EXPERIMENTAL WALL FLAME HEAT TRANSFER CORRELATIONS FOR THE ANALYSIS OF UPWARD WALL FLAME SPREAD

Yuji Hasemi

Building Research Institute, Ministry of Construction,
Tatehara 1, Oho-machi, Tsukuba-gun, Ibaraki-ken, 305, Japan

(Received, October 29, 1984)

ABSTRACT

Dependence of the height of turbulent wall flames and the wall flame heat transfer on the fuel conditions is studied by experiments using porous line burners against walls. The flame height is represented as a function of heat release rate and fuel width. According to the data of incident heat flux to the wall surface, the wall above a fire source appears to divide into four regions following the thickness and intermittency of a luminous flame. The distribution of the incident heat flux is represented as a function of the height normalized by a scale length representing the flame height. Exploratory analysis is made to explain this result from the heat transfer mechanism. The present analysis seems to be consistent with current experimental work on wall flames.

1. INTRODUCTION

Flame spread along a vertical combustible surface is counted among the most significant processes leading to a rapid fire growth in a compartment. In spite of the importance of controlling material properties related to wall flame spread, combustible materials are very often preferred to incombustible ones for esthetic reasons. Development of rational test methods for wall material is essential for the improvement of fire safety in buildings; however, most of the current test methods concerning flame spread do not seem to be based on a quantitative understanding of the relevant processes. For the wall flame spread to be characterized as a basis of testing and evaluation, some indices characterizing the speed of flame spread should be correlated with measurable material properties which can be determined regardless of the stoichiometry

of a material; the chemical composition of most materials used for interior linings and furniture is generally so complicated that a test based on stoichiometry may not be practical.

For flame spread analysis, most of theoretical work has assumed an ideal laminar flame without radiation [e.g. 1-5]; however, it is obvious that a wall flame during a building fire is generally turbulent, and its spread seems to be dominated mainly by flame radiation [e.g. 6-9]. While the turbulent flame spread may result from complicated interactions of chemical, kinetic and thermal processes [10], an engineering model of the velocity of steady turbulent wall flame spread has been formulated on the basis of a concept of ignition and flame spread as a result of inert heating of the solid to an ignition temperature^[9, 11]. Figure 1 displays a conception of the flame spread

Fig. 1 Schematic diagram of upward wall flame spread

model and its components.

The model assumes a one-dimensional flame spread in the x -direction and a one-dimensional thermal conduction in the solid normal to the surface. The location of the pyrolysis front is identified by x_p where the surface temperature of the solid has reached an ignition temperature, T_{ig} . The decrease of heat flux in the pyrolysis zone represents the blocking effect due to the injection of fuel gas from the wall surface^[12]. The pyrolysis zone may be regarded to have an infinite length normal to x, y directions. Ignoring the reradiation from the wall surface, the surface temperature at x , $T_w(x, 0)$, can be represented as

$$T_w(x, 0) - T_0 = \int_0^t \dot{q}_w''(x, t-\tau) \cdot \phi(\tau) d\tau \quad (1)$$

where $\dot{q}_w''(x, t-\tau)$ is the heat flux applied to the surface at $(t-\tau)$ after the ignition and $\phi(\tau)$ is the impulse response of the surface temper-

ature to the heat application. The functional form of $\phi(\tau)$ must depend on wall condition, i.e. $\phi(\tau) = (\pi k \rho c \tau)^{-1/2}$, for a semi-infinite wall. The location of the pyrolysis front is given as a function of time by solving

$$T_{ig} - T_0 = \int_0^t \dot{q}_w''(x, t-\tau) \cdot \phi(\tau) d\tau \quad (2)$$

For a steady flame spread along a semi-infinite thick wall, equation (2) yields

$$T_{ig} - T_0 = \int_0^x \frac{\dot{q}_w''[x, (x-x_p)/V_p]}{\sqrt{\pi k \rho c (x-x_p) V_p}} dx_p \quad (3)$$

While the incident heat flux should be dependent on external radiation, we will consider only flame spread without external radiation in this paper.

Assuming $\dot{q}_w''$ depends only on the distance above the top of the pyrolysis zone, the asymptotic value of V_p as $x \rightarrow \infty$ will become

$$V_p = \frac{[\int_0^\infty \dot{q}_w''(\xi+x_p)/\sqrt{\xi} d\xi]^2}{\pi k \rho c (T_{ig}-T_0)^2} \dots\dots\dots (4)$$

where ξ is the height above the pyrolysis front.

In equation (4), the thermal inertia of wall material, $k\rho c$, and the ignition temperature, T_{ig} , are the material properties to be determined by thermometric measurements. The numerator of equation (4) can be estimated by formulating the vertical distribution of wall heat transfer as a function of height above pyrolysis front. The numerator may be represented as $\dot{q}_{wc}''^2 \delta_f$ with a characteristic heat flux, $\dot{q}_{wc}''$, and a preheat distance, δ_f . Sibulkin and Kim^[9], and Quintiere and Fernandez-Pello^[11] represented the flame spread velocity as $V_p = \dot{q}_{wc}''^2 \cdot \delta_f / k\rho c (T_{ig}-T_0)^2$ with δ_f equivalent, or proportional, to the flame height.

