RESEARCH NOTE

AUTOMATIC ANALYSIS ALGORITHM FOR SIDE-SLOPE CUT-FILL BASED ON DIGITAL ELEVATION MODEL

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Abstract : In highway projects, balancing cut-fill is an important way to minimize economic payout. As the result, cut-fill calculation becomes a necessary work to the highway designer. Traditionally, cut-fill planning is not only a slow, laborious work, but also one of the problems in the feasibility analysis. With the large-scale products of Digital Elevation Model (DEM) shared up and put to use, automatic analysis for cut-fill projects is provided with reliable authenticity. This paper tries to develop a cut-fill algorithm on the basis of ARC/INFO GRID on the condition of the unknown hypsography resulted from the cut-fill diffusion model, provided with pre-cut-fill DEM and road coverage, engineering side-slope, road height and width. The aim of this paper is to propose the algorithm to calculate excavation- amount, fill-amount, and simulate the post-cut-filled DEM when the supposed cut-fill is done. This algorithm is feasible based on current GIS software, can save a lot time for the highway designers and offer a considerable cut-fill designing.

Key words : ARC/INFO, Cut-fill, DEM, Grid cell, Diffusion model

1. Introduction

Digital Elevation Model (DEM) is one of the most important methods to describe the characteristic of the earth surface. With the large-scaled products of 1:10,000 scale and even higher scale shared up and put to use, it will improve work efficiency of momentous engineering projects and lower designing and producing costs. We often face the problem of calculating road cut-fill volume, when engineers design, pick up line and build railway, highway, hydraulic engineering and all sorts of web tubes. Traditional method mostly depends on the work of measurement and data of profile's designing, making the cross-section through the centerline of road cut-fills alternately, calculating the area of each cross-section, and getting the cut-fill volume by promising two adjacent cross-sections as a flat column. Obviously, the traditional method is time-labor-consuming and in coarse precision (If the elevation of cut-fill area changes greatly, the promising is invalid, the data must be re-sampled densely), and unsuitable to pre-study, analysis for momentous engineering projects.

There are some researchers who have done about road cut-fill analysis, such as Li Weimin(1988), who exposed a method for designing road cut-fill software based on Triangulated Irregular Network (TIN: one format of DEM in ArcGIS software); Shi Shiyun(1998), who discussed an algorithm about road cut-fill in

unparallel area; Xu Shukun(1998) and other three researchers(Tang Rongxi, Wand Jiaye, Peng Qunsheng, 1990), who explored a method of transforming 3D problems to 2D problems and applied it to cut-fill analysis. ARC/INFO software also affords cut-fill analysis. However, the work above mentioned exists a fatal error that the post-cut-fill DEM must be known before cut-fill analysis. It is difficult to obtain the post-cut-fill DEM if the cut-fill project hasn't done. Obviously, it is invalid in solving practical problems. Even though there has a function of cut-fill analysis in ArcGIS software, the function without considering the side-slope is too simple to resolve the practice problem.

In order to overcome above shortcoming, we use ARC/INFO AML to compile an algorithm of road cut-fill based on DEM. Even more, we can show the result of the cut-filled region surface before the project being started.

2. Principle and algorithm of cut-fill analysis

2.1 Known conditions

Hypotheses are known as:

- 1) Pre-cut-fill DEM is TERLKGRID, road's centerline DEM is ROADGRID, and cell size is D ;
- 2) Road's width is H , road's altitude is Z , slope of safety

Fig. 1. Types of cut-fill

engineering is α , the road's start and end points are (X_1, Y_1) , (X_2, Y_2) .

We temporarily don't consider road-side's drain and scalar slope.

2.2 Types of road cut-fill

There are being five phenomena when roads pass over mountains (as Fig. 1a~e): Fig. 1a shows that the two sides are filled, Fig. 1b shows that only one side is filled, Fig. 1c shows that only one side is cut, Fig. 1d shows that the two sides are cut, Fig. 1e shows that one side is filled and the other side is cut. Road cut-fill project is classified as four types:

- (1) The first cut area (C1) is cut upward along mountain away from the road widths;
- (2) The second cut area (C2) is cut upward inside roads;
- (3) The second fill area (F2) is filled downward inside roads;
- (4) The first fill area (F1) is filled downward along mountains away from the road widths.

