MULTI-DIMENSIONAL STATISTICAL MODELING OF MINERAL ZONALITY AND SPATIAL DISTRIBUTION OF ORE OCCURRENCES

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INTRODUCTION

In geology seems to be very vital:

- quantitative geological forecasting;
- establishment of mineralogical - and - geochemical patterns of mineralization and evaluation of the degree of erosion of ore bodies and their extension depth;
- finding out spatial distribution of ore mineralization;
- reasonable substantiation of estimating prospective mineral resources of the P_3 , P_2 and P_1 categories and industrial ore reserves of the C_2 , C_1 , B and A categories.

Of great help is the information on the total guaranteed amount of prospective resources and reserves of the siderophile elements; Cr, Fe, Ti, V, Ni, Co; chalcophile elements; Cu, Pb and Zn; lithophile elements: Sn, W, Be, Mo, Sb, Hg, Ag and Bi; siderophile and chalcophile rare elements: Cd, Sc, Ga, Jn, Tl, Ge, Re and Se, Te; and also: Au, V, Th, Tr.

We distinguish seven way of estimation of prospective mineral resources (Kogan, 1984): regional metallogenic, geological statistical, expert, regional and local geochemical; geophysical, economic and direct calculation methods. Among various mathematical models applied to achieve the above targets are multi-dimensional mathematical statistics geo-statistics, patterns recognizing. Bull algebra, expert systems. Among them recommended in this paper is the system of multi-dimensional mathematical modeling (figs. 1,2).

The proposed system of multi-dimensional mathematical modeling has been adapted to universal geographical information system (GIS) named "Mineral (natural) Resources of Russia" to be created according to the order of the Committee of Geology and Use of Mineral resources under the Russian Federation Government.

GIS-system is intended for the management of the State Mineral Fund of Russia and working out the strategy of development and reproduction of the mineral raw materials base of Russia (Pitsersky, 1992).

The directions of the GIS-system application are follows:

- modeling forecasting and search for new sources of mineral resources ;
- compiling maps of the mineral base development dynamics and ecological situation;
- monitoring of the environment ecological examination of projects and plans of development territorial - industrial complex, modeling catastrophic situa-

tions of natural and technogenic origin;

- planning of the location of individual objects of mineral raw materials base under the terms of ecological. natural, population - related, ethnographical and other limitations;
- imitational modeling of the regional development strategy, analysis of development of territorial infrastructure.

We fully support the ideas on the role of mathematical modeling that have been brightly expressed by academician A. A. Samarsky (1989). Gain of new information, new knowledge is inconceivable without developed methodology. The core of this methodology lies in the triad: model-algorithm-program that puts to work all the achievements of theoretical and empirical knowledge coupled with computer processing of information. Computational experiment has no barriers: mathematical model of a geological object can be safely tested in any conceivable or unconceivable conditions. Hence, one should consider mathematical modeling not as a technocratic prescription, being a concern of a narrow group of experts but as a universal methodology, a major instrument for mathematization of scientific and technological progress. Foundation of mathematical modeling survey is now being projected in Russia. It might develop into a sort of mathematic industry which will become as usual as energy or transport industry. It should not be overlooked that the eleven countries had formed the European consortium "Mathematics and industry" which is financed by EC.

APPLIED GEOCHEMISTRY

In the field of applied geology (Barsukov, 1981; Beus, 1975; Grigorian, 1987; Kogan, 1984, 1986), confidence intervals and other recommended statistical techniques are tools in solving such vital problems as clarke and geochemical zonality theories actual in large-scale geochemical methods of prospecting using primary halos. Interval and related probabilistic statistical methods greatly contribute to differentiation of geochemical anomalies into the zones of dispersed and concentrated mineralization, as well as to evaluation the degree of erosion of ore-bearing and their extension in depth.

PROSPECTIVE MINERALOGY

In the field of prospective mineralogy (Ginzburg, 1981; Kogan, 1984, 1986) the application of robust interval estimates encompasses the main mineral components and impurities, elementary mineral cell,

Fig.1. The comparison of typical geological problems, which are decided by the tools of multi - dimensional mathematical statistics, pattern recognizing and bull algebra.

characteristic physical, thermal, electrical and magnetic mineral properties, which are practically helpful during large - scale prospecting, differentiation of mineral occurrences and evaluation the degree of erosion of ore - bearing and their extension in depth.