The wall heat flux in the flame is believed to be governed by flame radiation^[8], whereas the heat flux in the far field is mostly due to convection^[14]. This implies that flame height can be a scaling factor representing the distribution of wall heat transfer. On the analogy of unconfined fires, the flame height is expected to depend practically only on gross heat release rate per unit length of the pyrolysis zone, Q_g , and pyrolysis length, x_p [e.g. 13]; the experimental verification will be given in this paper. Heat release rate is proportional to volatilization rate, $\dot{m}_v''$, which is further to be determined by the heat balance around the fuel surface as

$$\dot{q}_w'' - \dot{q}_{rr}'' = \Delta H_G \cdot \dot{m}_v'' \dots\dots\dots (5)$$

The surface reradiation, $\dot{q}_{rr}''$, is related to the temperature of fuel surface, T_{ig} , and its emissivity. Therefore, heat release rate can be again correlated with such material properties as T_{ig}

and ΔH_G . Although the material properties appearing in this model may not be obtained directly for some composite materials, the model is still believed to be useful as a paradigm to develop a testing method concerning the wall flame spread.

The purpose of this study is to correlate the distribution of $\dot{q}_w''$ with such fuel properties as heat release rate and pyrolysis length by experiments using porous line burners. This is a first step toward modeling this poorly understood process, however, the analysis has led to an interesting result which may be useful for analyzing wall flame spread of materials and worth further investigation. The data from the pioneering experiments on turbulent wall fires by Faeth et al^[14, 15] and by Orloff et al^[7, 8] will be used for reference.

2. EXPERIMENTAL DESCRIPTION

Measurements of total heat flux to the wall surface and the visible flame height were made using porous, methane, line burners against isothermal and thermally thin walls.

The isothermal wall had a smooth black-painted front surface, and it was water-cooled through coils on its back surface. Flat water-cooled side panels helped to maintain a two-dimensional flow pattern along the test wall (Figure 2). The surfaces of the test wall and the side walls were kept at a constant temperature (approximately 60°C). Total heat flux to the wall surface was measured at six heights above the burner using the Gardon-type flux meters. The widths of the methane burners were 0.037m and 0.082m.

The thin-wall experiments were made using a porous methane burner 0.075m wide

Fig. 2 Wall flame heat transfer apparatus

and 0.92m in length against one wall of one 2.80m × 2.80m × 2.18m enclosure (Figure 3). The lightweight walls and ceiling of the enclosure were covered with a ceramic fiber insulation board to establish near-steady state quickly. A precise description of the enclosure is given elsewhere^[16]. Total heat flux was monitored at five heights above the burner by the Gardon-type flux meters.

The reported values of total heat flux are the average of data recorded for 5 minutes at the intervals of 10 seconds. Heat release rate was estimated from the flow rate of the fuel gas, assuming complete combustion. It should

Fig. 3 Experimental arrangement for thin wall tests

be noted that the thermal condition of the test walls is more ideal for the analysis than the one in actual buildings. However, the result of this experiment is believed to give a reasonable measure for the evaluation of such flame spread properties as mentioned in the section 1.

Visible flame was monitored by a video camera during both experiments. Data on the heights of flame tips and solid flame were obtained from observation of the videotape; the reported values of flame height are the average of the data observed for 3 minutes at intervals of 0.5 seconds. The heights of flame tips appeared to be more consistent among the observers than those of solid flame. The height of flame tips on the videotape appeared to be slightly less than that in the observations by eye during respective experiments.

3. FLAME HEIGHT CORRELATIONS

For unconfined turbulent diffusion flames, dimensional analysis based on the Froude number^[17] has established a dependence of the flame height on the size and intensity of fuel; $Q^* \equiv Q/\rho_\infty C_p T_{0g}^{1/2} D^{5/2}$ may be a domi-

nant dimensionless parameter concerning this problem^[18]. Similar analysis on an unconfined line fire results in a dependence of flame height on $Q_\ell^* \equiv Q_\ell / \rho_\infty C_p T_O g^{1/2} D^{3/2}$, since the square root of the inertial force to buoyancy ratio for a constant centerline flame temperature, θ_m , can be represented as

$$\left(\frac{T_O}{Dg\theta_m} \right)^{1/2} \cdot u_c = \frac{Q_\ell}{\rho_\infty C_p \theta_m D} \cdot \left(\frac{T_O}{Dg\theta_m} \right)^{1/2} \propto \frac{Q_\ell}{\rho_\infty C_p T_O (gD^3)^{1/2}} \quad (6)$$

where the characteristic velocity, u_c , is defined as $u_c \equiv Q_\ell / \rho_\infty C_p \theta_m D$. The order of Q^* or Q_ℓ^* for natural fires is $0(10^{-1}) - 0(10^0)$, unless the volatilization rate is accelerated by some heating mechanism other than the heat flux from the flame^[13]

Assuming the following functional expression for the flame height,

$$L_F \approx \gamma \cdot Q_\ell^{*n} \cdot D \quad (7)$$

the power of Q_ℓ^* representing L_F , n , should approach $2/3$ for a large value of Q_ℓ^* , since the fuel size effect on flame height must vanish in the $Q_\ell^* \rightarrow \infty$ limit. The parameter, γ , will take a particular value according to the definition of flame height, however γ is believed to be independent of the stoichiometry of fuel^[19]. This analysis will also hold for a line fire against a wall.

Figure 4 displays the dependence of L_F/D on Q_ℓ^* based on the height of flame tips obtained from the three burners against walls; the result shows that the height of wall flame is apparently proportional to $2/3$ power of Q_ℓ^* for $Q_\ell^* > 1$. The power of Q_ℓ^* representing the flame height appears to be slightly larger for $Q_\ell^* < 1$ than for $Q_\ell^* > 1$, i.e. $n \approx 0.8$. The parameter, γ , is approximately 6.0. The height of

solid flame was $L_F \approx 2.8 Q_\ell^{*n} D$; however, the data of solid flame height were not only less consistent among the observers but also more scattered.