2.3 Criteria to Distinguish road cut-fill

We have some hypothetic conditions (Fig. 2). Now, we can get a profile as an example to explain the distinguishing criteria. We also know that the intersection coordinate between the profile and road's centerline is (X_i, Y_i, Z_i) , the point on the common line between the profile and road is (X_{ij}, Y_{ij}, Z_{ij}) , it also is one of the point on the extending line which is upward with road's centerline, corresponding TERLGRID's coordinate is $(X_{ij}, Y_{ij}, Z'_{ij})$. They have the following relations:

If $Z_{ij} < Z'_{ij}$, then the area is due to cut, ($i=1, 2, 3 \dots n, j=1, 2, 3 \dots m$);

Fig. 2. Frame of cut-fill algorithm

Further, 1) The cut-volume of the first cut area:

Because the altitude of slope-line is:

$$Z_{ij+1} = Z_{ij} + \tan\alpha \times D \quad (j = 1, 2, 3 \dots m, Z_{i,1} = Z_i);$$

The corresponding altitude of mountain is $Z'_{i,j+1}$, so the volume of a cut cell is:

$$\Delta V = (Z'_{ij} - Z_{ij}) * D^2;$$

Then we can sum every cut cell until $Z_{ij} \geq Z'_{ij}$:

$$\sum_{Z_{ij} \leq Z'_{ij}} (Z'_{ij} - Z_{ij}) * D^2 (j = 1, 2, 3 \dots m).$$

2) The cut-volume of the second cut area:

It is easier to calculate the second cut area, because the altitude of road is Z_i invariantly. When the relational altitude of the mountain is Z'_{ik} , the sum-volume of every cut cell is:

$$\sum_k (Z'_{ik} - Z_i) * D^2 (k = 1, 2, 3 \dots s).$$

If $Z_{ij} > Z'_{ij}$, then the area is due to fill,

$$(i = 1, 2, 3 \dots n, j = 1, 2, 3 \dots m).$$

Further, 3) The fill-volume of the second filled area

The calculation of the second filled area is similar to the second cut area, because the altitude of road is Z_i invariantly. When the relational altitude of the mountain is Z'_{ik} , the sum-volume of every filled cell is:

$$\sum_{k'} (Z_i - Z'_{ik'}) * D^2 (k' = 1, 2, 3 \dots s').$$

4) The fill-volume of the first filled area

The calculation of the first filled area is similar to the first cut area. Because the altitude of slope-line is:

$$Z_{ij'+1} = Z_{ij'} - \tan \alpha * D (j' = 1, 2, 3 \dots m', Z_{i,1} = Z_i);$$

The relational altitude of the mountain is $Z'_{i,j'+1}$, so the volume of a filled cell is:

$$\Delta V = (Z_{ij'} - Z'_{ij'}) * D^2;$$

Then we can sum every filled cell until $Z_{ij'} \leq Z'_{ij'}$:

$$\sum_{Z_{ij'} \leq Z'_{ij'}} (Z_{ij'} - Z'_{ij'}) * D^2 (j' = 1, 2, 3 \dots m').$$

2.4 The method to design post-cut-fill DEM

Designing the post-cut-fill DEM is the process of making diffusion model. Cut-fill diffusion model is originated from a central line and extended along both the road surface and the side slope (shown as Fig. 3).

Fig. 3. Cut-fill diffusion DEM

(1) Using AML circular function, it is required to make certain the extended direction of the road centerline before we design the post-cut-fill DEM. Firstly, we calculate the extended azimuth angle β (shown as Fig. 4):

$$\tan \beta = (Y_2 - Y_1) / (X_2 - X_1).$$

Concretely, it can be divided into the following six types:

I) If $0 < \tan \beta < 1$, then:

$$X = X + 1;$$

$$Y = \tan \beta * X + 1;$$

$$a = 1;$$

$$b = \text{cal}[\text{bound}[Y - \text{truncate}[Y]]].$$

Among them, X, Y respectively represent the extended distance at both horizontal direction, and a, b represent cell moving. So the meaning of the expressions are: If $0 < \beta < 45^\circ$, according to Brashenham's algorithm, X -direction moves away from centerline cell by cell. Y -direction moves dependent on the round value of the Y -value's decimal. When it is 0, Y -direction doesn't move; else when it is 1, Y -direction does one cell.