TECHNOLOGICAL MINERALOGY

In the field of technological mineralogy (Ginzburg, 1981; Kogan, 1986) the application of interval and other related statistical procedures is justifiable in estimating the main technological properties of minerals, such as gravity, magnetic, electrical, optical, fluorescence, surface, sorbtion, thermal, chemical, radiation and bacterial for the purpose of resolving the main targets:

- mineral composition and all valuable components study, balancing their distribution, determining various ore types and their spatial occurrence;
- a study of technological properties of minerals depending upon mineral composition, micro - heterogeneity and fabrics;
- a study of mineral behavior during technological process and development of flow charts;

- a study of mineral products for the purpose of utilization and ecological viewpoint.

EXPLORATION AND RESERVOIR ESTIMATION

In the field of exploration and reservoir estimation (Shumilin and Vikentjev, 1982; Fedorchuk, 1991; Kazhdan, 1984; Kogan 1989) interval estimates and related procedures are applied to three groups of parameters:

- conditions indicating a certain economic effectiveness in exploitation, such as cut - off grade, etc.;
- conditions responding to pre - determined mining technology, such as minimum thickness of barren seams and ores of low economic value ;
- conditions meeting the requirements of ore dressing, such as maximum allowable content of deleterious impurities and minimum allowable content of valuable products.

ECONOMIC - GEOLOGICAL ESTIMATION

In the field of economic - geological estimation (beasibility studies) (Kazhdan, 1984; Kogan, 1984, 1986) the application of robust interval estimates (fig.

Fig.2. Principal block - scheme of multi - variate statistical analysis in establishing the spatial regularities of mineralization. Termination of block - scheme showing application of multivariate statistical analysis in establishing spatial regularities of mineralization.

Fig.3. Scheme of interval procedures for estimation of geological parameters.

3) is related to prospect and recoverable reserves, average contents of the primary and accessory mineral components, recovery ratios in ore dressing, cost of mineral raw, and the cost of one ton of product.

ECOLOGY AND ENVIRONMENT PROTECTION

In the field of ecology and environment protection (Pollard, 1982), the recommended statistical models and algorithms, (figs. 2-3) can be applied for the following important targets:

- type - design and computer modeling of geoecological events;
- Comprehensive, geoecological, ecologo - medicosanitary - geo - chemical automated mapping;
- integrated statistical processing of geochemical data related to environment problems;
- modeling of the harmful components migration and prediction of environment changes;
- modeling and estimation of ecogeochemical events: soil analysis for harmful components content and elaboration of practical recommendations on lower -

ing soil pollution by applying natural mineral mixtures; soil analysis for micro - and macro - components contributing to crop productivity; working out of recommendations on gaining ecological clean agricultural products

- informational, software, algorithmic, hardware support for environment monitoring;
- establishing and realization of the major trends in geological data bases development.

To make the most of the available information, the basic scheme of multidimensional statistical analysis computational algorithms includes 6 stages of geological exploration data integrated statistical processing (Kogan, 1989a,b).

Principal block - scheme, showing application of multi - variate statistical analysis in establishing the spatial features of mineralization, encompasses the following stages (Fig.2;3).

FIRST STAGE

The first stage consists of the analysis of selected

populations of geological data. The following items hold the most importance:

- checking the independence of geological observations by autocorrelation analysis. and in case of correlation, application the first- and second-order differences;
- this is followed by Smirnov-Grubbs and Titien-Moore methods of abnormal observations accompanied by robust evaluation methods, such as Tinzor estimates;
- reconstruction of missing observation applying one of six techniques proposed by Afifi and Azen (1982);
- Goldin method of checking the geological variable distributions under bimodality probability.

SECOND STAGE

The second stage initiates statistical analysis of geological data in three - dimensional geometrical geological space of deposits, ore-bearing zones, local ore accumulations, geological and mining blocks, etc.

It has following two sub - stages.

FIRST SUBSTAGE

The first of them is widely applicable Rodionov's method of dividing the geological objects into homogeneous parts. Three variants of dividing may be used in technological mapping. They are bordering the objects in plane in space, and than - spatial - volumetric approach between boreholes, profiles and ore body sections. In case of encountering abnormal observations, one should apply the robust modification of these methods based upon MAD - standard estimates:

$$MAD = \frac{1}{0.6745} \text{med} | x_i - \text{med } x |$$

where med is a median.