Fig. 4 Height of wall flame as a function of Q_ℓ^* (flame tips)

4. HEAT FLUX TO THE WALL SURFACE

Since the intermittency of the flame is believed to dominate wall flame heat transfer due to radiation, flame height, either for flame tips and for solid flame, is expected to be a measure to classify the regions of an upward flow for the analysis of wall flame heat transfer. A parameter characterizing the visible flame height for a wide range, $Q_\ell^{*2/3} \cdot D = (Q_\ell / \rho_\infty C_p T_O g^{1/2})^{2/3}$, may be a characteristic scale length. Figure 5 represents total heat flux to the isothermal wall against height normalized with $Q_\ell^{*2/3} D$ for the 0.082m wide burner. The flames for $Q_\ell^* \leq 0.71$ seemed to be laminar, while the flames for the other conditions were apparently turbulent. The data of total heat flux of Liburdy and Faeth^[15] for a CO burner are superimposed for refer-

ence. The arrows for the data of Liburdy and Faeth show the direction in which their data must move, because they used the local heat flux defined as

$$Q_x \equiv \int_0^\infty \rho C_p u \theta dy \dots\dots\dots (8)$$

instead of the heat release rate. The ratio of the local heat flux to the heat release rate, Q_x/Q_ℓ , for the data of Liburdy and Faeth is estimated to be as low as 0.74. The location of the limit of solid flame and flame tips in the figure indicates that the data covers quite a wide range of height above the fuel.

The data appear to cluster densely along one curve; the break of the data at $x/Q_\ell^{*2/3} D \simeq 1.2$ might be due to the change of flow characteristics in the solid flame from laminar to turbulent flow at $Q_\ell \simeq 20\text{kW/m}$. The data on the smaller burner may be more appropriate to look at the similar relationship for small values of the normalized height[†]. Figure 6 shows the distribution of total heat flux for the 0.037m wide burner; the flame for the reported heat input were evidently turbulent. The results show that total heat flux tends to increase with height for $x/Q_\ell^{*2/3} D < 1$ and it becomes almost constant for $1 \leq x/Q_\ell^{*2/3} D < 2.8$. The total heat flux in the latter region appears to be weakly dependent on heat input. For the analysis of the mechanism on wall flame heat transfer, it may be convenient to classify the wall surface into the following four distinct regions:

I) $x/Q_\ell^{*2/3} D < 1$

(lower part of the solid flame)

$\dot{q}_w''$ tends to increase with height. Since the value of Q_ℓ^* for usual wall fires is around unity,

this region may correspond roughly to the pyrolysis zone. According to the observation of wall flame heat transfer and flame thickness within the pyrolysis zone of a PMMA vertical slab by Orloff et al^[8], the flame thickness is proportional to the height above the bottom of pyrolysis zone. Their measurement also showed that the incident radiative flux to the slab surface is approximately proportional to $x^{0.2}$ for $x < 1.0\text{m}$ and to $x^{0.5}$ for $1.0 < x < 3.5\text{m}$ (Figure 9). Their result is believed to be more realistic than ours, since they used a vertical burning slab instead of a sand burner against a wall, and this may have some influence on the heat transfer in the near field to the fire source.

II) $1 \lesssim x/Q_\ell^{*2/3} D < 2.8$

(upper part of the solid flame)

$\dot{q}_w''$ is apparently constant, and it appears to be a weakly increasing function of Q_ℓ^* . The flame thickness appeared to be almost constant with height.

III) $2.8 \lesssim x/Q_\ell^{*2/3} D < 10$ (transition region)

The slope is the steepest among the four regions; all data fall quite densely on the curve represented as

$$\dot{q}_w'' \simeq 45 (x/Q_\ell^{*2/3} \cdot D)^{-5/2} \dots\dots\dots (9)$$

This region may be characterized by the intermittency of flame^{††}.

IV) $x/Q_\ell^{*2/3} D \gtrsim 10$ (plume region)

Although sufficient number of data have not been obtained for this region, the slope representing the dependence of heat flux on height is clearly less steep than in the transition region. Since the data of Liburdy and Faeth in Figure 5 must move slightly to the

[†] The total heat flux data for the isothermal wall are shown in Table 1.

^{††} Recent wall flame radiation measurement by Kulkarni^[20] using the same apparatus shows that the radiative heat flux to a wall surface decreases significantly in this region.

left in the figure, heat flux in this region will be represented approximately by

$$\dot{q}_w'' \simeq 2.5 (x/Q_\ell^{*2/3} \cdot D)^{-1.3} \dots \dots \dots (10)$$

A similar analysis was attempted on the 0.075m wide burner against a thin wall. Figure 7 displays the $x/Q_\ell^{*2/3} D$ dependence of total heat flux for the 0.075m wide burner. The straight line in the figure represents equation (9); the data for the regions II) and III)

seem to be still consistent with the results on the isothermal wall. Figure 8 shows the result of similar analysis of the experiment of Ahmad and Faeth^[15] using saturated vertical wicks of methanol, ethanol and propanol on an insulated wall, calculated using the height of the wicks in place of D. The result is almost identical to those of the burner flames.