II) If $\tan \beta = 1$, then:

$$X = X + 1;$$

$$Y = Y + 1;$$

$$a = 1;$$

$$b = 1.$$

III) If $\tan \beta > 1$, then:

$$Y = Y + 1;$$

$$X = 1 / \tan \beta * Y;$$

$$a = \text{cal}[\text{bound}[X - \text{truncate}[X]]];$$

$$b = 1.$$

IV) If $\tan \beta = 0$, then:

$$X = X + 1;$$

$$Y = 0;$$

$$a = 1;$$

$$b = 0.$$

V) If $-1 < \tan \beta < 0$, then:

$$X = X - 1;$$

Fig. 4. Central line's azimuth β

$$Y = \tan \beta * X;$$

$$a = -1;$$

$$b = -\text{cal}[\text{bound}[Y - \text{truncate}[Y]]].$$

VI) If $\tan \beta < -1$, then:

$$Y = Y - 1;$$

$$X = 1 / \tan \beta * Y;$$

$$a = -\text{cal}[\text{bound}[X - \text{truncate}[X]]];$$

$$b = -1.$$

(2) Within the road width, the cell value of the next extended DEM: OUTGRID[i] is Z which is equal to the value of OUTGRID[$i-1$]. Once a new extended DEM is generated, it must be put into pre-generated diffused-model: DESIGNGRID[$i-1$] and combined to the new diffused model: DESIGNGRID[i]. There have following cases:

I) If $X^2 + Y^2 \leq H^2 / 4$, then:

$$\text{OUTGRID}[i] = \text{OUTGRID}[i-1](a, b);$$

$$\text{DESIGNGRID}[i] = \text{con}(\text{isnull}(\text{DESIGNGRID}[i-1]),$$

$$\text{OUTGRID}[i], \text{DESIGNGRID}[i-1]).$$

$i = i + 1$, (i 's original value is 0, If $i = m$, then i is sufficed with the maximum value when it fits the looping condition ;)

Among above expressions, DESIGNGRID[i] represents diffused-model within the road width, OUTGRID[i] represents single file paralleling centerline DEM which is generated before diffused-model is created and extended from centerline.

Consequently, we can import some new GRID: DESIGNGRID1[j] represents the extended diffused-model which is upright the safety engineering slope; DESIGNGRID2[j] represents the extended diffused-model which is downgrade the safety engineering slope; OUTGRID1[j] represents the upright single file paralleling centerline DEM which is generated before diffused-model is created and extended from centerline; OUTGRID2[j] represents downgrade single file same as OUTGRID1[j]. They have a common feature that they are all out of the road width.

II) If $X^2 + Y^2 > H^2 / 4$, then OUTGRID1[j] (OUTGRID2[j]) and pre-generated OUTGRID1[$j-1$] (OUTGRID2[$j-1$]) have the following relations:

$$\text{OUTGRID1}[j] = \text{OUTGRID1}[j-1](a, b) + \tan \alpha * D;$$

$$\text{OUTGRID2}[j] = \text{OUTGRID2}[j-1](a, b) - \tan \alpha * D;$$

$$\text{DESIGNGRID1}[j] = \text{con}(\text{isnull}(\text{DESIGNGRID1}[j-1]),$$

$$\text{OUTGRID1}[j], \text{DESIGNGRID1}[j-1]);$$

$$\text{DESIGNGRID2}[j] = \text{con}(\text{isnull}(\text{DESIGNGRID2}[j-1]),$$

$$\text{OUTGRID2}[j], \text{DESIGNGRID2}[j-1]).$$

$j = j + 1$, (j 's original value is $m+1$, If $j = n$, then j is sufficed with the maximum value when it fits the looping condition;)

Above expressions meaning is: Out of the road width, the post-generated DEM OUTGRID1[j]'s and OUTGRID2[j]'s cell value are respectively equal OUTGRID1[$j-1$]'s value plus the product

between cell size and tangent of safety project slope, and OUTGRID2[$j-1$]'s value subtract the product between cell size and tangent of safety project slope. It is done until it coexists OUTGRID1[j]'s cell value exceeding TERLKGRID's maximum and OUTGRID2[j]'s bellowing TERLKGRID's minimum (at same time: $j=n$). Once a new extended DEM is generated, it must be put into pre-generated diffused-model: DESIGNGRID1[$j-1$] or DESIGNGRID2[$j-1$] and combined to new diffused model: DESIGNGRID1[j] or DESIGNGRID2[j].

