STEREOREOGRAPHIC PROJECTION OF MINERAL ASSEMBLAGES

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INTRODUCTION

In dealing with the phase equilibrium relations between coexisting minerals in four component system, the use of graphic representation of the paragenesis helps the understanding of compatible and incompatible assemblages. The projection of a tetrahedron onto a plane, however, brings tedious complication into the diagram, and hardly serves for quantitative use. Simplified triangular diagrams such as ACF and AKF by Eskola (1915) are widely used, and are useful in many practical purposes but they do not satisfy the requirements in detailed phase equilibrium considerations.

Based on phase-rule considerations, Thompson (1957) proposed a method of projecting the assemblages in a tetrahedron from a stoichiometric mineral onto a plane, thereby enabling us to deal with the equilibrium relations in several partial systems of the four component system in terms of a ternary equilibrium. There is, however, one demerit in this projection. The solid angle of projection is restricted to be less than $2\pi$, and practical solid angle is much more restricted.

The restriction of the solid angle in the plane projection can be avoided by projecting mineral assemblages onto a sphere and by stereographic representation of the spherical geometry.

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The mineral assemblages in a four component system are generally represented by a tetrahedron. Partial systems of this which contain a stoichiometric mineral $O$ can be dealt with the theory of ternary system, if the relative amount of $O$ in the partial systems can be ignored. Then, if we draw a sphene with the origin at the point for $O$, and project the paragenetic relation onto this sphene, the phase relations involving $O$ can be represented by the spherical geometry. Therefore, the stereographic projection of this sphere makes it possible to deal the paragenetic relation including the whole $4\pi$ solid angle, if the North-and South Pole projections are being used simultaneously. Further, by proper rotation of the sphene, we can avoid $4\pi$ geometry and need only deal with the projection from one pole in many cases.

The algebra of the projection is as follows.

Let us consider the equilibrium relations involving mineral $O$, having a stoichiometric composition, in a four component system.

We first transform the coordinates of tetrahedron to ordinary rectangular ones.
Stereographic projection of mineral assemblages

The relation between two coordinate systems is shown in Fig. 1. The rectangular coordinates of a point with the tetrahedral coordinate \((a, b, c, d)\) are given by

\[
x = \frac{a + 2c + d}{2}
\]

\[
y = \frac{\sqrt{3}}{6} a + \frac{\sqrt{3}}{2} b
\]

\[
z = \frac{\sqrt{2}}{3} a
\]  

Let us denote the rectangular coordinates of the mineral 0 by \((x_0, y_0, z_0)\) and those of a coexisting mineral 1 by \((x_1, y_1, z_1)\). The direction of a vector 0–1 is then expressed in polar coordinates by

\[
\theta = \tan^{-1} \frac{\Delta y_1}{\Delta x_1}
\]

\[
\phi = \tan^{-1} \frac{\Delta x_1}{\sqrt{\Delta x_1^2 + \Delta y_1^2}}
\]  

where \(\Delta x_1 = x_0 - x_1\) etc.

The projection of the tie line between minerals 1 and 2, both coexisting with 0 is a great circle, and a four phase space 0–1–2–3 is represented by a spherical triangle. Thus, the paragenetic relations on the stereographic projection are shown in terms of great circles and spherical triangles. The range of solid solutions and the partition relation are quantitatively dealt with in this projection.

There is, however, one demerit in this projection. By expanding the solid angle of projection, we have sacrificed a simple geometrical relation between the ratio of coexisting minerals 1, 2 and 3 and the bulk chemical composition in regard to these components, which is quite simple in the plane projection. The algebraic relations between the polar coordinates of minerals 1, 2 and 3 and the molecular ratio among these are given in the appendix.

A few examples of the use of this projection are given below.

Example 1 Paragenesis involving almandine garnet

The pelitic schists of the albite-epidote amphibolite facies generally contain almandine garnet. Associated minerals are muscovite, biotite and chlorite. The stereographic projection from the almandine composition makes it possible to represent the paragenesis involving muscovite-phengite, biotite, chlorite, microcline and paragonite as shown in Fig. 2. The data are taken from Kurata (1972).

Example 2 Paragenesis of peridotite

The phase equilibrium relations in Mg-rich and Na2O poor portion of eclogitic and peridotitic assemblages can be dealt with the four component system SiO2-Al2O3-MgO-CaO. The projections of assemblages from diopside and olivine are shown in Figs. 3 and 4a. The applicability of the projection from diopside depends on the adequacy of assuming stoichiometric diopside. Thus, the paragenesis in CaO-rich side of the diagram, for which Ca-rich clinopyroxene should be diopside-Ca-Tschermakite solid solution, is rather schematic.
Fig. 2. Mineral assemblages of pelitic schists of the epidote amphibolite facies terranes of the Sazare area. Stereographic projection from almandine.

Fig. 3. Stereographic projection of peridotitic assemblages in SiO$_2$-Al$_2$O$_3$-CaO-MgO system from diopside. Qz: quartz, En: enstatite, Fo: forsterite, Ak: akermanite, Gl: gehlenite, Sp: spinel, An: anorthite, Wo: wollastonite

For the assemblages projected from olivine, the stereographic projection is compared with the plane projection. When the solid angle of projection is small, the plane projection is preferred.

**APPENDIX**

1. Geometrical relations between the coexisting minerals, 1, 2 and 3, and the bulk chemical composition, P, are shown in Fig. 5. Point Q is defined as the intersection of two great circles 1P and 23. Denoting the angles of 1P and PQ by $\varphi_1$ and $\varphi_2$, the amount of mineral 1, in this partial system is given by

$$X_1 = \frac{\tan \varphi_1}{\tan \varphi_1 + \tan \varphi_2}$$

These angles can be calculated using...
Fig. 5. Relationships between the spherical angle and the modal compositions (see text).

spherical trigonometry, but they can easily be read on the stereographic net.

2. The coordinates of point \( P \) in the plane projection is determined as follows. We denote the coordinates of the origin and given point \( P \) by \( (x_0, y_0, z_0) \) and \( (x_p, y_p, z_p) \), respectively.

The projection of \( P \) onto a plane passing through \( (x_1, y_1, z_1) \), \( (x_2, y_2, z_2) \) and \( (x_3, y_3, z_3) \) is given by

\[
Ax + By + z = D \tag{7}
\]

where

\[
\begin{pmatrix}
x_1y_1 - 1 \\
x_2y_2 - 1 \\
x_3y_3 - 1
\end{pmatrix}
= \begin{pmatrix}
A \\
B \\
C
\end{pmatrix}
\begin{pmatrix}
-z_1 \\
-z_2 \\
-z_3
\end{pmatrix} \tag{8}
\]

for the line passing through \( O \) and \( P \), we have

\[
Z = \frac{\Delta z}{\Delta x} x + (z_0 - \frac{\Delta z}{\Delta x} x_0) \tag{9}
\]

The plane being in question intersects the coordinate lines at \( (-\frac{D}{A}, 0,0) \), \( (0, \frac{D}{B}, 0) \) and \( (O,O,D) \). Therefore, the coordinate of \( P \) in regard to the triangular diagram having these points at the apices are given as follows;

\[
X = \frac{A}{D} x, \quad Y = \frac{B}{D} y, \quad Z = \frac{z}{D}
\]

when \( X + Y + Z = 1 \).

ACKNOWLEDGEMENTS

The author is indebted to Prof. M. Yamasakai who kindly wrote a FORTRAN IV program for this projection, and critically read the manuscript.

REFERENCES