Table 1 Data of wall heat flux

(a) Total wall heat flux (D = 0.082m, Isothermal wall)

Heat release rate per unit length (kW/m)	Total heat flux (W/cm ²)						
	height (m)	0.070	0.295	0.520	0.745	0.970	1.195
16.7	1.684	0.368	0.130	0.081	0.069	0.060	
18.5	1.839	0.404	0.133	0.086	0.057	0.048	
22.2	2.079	0.565	0.171	0.102	0.078	0.069	
24.1	2.165	0.619	0.194	0.108	0.077	0.066	
27.8	2.264	0.822	0.261	0.133	0.090	0.078	
40.7	2.632	1.331	0.501	0.223	0.134	0.110	
53.7	2.686	1.611	0.702	0.323	0.186	0.149	
66.7	2.701	1.916	0.987	0.477	0.253	0.172	
81.5	2.665	2.159	1.284	0.679	0.335	0.205	
92.6	2.583	2.320	1.453	0.812	0.441	0.269	

(b) Total wall heat flux (D = 0.037m, Isothermal wall)

Heat release rate per unit length (kW/m)	Total heat flux (W/cm ²)						
	height (m)	0.070	0.295	0.520	0.745	0.970	1.195
29.6	1.981	1.466	0.633	0.243	0.125	0.076	
29.6	1.973	1.472	0.640	0.242	0.124	0.075	
29.6	1.976	1.170	0.528	0.230	0.115	0.075	
55.6	1.901	1.827	1.391	0.908	0.426	0.213	
55.6	2.196	1.939	1.535	0.901	0.405	0.205	
55.6	2.217	2.061	1.593	0.890	0.374	0.187	
74.1	1.485	2.136	1.748	1.037	0.504	0.259	
74.1	1.537	2.088	1.748	1.215	0.564	0.293	
74.1	1.609	2.162	1.759	1.167	0.554	0.280	
92.6	1.595	2.244	1.940	1.503	0.804	0.443	
92.6	1.738	2.217	1.913	1.444	0.762	0.417	
92.6	1.629	2.257	1.971	1.387	0.725	0.384	
145.5	0.698	2.404	2.432	2.099	1.419	0.906	
218.2	0.658	2.448	2.584	2.694	2.177	1.674

Fig. 5 Total wall heat flux vs. height normalized with $Q_l^{*2/3} D$ ($D = 0.082\text{m}$)

Fig. 7 Total wall heat flux vs. height normalized with $Q_l^{*2/3} D$ ($D = 0.075\text{m}$, thin wall)

Fig. 6 Total wall heat flux vs. height normalized with $Q_l^{*2/3} D$ ($D = 0.037\text{m}$)

Fig. 8 Total wall heat flux vs. height normalized with $Q_l^{*2/3} D$ (vertical saturated wicks, Ahmad et al)

5. IMPLICATION TO WALL FLAME HEAT TRANSFER

Although the normalization of height by $Q_l^{*2/3} D$ has led to an apparently good correlation on the distribution of wall flame heat flux, it is not yet clear how this result can be explained from the mechanism of heat trans-

fer. Since the measurement of wall flame heat transfer within the pyrolysis zone by Orloff et al [8] using a large PMMA vertical slab has resulted in larger total heat fluxes than in the present measurement (Figure 9), the present result may not hold for larger fuels. The purpose of this section is to correlate the analysis

in the previous section with fluid dynamic and radiative properties of turbulent diffusion flames. Discussion will be made of the pyrolysis zone, the solid flame and the plume region.

Fig. 9 Total wall heat flux vs. height in pyrolysis zone of PMMA (Orloff et al)

5-1 Pyrolysis zone

The integrated form of the conservation equations of momentum and heat of the upward combusting current along a vertical wall can be described as

$$\frac{d}{dx} \int_0^\infty \rho u^2 dy = \int_0^\infty g(\rho_\infty - \rho) dy - \tau_w \simeq \rho_\infty \int_0^\infty g\beta\theta dy - \tau_w \quad (11)$$

$$\frac{d}{dx} \int_0^\infty \rho C_p u \theta dy = \dot{q}'_c - \dot{q}''_r - \dot{q}''_c \quad (12)$$

where τ_w , $\dot{q}''_c$, $\dot{q}'_c$ and $\dot{q}''_r$ represent the wall shear stress, the convective heat flux to the wall surface, the heat release rate due to combustion and the heat loss due to the flame radiation respectively. Assuming that a fixed fraction of the oxygen entrained into the flame at a given height is immediately con-

sumed at that height, $\dot{q}'_c$ is formulated as

$$\dot{q}'_c = \frac{q_f c_T Y_{Ox}}{m} \cdot \frac{d}{dx} \int_0^\infty \rho u dy \quad (13)$$

where q_f is the heat of reaction per mass of fuel, and c_T is the stoichiometric fuel/oxygen mass ratio. Following the analysis of Ahmad and Faeth on a turbulent wall flame^[15], the wall shear stress and the convective heat flux to the wall surface can be represented as

$$\tau_w \simeq \frac{C_f \rho_\infty u_m^2}{2} \quad (14)$$

$$\dot{q}''_c \simeq St \cdot \rho_\infty C_p u_m \theta_m \quad (15)$$

where C_f is the friction coefficient and St is Stanton number.