SECOND SUBSTAGE

The second substage involves hierarchical agglomerative cluster procedures of complete classification of geological objects using parametrical and free of distribution range criteria of Welsch, James - Su, Puri - Sen - Tamura and Gauer associativity coefficients.

THIRD STAGE

The third stage envisages visualization of geological characteristics within quasi-uniform areas outlined at the second stage. Delineation of isolines (equal contents of the ore components, isopachs, etc.) specifies application of Guildenbrant and Pollard averaging coefficients which are preferable in comparison to trend procedures and saline approximations.

FOURTH STAGE

Prediction of necessary geological variable (response) at any coordinate point of quasi-uniform area come at the fourth stage. For estimation of the predicted geological variable - response one has to apply perspective algorithm of linear regression analysis, construct confidence intervals for dependent vari-

able - response (Sachs, 1976). However, it should be borne in mind that classical regression analysis is based on least squares method and seven assumptions, which are frequently violated. Therefore, we recommend to use modified procedures of regression analysis, such as:

- considering linear limits of regression coefficients;
- centering and standardization of regressions - functions of independent variables;
- checking all possible regressions;
- using a method of regressions including and excluding;
- using step regression;
- using weighed method of the least squares.

Bolgarian scientific team (I. Vuchkov, L. Boyadjieva and E.Solakov,1987) proposed modified procedures of regression anneals, such as: problems of ill-conditioned matrix;

- regression analysis; at inhomogeneous and correlated parameters;
- regression anneals with errors in independent variables;
- regression analysis than the assumption of normal distribution is violated.

FIFTH STAGE

The fifth stage comprises statistical methods of establishing informative combinations of geological variables for finding reliable identifications of mineralization. Here, one should be guided by four principles:

- retrieval of representative combination of geological parameters by fixed steps;
- retrieval of non - informative combination of geological parameters by fixed steps;
- exhaustive search of combinations;
- combined strategy (retrieval of 5 - 6 informative geological characteristics by a complete exhaustive search followed by the first principle.

SIXTH STAGE

The sixth stage means estimation of geological parameters and construction of confidence intervals for them (Figs. 2 - 3). The fifth and sixth stages characterize the statistical analysis of geological data not only in three - dimensional space, but reveals the relationships of geological variables by estimation of probabilistic values, as the upper and lower confidence limits of geological characteristics. Confidence intervals or interval estimates of geological variables should be distinguished in three important cases (Fig. 3), such as:

- for simple univariate geological variables, such as contents of economic and deleterious components, ore body thicknesses, physical and mechanical properties of rocks, minerals, etc.
- for composite geological variables as a product of simple variables, such as productivity of geochemical halos, prospective resources, reserves, etc.
- for composite geological variables as a ratio of sim -

Fig.4. Chorux - Daronskoe W - Mo deposit of Tadzhikistan.

A. Vertical plane projection of molybdenite ores.

B. Vertical plane projection of tungsten ores.

C. Vertical plane projection of molybdenite, tungsten and complex ores grades (types)

Legend: 1 - outline of ore zone; 2 - producing blocks with adjacent workings; 3 - areal boundary of predominant development of biotite-quartz-oligoclase metasomatites (south-western wing) and skarnated rocks (north-eastern wing); 4 - overburden; 5 - sub-economic ores (average conventional unit content of Mo - 2.7, WO₃ - 8.7); 6 - economic molybdenite ores (average Mo content 12.4); 7 - rich molybdenite ores (average Mo content - 58.4); 8 - super-rich molybdenite ores (average Mo content - 154.6); 9 - economic tungsten ores (average WO₃ content - 25.3); 10 - rich tungsten ores (average WO₃ content - 56.2); 11 - super-rich tungsten ores (average WO₃ content - 131.6); 12 - complex ores (average Mo:WO₃ content ratio = 12.4:25.3); 13 - super-rich complex ores (average ratio of Mo:WO₃ content = 58.4:131.6)

ple variables, such as relations between the primary and accessory mineral components, multiplex and additive characteristics of geochemical halos, etc.

One should remember that algorithms of certain confidence intervals depend upon give types of initial geological information:

1. Scalar and vector univariate geological variables are estimated at:

- grouping of N initial observations in population into m groups as $N = M \times 1$, so making $N = (m=n)$

$\times (n=1)$;

- grouping of N initial data into m groups making n observations in each $N \approx m \times n$, provided that $m \gg n$;
- in constructing orientation confidence intervals using Guterman formula as: $\sqrt{n/(n-1)}$;
- in applying Mardia (1978) recommendations.

