NOTE

Vertical diffusivity and consumption rate of dissolved oxygen in the thermocline

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Abstract—On the basis of the vertical distribution of $^3$H observed at the Geosecs station (1969) in the Northeast Pacific, the vertical eddy diffusivity in the thermocline was evaluated to be $0.16 \text{cm}^2/\text{sec}$. By using this value and dissolved oxygen profile, the consumption rate of oxygen in the thermocline was estimated to be $3.9 \times 10^{-2} \text{ml/kg/y}$ on an average.

The intermediate layer below the surface layer is a region where the temperature decreases steeply with depth, and is called thermocline. The thermocline is a buffer layer for the overlying surface layer against mixing with the deep layer. The interchange between surface and deep water through the thermocline plays a most significant role for the distribution of nutrients and food chain of marine biota. The time scale of the mixing process through the thermocline appears to be of the order of ten years (SUESS, 1969), but unfortunately the precise value relevant to the mixing process is not well known.

Small amounts of tritium (half-life = 12.26y) are constantly being produced by cosmic rays. Large quantities of tritium, however, were released artificially in the atmosphere during 1961 and 1962 (MARTELL, 1963), and added to the surface water of the oceans. Therefore, measurements of tritium profile provide a fine clue to elucidating the mixing process in the thermocline. BAINBRIDGE (1963) analyzed tritium in the Pacific and utilized tritium as a tracer in a box model. ÖSTLUND et al. (1969) have analyzed tritium in the vertical samples in the equatorial Atlantic collected on the cruise of Equalant III, 1964. DOCKINS (1969) has shown a tritium profile in the samples from 0 to 400m in the mid-Pacific collected in May, 1965. In these regions of the Atlantic and the Pacific, small amounts of tritium were found below the surface layer. ROETHER et al. (1970) have measured a detailed tritium profile at the Geosecs station (1969) in the Northeast Pacific, and obtained the result as shown in Fig.1.

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ROETHER et al. (1970) calculated the vertical eddy diffusivity in the thermocline from the steep decrease of tritium in the profile using the following equations;

\[ \frac{\partial M}{\partial t} = D_Z \frac{\partial^2 M}{\partial z^2} \]  

(1)

and

\[ D_Z \frac{\partial^2 M}{\partial z^2} + w \frac{\partial M}{\partial z} - \lambda M = 0 \]  

(2)

where \( M \) denotes the concentration of tritium, \( t \) the time, \( D_Z \) the vertical eddy diffusivity, \( z \) the depth, \( w \) the upward advection velocity and \( \lambda \) the decay constant of tritium. Eq.1 represents a simple non-steady state condition and eq.2 a steady state. They have obtained the values of \( D_Z = 0.15 \, \text{cm}^2/\text{sec} \) by the use of eq.1 and \( 0.2 \, \text{cm}^2/\text{sec} \) by eq.2 under the assumption of \( w = 3.3 \times 10^{-7} \, \text{cm/sec} \) (10 m/y) as the upward advection velocity.

If the one-dimensional model can be applied to the tritium profile in the thermocline, we can write the following equation;

\[ \frac{\partial M}{\partial t} = D_Z \frac{\partial^2 M}{\partial z^2} + w \frac{\partial M}{\partial z} - \lambda M. \]  

(3)

A general solution of eq.3 is given by LERMAN (1971). The solution of eq.3 with the initial and boundary conditions; at \( t = 0, M = 0 \) in \( z > 0 \), at \( t > 0, M = M_0 \) at \( z = 0 \) and \( M = 0 \) at \( z = \infty \), is
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\[
2M/M_0 = \exp\left[(-w + \sqrt{w^2 + 4\lambda D_z}z/2D_z)\text{erfc}\left[z/2\sqrt{D_zt} + \sqrt{(w^2/4D_z + \lambda)t}\right]
+ \exp\left[(-w - \sqrt{w^2 + 4\lambda D_z}z/2D_z)\text{erfc}\left[z/2\sqrt{D_zt} - \sqrt{(w^2/4D_z + \lambda)t}\right]\right]
\]

(4)

where \(z = 0\) stands for the top of the thermocline.

Before the best-fit of eq.4 to the real profile of tritium is made, in order to check the problem whether the one-dimensional model can be applied to the profile of tritium or not, the following consideration was made. A linear relationship in T–S diagram in a vertical column can be attributed to the vertical mixing of two different types of water in different proportion (SVERDRUP et al., 1942). Therefore, the linear relationship in T–S diagram gives us an indication of the applicability of the one-dimensional model to a vertical column. If the one-dimensional model can be applied to a vertical column, the profile of the stable conservative tracer such as salinity or potential temperature, i.e., corrected temperature for adiabatic temperature change, can be expressed by eq.5 under the assumption of steady state.

![T-S diagram at the Geosecs station with depth (m).](image-url)

**Fig.2.** T–S diagram at the Geosecs station with depth (m).
\[ D_z \frac{\partial^2 M}{\partial z^2} + w \frac{\partial M}{\partial z} = 0. \]  \hspace{1cm} (5)

The solution of eq.5 is given by

\[ \frac{M - M_1}{M_2 - M_1} = \frac{\exp[-w/D_z \cdot (z - z_1)] - 1}{\exp[-w/D_z \cdot (z_2 - z_1)] - 1} \]  \hspace{1cm} (6)

where \( M_1 \) and \( M_2 \) represent the boundary values at \( z_1 \) and \( z_2 \) cm depths, respectively. As it is obvious from eq.6, the feature of a profile of conservative tracer is described only by the value of \( w/D_z \).