u_m and θ_m are the maximum upward velocity and the maximum excess temperature at each height respectively. According to the measurement of the distribution of these properties by Ahmad and Faeth^[15], a parameter representing buoyancy, $u_m^2/2g\beta\theta_m x$, is nearly constant ($u_m^2/2g\beta\theta_m x \simeq 0.9$) in this region; this implies that the distribution of u and θ can be represented as (19)

$$u = u_m \cdot U(\eta) \quad (16)$$

$$\theta = \theta_m \cdot \Theta(\eta) \quad (17)$$

$$\eta = y/\delta_b$$

where δ_b is the boundary layer thickness. Assuming that u_m , θ_m and δ_b are proportional to x^μ , x^ν and x^λ respectively, equations (11) and (12) will yield

$$\alpha u_m^2 (2\mu + \lambda) x^{\lambda-1} \cdot \int_0^\infty \rho U(\eta)^2 d\eta =$$

$$\alpha \rho_\infty g \beta \theta_m x^\lambda \cdot \int_0^\infty \Theta(\eta) d\eta - \frac{\rho_\infty C_p u_m^2}{2} \quad (18)$$

$$\alpha C_p u_m \theta_m (\mu + \nu + \lambda) x^{\lambda-1} \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta =$$

$$\frac{\alpha q_f c_r Y_{Ox} (\mu + \lambda) u_m}{m} \cdot x^{\lambda-1} \cdot \int_0^\infty \rho U(\eta) d\eta$$

$$- \dot{q}_r'' - St \cdot \rho_\infty C_p u_m \theta_m \dots \dots \dots (19)$$

The integrals appearing in these equations are constant if velocity and temperature field is described by equations (16) and (17). For equations (18) and (19) to hold, the power of x of each term must be equal. Since the friction factor and Stanton number are constant for a rough wall flow and proportional to $x^{-0.3}$ for a smooth wall [21, 15], equations (18) and (19) imply that the boundary layer thickness must be proportional to x for a rough wall and to $x^{0.7}$ for a smooth wall. Assuming a constant maximum temperature from the

temperature measurements by Ahmad and Faeth [15], μ and ν are to be 1/2 and 0 respectively. These require that $\dot{q}_r''$ is proportional to $x^{1/2}$ for a rough wall and $x^{0.2}$ for a smooth wall for equations (18) and (19) not to fail. Comparing this presumption with the wall heat flux measurement by Orloff et al [8], the rough wall assumptions and the smooth wall assumption are expected to hold for $x > 1m$ and $x < 1m$ respectively.

For a rough fuel surface, equations (18) and (19) yield

$$2\alpha u_m^2 \cdot \int_0^\infty \rho U(\eta)^2 d\eta = \alpha \rho_\infty g \beta \theta_m \cdot$$

$$\int_0^\infty \Theta(\eta) d\eta - \frac{\rho_\infty C_f u_m^2}{2} \dots \dots \dots (20)$$

$$\frac{3\alpha C_p u_m \theta_m}{2} \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta = \frac{3\alpha q_f c_r Y_{Ox} u_m}{2m} \cdot \int_0^\infty \rho U(\eta) d\eta - \dot{q}_r'' - St \cdot \rho_\infty u_m \theta_m \dots \dots \dots (21)$$

u_m and θ_m are solved as

$$u_m = \left[\frac{\left\{ \frac{3\alpha q_f c_r Y_{Ox}}{2m} \cdot \int_0^\infty \rho U(\eta) d\eta - C_{qu} \right\} \int_0^\infty \Theta(\eta) d\eta \cdot g \beta \alpha \rho_\infty}{\left\{ \frac{3\alpha C_p}{2} \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta + St \cdot \rho_\infty C_p \right\} \left\{ 2\alpha \int_0^\infty \rho U(\eta)^2 d\eta + \frac{\rho_\infty C_f}{2} \right\}} \right]^{1/2} \dots \dots \dots (22)$$

$$\theta_m = \frac{\frac{3\alpha q_f c_r Y_{Ox}}{2m} \cdot \int_0^\infty \rho U(\eta) d\eta - C_{qu}}{\frac{3\alpha C_p}{2} \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta + St \cdot \rho_\infty C_p} \dots \dots \dots (23)$$

where $C_{qu} = \dot{q}_r''/u_m$ is constant with height. Since the reaction heat per unit mass of oxygen, $q_f c_r$, is nearly constant for most of the natural fuels [22], u_m and θ_m are thought to

depend mainly on the heat transfer to wall surface and wall friction.

For a smooth fuel surface ($\lambda = 0.7$ and $\dot{q}_r'' \propto x^{0.2}$), equations (18) and (19) yield

$$1.7\alpha u_m x^{-0.3} \int_0^\infty \rho U(\eta)^2 d\eta = \alpha \rho_\infty g \beta \theta_m x^{-0.3} \int_0^\infty \Theta(\eta) d\eta - \frac{\rho_\infty C_f u_m^2}{2} \dots \dots \dots (24)$$

$$1.2\alpha C_p u_m \theta_m x^{-0.3} \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta = \frac{1.2\alpha q_f c_r Y_{Ox} u_m}{m} \cdot x^{-0.3} \cdot \int_0^\infty \rho U(\eta) d\eta - \dot{q}_r'' - St \cdot \rho_\infty C_p u_m \theta_m$$

$$\dots \dots \dots (25)$$

The friction factor for a smooth vaporizing surface is

$$C_f = C_{f0} \cdot \ln(1+B)/B \dots \dots \dots (26)$$

where B is the mass transfer number. $C_{f0} = 0.0450(u_m \delta_b / \nu_\infty)^{-1/4}$ represents the reference friction factor for a smooth wall^[21], and $\ln(1+B)/B$ the blocking effect due to fuel injection^[12].

Stanton number is estimated from the Reynolds-Colburn analogy as

$$St \cdot Pr^{2/3} = \frac{Ro \cdot C_{f0}}{2} \dots \dots \dots (27)$$

where Ro is Rotta's correction factor^[23].

