in a two-layer environment of a room fire.

For simulating the heat detector's response, a modified RTI-C model which uses two RTI's is introduced. This method shows better results than the original RTI-C model, in estimating the heat detector's response. In addition, a calculation method for predicting

the response of rate-of-rise detector is introduced. This method can be used either for the conventional pneumatic type detector or the electric type comprising a temperature sensor and a differential circuit.

For simulating the response of photoelectric smoke detector, a method utilizing a response threshold in the optical smoke concentration and a response threshold in the air velocity is introduced. This method is found useful for engineering estimations in both the flaming fire and the smoldering fire. A modification in the model may be needed to increase the accuracy in smoldering fires.

NOMENCLATURE

b	1/e width of plume [-]
C	conductive parameter in RTI-C model [$\text{m}^{1/2} \text{s}^{-1/2}$]
C_s	smoke particulate (mass) concentration [kg/m^3]
ΔC_s	particulate mass concentration difference from ambient [kg/m^3]
C_p	specific heat (constant pressure) [$\text{kJ kg}^{-1} \text{K}^{-1}$]
C_v	specific heat (constant volume) [$\text{kJ kg}^{-1} \text{K}^{-1}$]
E	entrainment function of ceiling-jet [-], E_0 is set as $E_0=0.05$ in this paper
f_w	ceiling friction factor [-], set as $f_w=0.025$ in this paper
g	gravitational acceleration [m/s^2]
h	ceiling-jet thickness for top-hat profile [m]
h_w	ceiling heat transfer coefficient [$\text{kW m}^{-2} \text{K}^{-1}$]
H	ceiling-height [m]
H_c	heat of combustion [kJ/kg], Equation 60
K	extinction coefficient (optical smoke density) [$1/\text{m}$]
k_p	ratio of the convective heat flux of plume to the heat release rate of fire [-], ($k_p \equiv Q_p/Q_f$, $Q_p \equiv Q_f - Q_R$)
k_s	parameter of smoke density [m^2/g] (Equation 50)
k_w	parameter of ceiling thermal boundary condition at ceiling [-] (Equation 37), set as $k_w=1$ in this paper
l	wall thickness [m]
L	characteristic length [m]
P	photoelectric detector's output [$1/\text{m}$]
ΔP	pressure rise from ambient [Pa]
Pr	Prandtl number
q	rate of heat gained or lost by convective heat transfer [kW]
Q	heat release rate or heat flux [kW]
Q_s	smoke (mass) production rate or smoke mass flux [kg/s]
r	radial distance from fire axis [m]
Ri	Richardson number
RTI	response time index [$\text{m}^{1/2} \text{s}^{1/2}$]

S	shape parameter of ceiling-jet [-] (<i>Equations 26 and 30</i>), set as $S \approx 1.0$ in this paper
St	Stanton number
T	absolute temperature [K]
ΔT	temperature difference from ambient [K]
u	vertical velocity of plume [m/s]
v	radial velocity of ceiling-jet [m/s]
V	volume [m ³]
W	volumetric flow rate [m ³ /s]
Y_s	smoke yield (ratio of smoke production rate to mass loss rate of fuel) [-] (<i>Equation 60</i>)
z	vertical coordinate [m]

Greek letters

α	plume entrainment constant [-], set as $\alpha=0.118$ in this paper
β	parameter of entrainment [-] (<i>Equation 28</i>), set as $\beta=3.9$ in this paper
ρ	density [kg/m ³]
λ	Gaussian width ratio [-] (<i>Equation 12</i>), set as $\lambda^2=1.157$ in this paper
λ_c	conductivity [kW m ⁻¹ K ⁻¹]
τ	characteristic response time of rate-of-rise heat detector [s]

Superscripts

—	spatially averaged quantity
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Subscripts

0	initial value
a	ambient
b	detector body
c	combustion, conduction, convection
e	equivalent, exit
f	fuel
g	gas
i	interface
l	lower layer
m	maximum value
p	plume
P	pressure
r	reference, response
R	radiation
s	sensor, smoke particulate
th	threshold
u	upper layer
w	wall

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