As shown in Fig.2, since the \( T-S \) relationship in the thermocline at the Geosecs station is almost linear between 250 and 400m, the one-dimensional model is applicable to the column. As shown in Fig.3, the potential temperature profile can be described by the value of \( w/D_z = 1 \times 10^{-5} \text{ cm}^{-1} \). On these bases, the tritium profile given by ROETHER et al. (1970) can be treated by the one-dimensional model between 150 and 400m depths.

First, we have to estimate whether the advection term and/or decay term are negligible or not in the mixing process. ROETHER et al. (1970) reported that the tritium concentration below 160m is expressed by

![Fig.3. Potential temperature profile in the thermocline at the Geosecs station.](image-url)
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\[ M = 23 \exp[-z/(4.4 \times 10^3)] \]  \hspace{1cm} (7)

where $M$ denotes the tritium concentration by T. U., and $z$ the depth by cm below 160m level. Inserting eq.7 into eq.3, we have

\[ \frac{\partial M}{\partial t} = M(5.2 \times 10^{-8}Dz - 2.3 \times 10^{-4}w - 1.8 \times 10^{-9}). \]  \hspace{1cm} (8)

It is obvious from eq.8 that the advection term is negligible because the value of $w/Dz$ is $1 \times 10^{-5}$ cm$^{-1}$. Therefore, eq.8 is rewritten

\[ \frac{\partial M}{\partial t} = M(5.2 \times 10^{-8}Dz - 1.8 \times 10^{-9}). \]  \hspace{1cm} (9)

As it is obvious from eq.9, the decay term is not negligible when the value of $Dz$ is $10^{-1}$ cm$^2$/sec or less. When $w = 0$ in eq.4, it follows,

\[ \frac{2M}{M_0} = \exp \left( \frac{z\sqrt{\lambda}/Dz}{2\sqrt{Dzt + \sqrt{\lambda}t}} \right) \]
\[ + \exp \left( -\frac{z\sqrt{\lambda}/Dz}{2\sqrt{Dzt - \sqrt{\lambda}t}} \right). \]  \hspace{1cm} (10)

The numerical value of $Dz$ was evaluated to be 0.16 cm$^2$/sec from the best-fit of eq.10 to the real profile of tritium, when $t$ is set at 6y. Since the tritium concentration in North Pacific surface water had steeply risen in 1963 (DOCKINS et al., 1967), the author applied 6y to the sample collected in 1969 as ROETHER et al. (1970) did.

When $t$ becomes large, errfc$\sqrt{\lambda}t$ approaches unity, and eq.10 turns out to be identical with eq.11. Eq.11 is the solution of eq.3 under the condition of $\frac{\partial M}{\partial t} = w = 0$,

\[ M = M_0 \exp[-z\sqrt{\lambda}/Dz]. \]  \hspace{1cm} (11)

Although the application of eq.11 to the tritium profile at the Geosecs station is not suitable because the elapsed time is not large enough, the fitting of eq.11 to the profile gives $Dz = 0.035$ cm$^2$/sec. This value in a steady state gives the lower limit. The value of $Dz = 0.2$ cm$^2$/sec calculated by ROETHER et al. (1970) by the use of eq.2 (steady state) is derived from the assumption of an extremely high advection velocity of $w = 3.3 \times 10^{-5}$ cm/sec.

The value of $Dz$ calculated on the basis of the tritium profile at the Geosecs station in the Northeast Pacific in the region between 150 and 400m seems to be considerably smaller than many earlier estimates based on the other methods. ROBINSON and STOMMEL (1959) reported that $Dz$ at the bottom of the thermocline was calculated to be 1cm$^2$/sec by the use of geostrophic dynamics model. BIERI et al. (1966) reported that $Dz$ in the thermocline was calculated to be 0.3 cm$^2$/sec based on the noble gas profile and laminar layer model for gas interchange. BROECKER (1966) suggested the value of $Dz = 1$ cm$^2$/sec in the main thermocline on the basis of a complicated box model.

The dissolved oxygen is an appropriate tracer to study the oxidation of organic matter in sea water. However, the consumption rate of oxygen in the thermocline is not well known. As WYRTKI (1962) showed, the consumption rate of oxygen $(J)$ in sea water depends on depth exponentially, i.e.,
\[ J = J_0 e^{-\alpha z}. \]  

(12)

Therefore, as to the oxygen in the thermocline at the Geosecs station, we can write the following equation,

\[ D_z \frac{\partial^2 M}{\partial z^2} + w \frac{\partial M}{\partial z} - J_0 e^{-\alpha z} = 0. \]  

(13)

The oxygen profile at the Geosecs station (CRAIG and WEISS, 1970) is shown in Fig.4. According to them, the oxygen concentration between 250 and 400m is empirically expressed by

\[ M = 17 \exp[-6.7 \times 10^{-5} z] \text{ml/kg}. \]  

(14)

Inserting eq.14 and the values of \( D_z \) and \( w \) obtained previously into eq.13, the consumption rate of oxygen is obtained to be

\[ J_0 e^{-\alpha z} = 0.33 \exp[-6.7 \times 10^{-5} z] \text{ml/kg} \text{/y}. \]  

(15)

The average consumption rate of oxygen in the region from 250 to 400m obtained by integration of eq.15 is

\[ \langle J \rangle = 3.9 \times 10^{-2} \text{ml/kg} \text{/y}. \]

This rate is about ten times as large as that in the deep Pacific water (CRAIG, 1971; TSUNOGAI, 1972).
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