Then, u_m and θ_m are given as the solution of the following simultaneous equations

$$u_m = \left[\frac{\alpha \rho_\infty g \beta \theta_m \cdot \int_0^\infty \Theta(\eta) d\eta}{1.7 \alpha \int_0^\infty \rho U(\eta)^2 d\eta + \frac{\rho_\infty C_f x^{0.3}}{2}} \right]^{1/2} \dots \dots \dots (28)$$

$$\theta_m = \frac{\frac{1.2 \alpha q_f c_r Y_{Ox}}{m} \cdot \int_0^\infty \rho U(\eta) d\eta - \frac{\dot{q}_r'' \cdot x^{0.3}}{u_m}}{1.2 \alpha C_p \cdot \int_0^\infty \rho U(\eta) \Theta(\eta) d\eta + St \cdot \rho_\infty C_p x^{0.3}} \dots \dots \dots (29)$$

where $\dot{q}_r'' x^{0.3}/u_m$ and $St \rho_\infty C_p x^{0.3}$ are constant in this region.

Combining this analysis with the flame height correlations described in the section 4, we can formulate the dependence of flame height on the height of pyrolysis zone, Substituting equation (5) into equation (7), flame height is correlated with incident heat flux to the fuel surface as

$$L_F \simeq \gamma \cdot Q_\ell^{*n} \cdot x_p$$

$$\begin{aligned} &= \gamma \cdot \left\{ q_f \cdot \int_0^{x_p} \Delta H_G \cdot \dot{m}_v'' dx / \rho_\infty C_p T_o \sqrt{g x_p^3} \right\}^n \cdot x_p \\ &= \gamma \cdot \left\{ q_f \cdot \int_0^{x_p} (\dot{q}_w'' - \dot{q}_{rr}'') dx / \rho_\infty C_p T_o \sqrt{g x_p^3} \right\}^n \cdot x_p \\ &\dots \dots \dots (30) \end{aligned}$$

where n is approximately 0.8 for $Q_\ell^* < 1$ and $2/3$ for $Q_\ell^* \geq 1$.

Equation (30) means that flame height is approximately proportional to $(\int_0^{x_p} \dot{q}_w'' dx)^n \cdot x_p^{1-3n/2}$ if reradiation from the fuel surface is insignificant.

Representing $\dot{q}_w'' = \dot{q}_{wo}'' x^j$ where $\dot{q}_{wo}''$ is constant and j is approximately 0.2 for $x < 1m$ and 0.5 for $x > 1m$, the proportionality of flame height to x_p can be described as

$$L_F \propto x_p^{n(j-1/2)+1} \dots \dots \dots (31)$$

For $x_p < 1m$, this relationship will become

$$\begin{aligned} L_F &\propto x_p^{0.76}, \quad (Q_\ell^* \lesssim 1) \\ L_F &\propto x_p^{0.80}, \quad (Q_\ell^* \gtrsim 1) \end{aligned} \dots \dots \dots (32)$$

The asymptotes of the proportionality at $x_p \rightarrow \infty$ is $L_F \propto x_p$.

This relationship implies that flame height is proportional to 0.76 ~ 0.80 power of pyrolysis height for $x_p < 1m$, and the power of x_p representing L_F tends to increase to approach unity if the pyrolysis zone grows over $x_p > 1m$. Orloff et al^[7] found $L_F \propto x_p^{0.781}$ for $0.18 < x_p < 0.85m$ of a PMMA slab, and Kishitani et al^[24] obtained recently $L_F \propto x_p^{0.774}$ for $0.05 < x_p < 0.50m$ also of a PMMA slab. These experimental correlations of flame height seem to be very close to the present analysis.

5-2 Solid flame

On the analogy of the radiative flux to mean beam length in the pyrolysis zone, the constant radiative heat flux in this region is believed to be due to the constant flame thickness. Figure 10 shows the correlation between $\dot{q}_w''(Q_\ell^{*2/3} D)^{1/2}$ and $x/Q_\ell^{*2/3} D$ in this region, and it implies that the flame thickness is proportional approximately to $(Q_\ell^{*2/3} D)^{1/2}$.

5-3 Plume region

For a turbulent plume along an isothermal vertical wall, Ahmad and Faeth^[15] have found the following dimensional relationship for the incident heat flux to the wall surface.

$$x \cdot \dot{q}_w'' \cdot \text{Pr}^{1/3} / Q_x = 1.9 \text{ Ra}_x^{-0.095},$$

$$\text{Ra}_x \equiv g\beta x^3 Q_x \cdot \text{Pr} / \rho_\infty C_p \nu_\infty^3 \dots \dots \dots (33)$$

where Q_x is the local heat flux in the plume, and its conservation is described as

$$\frac{dQ_x}{dx} = \frac{d}{dx} \int_0^\infty \rho C_p \theta u dy = -\dot{q}_w'' \simeq -\dot{q}_c'' \dots (34)$$

The local heat flux must be equivalent to the convective fraction of heat release rate at the height of the solid flame; e.g. $Q_x = kQ$ at $x = 2.8Q_\ell^{*2/3} D$. Let us discuss if the analysis in the previous section is consistent with their result. Defining $\mathcal{L} \equiv 1.9 (\rho_\infty C_p \nu_\infty^3 / g\beta \text{Pr})^{0.095} / \text{Pr}^{2/3}$ just for simplicity, the incident wall heat flux can be represented as a function of Q_x and x as

$$\dot{q}_w'' = \mathcal{L} \cdot Q_x^{0.905} / x^{1.285} \dots \dots \dots (35)$$

