ON FORMULAE FOR THE MOVEMENT OF SMOKE IN FIRES

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Introduction

There are various approaches to the design of smoke protection systems and various methods of calculating smoke movement. There is necessarily much agreement in principle on the exploitation of concepts such as those relating to entrainment, plumes, hot and cold layering but there are several differences in detail, their approximation and in their interpretation. It is not always clear what importance should be attached to these or what precision should be expected.

In the near future many differences in calculation methodology will be resolved by numerical fluid dynamics i. e. "field modelling" but some aspects of these techniques have yet to be validated themselves. In the meanwhile it is essential that judgements about differences between design methods be made in the context of their limitations and lack of internal consistency. This brief note lists a few examples of such limitations, especially those in conventional plume theory and its applications.

Comments

1 Thomas, Hinkley et al.^[1] based their calculations for roof venting on experimental data for the mass flow in a plume, taken from the work of Yih^[2]. He gave

$$\dot{m} = 0.15 \rho_A \left(\frac{g Q}{\rho_A C_p T} \right)^{1/3} Z^{5/3} \quad (1)$$

where $\dot{m}$ is the mass flow. T_A is the ambient temperature, Z is the height above the effective point source of the plume, and ρ_A and C_p are the density and specific heat of the gases, both taken as uniform. g is the acceleration due to gravity.

In later work Zukoski et al.^[3] gave 0.21 for the coefficient but this has yet to be incorporated into some calculation methods for design. This 35% difference

needs to be judged in relation to other differences, some quantitative and some qualitative.

2 Entrainment into a plume is a concept associated with flow similarity, be it of velocity, temperature or density difference. None of these is 'a priori' similar near to a finite hot source. In principle, however, it is only in a region where cross sectional distributions are conserved that can one strictly define E , a coefficient of entrainment. For axi-symmetric plumes this is usually

$$E = \frac{1}{PW_c} \frac{d\dot{m}}{dZ} \quad (2)$$

where P is an effective perimeter for the plume and W_c (Z) is the peak, axial, velocity.

Clearly P has to be defined by reference to a characteristic dimension e.g. $P = 2\pi b$ where 'b' is the "scale" of the assumed distribution, which is usually taken to be Gaussian.

For a 2 dimensional line plume

$$E = \frac{1}{2W_c} \frac{d\dot{m}'}{dZ} \quad (3)$$

where $\dot{m}'$ is the mass flow per unit length of line. No dimension corresponding to 'b' appears; a difference between axi-symmetric and line plumes.

Experiments by Ricou & Spalding^[4] on an axi-symmetric jet resulted in a modification to the concept of a constant coefficient of entrainment relating velocities.

They showed that the coefficient of entrainment is not independent of the density difference between the plume and the surroundings, although most plume theories treat it as such.

Data obtained by Heskestad^[9], Kung and Stravrianidis^[10], You and Kung^[11] confirm Morton's prediction that correlating temperature rises in axi-symmetric plumes may be better in terms of $\Delta T / T_A$ than in terms of $\Delta T / T_p$ where T_p is the absolute peak temperature in the plume, T_A that of the surroundings

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and ΔT is $T_P - T_A$. However, the arguments employed by Morton cannot apply unmodified to line plumes; another difference between the two kinds of plume geometry.

Consideration of the secondary effects of density differences suggests (see Appendix) that one might be able to justify correlations in terms of $\Delta T / T_A$ i.e. temperatures lower than those expected assuming correlations are in term of $\Delta T / T_P$; however the arguments are not the same as for axi-symmetric plumes.

3 An equation for the mass flow in a flame, used by Thomas, Hinkley et al^[1] in what is called "large fire" theory for roof venting, has been shown to be useful for plumes^[12]. It describes the variation of mass flow ' m ' with height ' h ' as

$$m \propto h^{3/2} \quad (4)$$

A theoretical justification is still awaited for this widely exploited extension of a simple flame correlation.

4 Morgan and Marshall^[13] have considered the rotation of a layer of hot gas, flowing from under a horizontal balcony to become a vertical (2 dimensional) plume.

They argued that their data showed that there was an entrainment much greater than expected. Instead of the horizontal, inward, entrainment velocity being the conventional 0.16 times the vertical component of the central plume velocity, they found that to get good agreement a ratio of 0.9 had to be assumed as the plume turned.

The basis of their model was the assumption of uniform horizontal flow and, that on the level of the balcony there was an effective horizontal source for the vertical plume having a Gaussian distribution of velocity and of temperature, i.e. their "Equivalent Gaussian Source". They adopted the Lee and Emmons model for the vertical plume.