(3) Calculate cut-fill through looping replacing: Firstly, we execute second-combined operation: combine DESIGNGRID[m] with DESIGNGRID[n] into DESIGN1, combine DESIGNGRID[m] with DESIGNGRID2[n] into DESIGN2; And in the DOCELL block, if TERLKGRID > DESIGN1, the computer can automatically do-cell searching and comparing for all cells. It will have following expression:

$$\text{CUTGRID} = \text{TERLKGRID} - \text{DESIGN1};$$

$$\text{CUT} += \text{CUTGRID}.$$

Among above expression, CUTGRID represents the DEM composed with cut value and CUT represents actual cut. Else if TERLKGRID < DESIGN2, then :

$$\text{FILLGRID} = \text{DESIGN2} - \text{TERLKGRID};$$

$$\text{FILL} += \text{FILLGRID}.$$

FILLGRID represents the DEM composed with fill and FILL represents actual fill. At last, we can obtain the well-designed DEM of post-cut-fill:

$$\text{C-FGRID} = \text{con}(\text{isnull}(\text{CUTGRID}), -\text{FILLGRID}, \text{CUTGRID});$$

$$\text{DESIGN} = \text{con}(\text{isnull}(\text{C-FGRID}), \text{TERLKGRID}, \text{TERLKGRID} - \text{C-FGRID}).$$

C-FGRID represents the DEM of cut-fill whose cell value is given negative if it is in the cut area and the cell value is given positive if in the filled area. DESIGN represents actual post-cut-fill the surface DEM.

3. Case inspection

A certain area, its original surface DEM is TERRAIN, and its parameters are: cell size is $1*1 \text{ m}^2$, X -direction has 340 cells, Y -direction has 426 cells, its maximum altitude is 2028m, the minimum altitude is 1519m.; The centerline GRID is MIDROAD, its altitude is 1750m, the other parameters are the same as the TERRAIN (Especially, the growing point and cell count must be coherent between TERRAIN and MIDROAD), the safety engineering slope is 25° the road's pitch depends on its start point and end point, the road's width is 30m (Fig. 5a).

Secondary DEM used in diffused-model: OUTGRD(i) are series single line DEM along road, DESIGNGRD are series combining DEM of OUTGRD(i); OUTGRID1(i) are series upright 25° single line DEM out of the road width, DESIGNGRD1 are series combining DEM of OUTGRD1(i); OUTGRD2(i) are series

Fig. 5. Areas of cut-fill.

downgrade 25° single line DEM out of the road width, DESIGN-GRD2 are series combining DEM of OUTGRD2(i). OUT1 is cut DEM, OUT2 is fill DEM. When these DEMs aren't being used again, they will be deleted to free some computer space.

Results: the amount variable of cut-fill effectively is CUT, FILL, designing DEM of post-cut-fill is DESIGNDEM, which is our end result, and also is our cardinal work.

The following are comparative graphs between TERRAIN and DESIGNDEM:

In this project, cut amount is 428221 m^3 and fill amount is 353578 m^3 .

Comparing above two graphs, we can see that cut-fill operation is obvious, the cut or filled area keep α -slope or less slope.

4. Discussion

We have calculated the cut-fill and compared original DEM with post-cut-fill DEM through designing and simulating post-cut-fill DEM, and it is available to put the algorithm into practice. Comparing with traditional method, we can see that the work which is done by five peoples in 20 days before can be done by one people and one computer in few hours.

We only picked one simple road as example. In general, we can also read a file that contains many points, and connect these points to create a few lines until faced with 'END' character. There are many vertexes in multiple roads, by the same way, generate the point diffusion DEM, pulsing the line diffusion DEM, we can simulate the multiple lines cut-fill analysis.

Fig. 6. Comparing DEMs between pre and post cut-fill operation.

5. Conclusion

We developed a cut-fill algorithm on the basis of ARC/INFO GRID on the condition of the unknown hypsography resulted from the designed cut-fill, provided with pre-cut-fill DEM and road coverage, engineering side-slope, road height and road width. Obviously, our designing algorithm is reasonable and it can be improved in practice.

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要 旨

DEM に基いて斜面切り盛り自動解析算法

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高速道路など大規模な建設工事するとき斜面の切盛土工の実施は不可欠である。このとき必要な要件は計画切盛土量をバランスさせることである。しかし、従来切り盛り土量の計画は手動で行なっているため、非常に効率が悪く、多くの時間と費用を要し、その改善が求められてきた。近年、大縮尺のデジタル標高モデル(DEM)の応用によって従来行なわれてきた切盛土量計算を自動的に解析することが可能になってきた。本論文では ARC/INFO GRID に基づき、調査地域の DEM と道路に関するパラメータ(切盛法面の勾配、路面標高、幅員)を利用して道路の切盛土量の計画を実施した。この解析により斜面の切盛土工を模擬し、切盛土量をすばやく、簡単に計算することができる。

キーワード：ARC/INFO, 切盛土工, DEM, グリッドセル, 拡散モデル

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