Substituting equation (35) into equation (34), we have the distribution of Q_x as

Fig. 10 Total heat flux divided by $(Q_\ell^{*2/3} D)^{1/2}$ vs. height normalized with $Q_\ell^{*2/3} D$

$$Q_x = \left\{ \frac{\zeta}{3x^{0.285}} - \frac{\zeta}{4.0(Q_\ell^{*2/3} \cdot x_p)^{0.285}} + (kQ_\ell)^{0.095} \right\}^{1/0.095} \dots \dots \dots (36)$$

Substituting equation (36) into equation (35), the wall heat flux can be solved as

$$\dot{q}_w'' = \frac{\zeta}{x^{1.285}} \cdot \left\{ \frac{\mathcal{L}}{3x^{0.285}} - \frac{\zeta}{4.0(Q_\ell^{*2/3} \cdot x_p)^{0.285}} + (kQ_\ell)^{0.095} \right\}^{9.53} \dots \dots \dots (37)$$

For $\text{Pr} \simeq 1$, the order of $\mathcal{L}$ is approximately $10^{-2} \sim 10^{-1}$. In this case, the first two terms of the RHS of equation (37) are much smaller than the third term in the plume region. Ignoring the insignificant terms, equation (37) yields

$$\begin{aligned} \dot{q}_w'' &\simeq \zeta \cdot (kQ_\ell)^{0.05} \cdot \left\{ (kQ_\ell)^{2/3} / x_p \right\}^{1.285} \\ &\propto Q_\ell^{0.05} \cdot (x / Q_\ell^{*2/3} \cdot x_p)^{-1.285} \dots \dots (38) \end{aligned}$$

Equation (38) shows that $\dot{q}_w''$ depends almost only on $x/Q_\ell^{*2/3} D$ and Q_ℓ itself has only trivial influence on the distribution of $\dot{q}_w''$. The power of $(x/Q_\ell^{*2/3} D)$ to represent $\dot{q}_w''$ is consistent with that in the experimental relationship described by equation (10).

Terminology

B	: mass transfer number
C_f	: friction factor
C_{fo}	: reference friction factor
C_p	: specific heat of air
D	: characteristic fuel size (burner width or pyrolysis height)
L_F	: flame height
Q	: heat release rate
Q^*	: nondimensional heat release rate ($Q/\rho_\infty C_p T_O \sqrt{gD^3}$)
Q_ℓ	: heat release rate per unit length of fuel
Q_ℓ^*	: nondimensional heat release rate per unit length ($Q_\ell/\rho_\infty C_p T_O \sqrt{gD^3}$)
Q_x	: local heat flux
T_{ig}	: ignition temperature
T_O	: ambient absolute temperature
T_w	: wall temperature
$U(\eta)$	: function characterizing the profile of upward velocity
V_p	: flame spread velocity
Y_{ox}	: oxygen concentration
Pr	: Prandtl number
Ra_x	: local Rayleigh number
Ro	: Rotta's correction factor for the Reynolds-Colburn analogy
St	: Stanton number
c	: specific heat of wall material
c_T	: stoichiometric fuel/oxygen ratio
g	: gravitational acceleration
k	: thermal conductivity
m	: excess air ratio
$\dot{m}_v''$	: fuel vaporization rate

$\dot{q}_c'$	: heat generation rate due to combustion
$\dot{q}_c''$	: convective heat flux to the wall surface
q_f	: heat of combustion per unit mass of fuel
$\dot{q}_r''$	: heat loss due to the flame radiation
$\dot{q}_{rr}''$	: surface reradiation
$\dot{q}_w''$	: incident heat flux to the wall surface
t	: time
u	: upward velocity
x	: height from the bottom of pyrolysis zone
x_p	: pyrolysis height
y	: horizontal coordinate normal to the wall surface
ΔH_G	: gasification heat of fuel
$\Theta(\eta)$	: function characterizing the profile of excess temperature
$\phi(\tau)$	: impulse response of temperature of wall surface to heat application
δ_b	: thickness of boundary layer
δ_f	: preheat distance
β	: expansion coefficient
ρ	: density
ρ_∞	: density of ambient air
θ	: excess temperature
η	: normalized horizontal distance
ξ	: height from the pyrolysis front
ν_∞	: kinematic viscosity
τ	: time
τ_w	: shear stress
$C_{qu}, j, n, \alpha, \gamma, \zeta, \lambda, \mu, \nu$	: constants

Suffix

c	: characteristic value
m	: maximum

6. CONCLUSIONS

Approximate relationships between the practical wall flame properties concerning the

turbulent upward flame spread and the fuel properties are derived from experiments using sand line burners against isothermal and thermally thin walls. The physics of the relevant processes seems to need further investigation; however, the following conclusions can be drawn.

- 1) The flame height against a wall is proportional to $Q_\ell^{*n} D$ where $Q_\ell^* \equiv Q_\ell / \rho_\infty C_p T_{OG}^{1/2} D^{3/2}$. The value of n is approximately 0.8 for $Q_\ell^* < 1$ and $2/3$ for $Q_\ell^* > 1$. This implies that the fuel size effect on the flame height vanishes for $Q_\ell^* > 1$.
- 2) The incident heat flux to the wall surface is represented approximately as a function of $x/Q_\ell^{*2/3} D$. The wall above the pyrolysis zone appears to divide into four distinct regions, the lower and upper parts of the solid flame, the transition region, and the plume region.
- 3) The experimental relationship between the incident heat flux to the wall surface and the normalized height, $x/Q_\ell^{*2/3} D$, is explained from the relevant heat transfer mechanism. This analysis seems to explain also the flame height correlations by current experimental work. For further analysis of upward flame spread, measurements and mathematical modeling of the flame radiation should be encouraged.