Lee and Emmons did not consider the transitional flow from a real finite source to a Gaussian plume. When examining their experiments they fitted their data to a theoretical relationship involving a dimensionless parameter dependant on

(1) the initial velocity of the Gaussian plume

(2) its density defect i.e. its temperature rise, and
(3) its characteristic width scale.

Morgan and Marshall match

(1) the heat flow of the actual layer moving horizontally
(2) its mass, and
(3) its kinetic energy to those of the vertical plume at its "Equivalent Gaussian Source".

Irrespective of the inappropriateness of the use of energy instead of vertical momentum, this means the average temperature rise of the actual flow is equal to the average temperature rise of the Equivalent Gaussian Source.

Now if m_o is the horizontal layer flow having a uniform temperature rise θ_o and if the vertical flow from the Gaussian Source is m_l ($\gg m_o$) at a mean mass temperature rise $\bar{\theta}_l$ then neglecting all heat loss

$$m_o \theta_o = m_l \bar{\theta}_l$$

But for a Gaussian distribution of velocity e^{-x^2/b^2} and a Gaussian distribution of temperature rise $e^{-x^2/b^2\lambda^2}$ where x is the distance from central plane of the plume, b is the characteristic width scale of the plume and λ a parameter shown by Lee and Emmons^[14] to be 0.9 and taken as such by Morgan et al

$$\begin{aligned} \bar{\theta}_l &= \theta_{peak} \frac{\lambda}{\sqrt{1+\lambda^2}} \\ &= 0.67 \theta_{peak} \end{aligned} \quad (5)$$

where θ_{peak} is the maximum temperature in the vertical stream.

The concept of the Equivalent Gaussian Source as used here therefore requires a hypothetical increase in the peak temperature of the flow-but this cannot be observed in actual temperature rises which may become interpreted as a cooling induced by extra entrainment which must be 50% even if there is no actual entrainment at all.

5 Lastly we consider what might happen if a plume is disturbed along its vertical axis. Conventional plume theory treats the local horizontal velocity as proportional to the local vertical velocity which for a line source is constant in the far field. If such a plume were interrupted by, say, a horizontal wire grid which did not deflect the central axis there would necessarily be a

loss of vertical momentum and velocity and so of entrainment. The lesser dilution implies that the temperature at some level above the disturbance in the new far field would be raised! Does this happen? Are the conventional assumptions governing mean values, similarity, neglect of pressure etc. sufficiently robust to describe this relatively simple effect?

Conclusion

The above comments all refer to matters which are the subject of approximation (or in some cases of neglect) in smoke safety design.

Maybe some of them are trivial in their effect or perhaps cancel the effects of others. However, their role may sometimes be important and lessen the assumed precision of some simplified theories.

Engineering applications of plume theories, based on mean flow and the conventional free boundary self-similar solutions, are of great usefulness but their limitations must not be overlooked.

Appendix

The line plume equations analogous to those used by Morton^[6], for axisymmetric plumes relate strong to weak plumes, but retaining a constant entrainment coefficient, are

$$\text{for mass} \quad \frac{d}{dz}(ub\rho_P) \propto \alpha u\rho_A \quad (\text{A1})$$

$$\text{for momentum} \quad \frac{d}{dz}(ub^2\rho_P) \propto g\Delta\rho b \quad (\text{A2})$$

$$\text{and for energy} \quad C_P\rho_P u\Delta T b \propto Q' \quad (\text{A3})$$

where Q' is the convected energy output (heat) per unit length of line; C_P is the specific heat, b is the width at height z , u is the vertical velocity and g is the acceleration due to gravity.

The Lee & Emmons^[15] treatment in effect puts ρ_P as the constant ambient density in the first two equations, but uses the gas law to make the third equation into

$$C_P\Delta\rho u T_A b \propto Q' \quad (\text{A4})$$

Lee & Emmons thereby remove T and ΔT and develop the theory in terms of $\Delta\rho/\rho_A$

The substitution

$$b\rho_P/\rho_A = B$$

in equation (A1) (A2) (A3)

removes ρ_P from (A1), makes the right hand side of (A2) into $g\Delta\rho B/\rho = gB\Delta T/T_A$ and makes (A3) into $C_P U\rho_A\Delta TB \propto Q'$.

It thus allows the theory to develop with fewer approximations using $\Delta T/T_A$ rather than $\Delta T/T_P$.

The approximation of a top hat profile, the arbitrary assumption that in strong plumes it is density difference not temperature rise that is distributed in a Gaussian fashion means that the Lee & Emmons theory can be used to justify neither the use of θ/T_A nor of θ/T_P : experimental justification is required.

A fuller discussion of this will appear elsewhere.

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