ACKNOWLEDGEMENTS

This work was mostly conducted at the Center for Fire Research, U.S. National Bureau of Standards. The experimental apparatus was built by the Center for Fire Research. The study was basically supported by NBS. The author wishes to thank Dr. J.G. Quintiere, and Mr. K.D. Steckler for the arrangement of the experiments. The author is also indebted

deeply to Mr. W. Rinkinen, Mrs. M. Harkleroad and Mr. D. Waggoner for their support in the experiments. Discussions of Dr. J.G. Quintiere, Dr. A. Robertson, and Dr. K. Saito of Princeton University are gratefully appreciated.

REFERENCES

- [1] de Ris, J. (1968). Spread of a laminar diffusion flame. Twelfth Symposium (International) on Combustion, pp. 241–252.
- [2] Ohki, Y., and Tsuge, S. (1974). On the flame spreading over a polymer surface. Combustion Science and Technology, 9, 1.
- [3] Fernandez-Pello, A.C. (1978). A theoretical model for the upward laminar spread of flames over vertical surfaces. Combustion and Flame, 31, 135.
- [4] Annamalai, K., and Sibulkin, M. (1979). Flame spread over combustible surfaces for laminar flow systems (Part I: Excess fuel and heat flux). Combustion Science and Technology, 19, 167.
- [5] Fernandez-Pello, A.C., and Pagni, P.J. (1983). Mixed convective burning of a vertical fuel surface. ASME-JSME Thermal Engineering Joint Conference, pp. 295–301.
- [6] de Ris, J., and Orloff, L. (1974). The role of buoyancy direction and radiation in turbulent diffusion flames on surfaces. Fifteenth Symposium (International) on Combustion, pp. 175–182.
- [7] Orloff, L., de Ris, J., and Markstein, G.H. (1974). Upward turbulent fire spread and burning of fuel surface. Fifteenth Symposium (International) on Combustion, pp. 183–192.
- [8] Orloff, L., Modak, A.T., and Alpert, R.L. (1976). Burning of large-scale vertical

- surfaces. Sixteenth Symposium (International) on Combustion, pp. 1345–1354.
- [9] Sibulkin, M., and Kim, J-B. (1977). The dependence of flame propagation on surface heat transfer II. Upward burning. *Combustion Science and Technology*, 17, 39.
- [10] Fernandez-Pello, A.C., and Hirano, T. (1983). Controlling mechanism of flame spread. *Combustion Science and Technology*, 32, 1.
- [11] Quintiere, J.G., and Fernandez-Pello, A.C. (1982). A simplified model of radiating turbulent upward flame spread over the surface of a charring combustible. Technial meeting. Eastern section, Combustion Institute.
- [12] Marxman, G.A. (1964). Combustion in the turbulent boundary layer on a vaporizing surface. Tenth Symposium (International) on Combustion, pp. 1337–1349.
- [13] Hasemi, Y., and Tokunaga, T. (1984). Some experimental aspects of turbulent diffusion flames and buoyant plumes from fire sources against a wall and in a corner of walls. *Combustion Science and Technology*, 40, 1.
- [14] Liburdy, J.A., and Faeth, G.M. (1977). Fire induced plume along a vertical wall: Part I. The turbulent weakly bulyant region. Prepared for U.S. Department of Commerce, NBS, Center for Fire Research, Grant No. 5-9020.
- [15] Ahmad, T., and Faeth, G.M. (1978). Fire induced plumes along a vertical wall: Part III. The turbulent combusting plume. Prepared for U.S. Department of Commerce, NBS, Center for Fire Research, Grant No. 5-9020.
- [16] Steckler, K.D. (1981). Fire induced flows through room openings-Flow coefficients. Technical Research Report, Armstrong World Industries.
- [17] Thomas, P.H., Webster, C.T., and Raftery, M.M. (1961). Some experiments on buoyant diffusion flames. *Combustion and Flame*, 5, pp. 359.
- [18] Zukoski, E.E., Kubota, T., and Cetegen, B. (1980/81). Entrainment in fire plumes. *Fire Safety Journal*, 3, pp. 107
- [19] Hasemi, Y., and Tokunaga, T. (1983). Modeling of turbulent diffusion flames and fire plumes for the analysis of fire growth. Twentyfifth National Heat Transfer Conference, Fire Dynamics and Heat Transfer, ASME, pp. 37–45.
- [20] Kulkarni, A.K. (1984). Private Communication.
- [21] Eckert, E.R.G., and Jackson, J.W. (1951). Analysis of turbulent free-convection boundary layer on flat plate. NACA Technical Report, No. 1015.
- [22] Hugget, C. (1980). Estimation of heat release by means of oxygen consumption measurements. *Fire and Materials*, 4, 2, pp. 61–65.
- [23] Rotta, J.C. (1964). Temperaturverteilungen in der Turbulenten Grenzschicht an der ebenen Platte. *International Journal of Heat and Mass Transfer*, 7, pp. 215.
- [24] Kishitani, K., Sugawara, S., and Hamada, K. (1984). Upward flame spread of building materials-Basic analysis on PMMA and plywood slabs. Saigai no Kenkyu (Collected Reports on Disaster Research), Vol. 15. (in Japanese).