2. In constructing confidence intervals for extreme values using three types of Gumbel distributions (Kogan, 1984).

3. Composite geological variables leading to one variable, such as reserves, indicator elements ratios, etc.; are estimated by constituting N set of $n_1, n_2, n_3, \dots$ simple variables:

$N = n_1 \cdot n_2 \cdot n_3 \dots$ followed by application of:

$N = m \times 1$ and $N = m \times n$ groups or Gueerman formula,

or using N set of composite variable by constituting three values (the upper and the lower confidence limits $x \pm \lambda$ and probable $\bar{x}$ value) of each simple variable followed by Guterman formula.

4. Groups of geological variables, which do not lead to one variable, such as mineral composition - property, are estimated by constructing composite Confidence regions.

5. Qualitative geological data based on experts conclusions may be estimated Johnson distribution and Monte - Carlo method.

Amund M, L and R robust evaluation procedures we recommend Winzor and MAD - standard estimates.

We don't give in this paper mathematical procedures and specific algorithms of the above - described system of multidimensional statistical analysis since they have been published more than once (see reference list at the end of the paper).

Concluding consideration of the multidimensional statistical analysis computational algorithms, we should note the following. Computational algorithms as well as the whole GIS system are integrated with VIEWS - developed technology of cartographic modeling (Mirónov, Rechkin, 1992).

A technology of cartographic modeling (TECAM) is specified for:

- designing cartographic digital basis;
- preparation of coordinate-tied factual data from geochemical, geophysical, ecological and other surveys;
- plotting maps using mathematical modeling geofields;
- preparation of synthetic electronic maps, including mapping the results;
- displaying maps and obtaining "hard" copies using plotters, printers, etc.

TECAM is a program - technical complex based on PC/AT. PC/AT - 386 computers provided with input-output devices (digitizers, plotters, matrix and laser printers).

TECAM is designed for users who are not professional programmers.

Fig.5. Prioskolskoye ferrous quartzite deposit of the Kursk Magnet Anomaly, Russia.

A portion of the magnetite iron distribution map based on technological sampling data composed with the help of the Rodionov's statistical distinguishing objects method.

- 1: Low - mineralized quartzite
- 2: Silicate - magnetite quartzite
- 3: Magnetite quartzite

5268: Drill - holes

Boundary of a quarry

Average, lower and upper interval estimations magnetite iron content (%) at the Prioskolskoye deposit, obtained by the Kogan's method.

I 23.74 ± 2.39

II 28.83 ± 1.08

III 33.73 ± 1.90

REFERENCES

- Aivazyanyan, S. A., Yenyukov, I. S., and Meshalkin, Z. D. (1983) Applied statistics. The basical modelling and primary processing of data. Moscow, Finansy i Statistika Press.
- Aivazyanyan, S. A., Buchstaber, V. M., Yenyukov, I. S., and Meshalkin, Z. D. (1989) Applied - statistics. Classification and reflection of dimensionality. Moscow, Finansy i Statistika Press.
- Afifi, A., and Azen S. (1982) Statistical analysis. A computer approach. Moscow, Mir Press.
- Barsukov, V. L., Grigoriyaal, S. V., and Ovtchinlikov, L. N. (1981) Geochemical methods of survey of ore deposits. Moscow, Nauka Press.
- Beus, A. A., and Grigoriyan, S. V. (1975) Geochemical exploration and prospecting of mineral deposits. Moscow, Nedra Press.
- Bolshev, V., and Smirnov, N. V. (1983) Mathematical statistics tables, 3rd Edition. Moscow, Nauka, Press.
- Bugaets, A. N., Vostroknutov, E. P., and Vostroknutova, A. I. (1989) Geological forecast making and artificial intelligence methodology. Proceedings of the XXVII International Geological Congress. History of Geology, Geological education, Mathematical geology, Moscow, Nauka Press.

- David, M. (1980) Geostatistical methods in ore reserves estimation. Leningrad, Nedra Press. (in Russian)
- Dvornichenko, G. K. (1984) Application of ECG computer software in infrared resources appraisal using Monte - Carlo technique: Algorithms and programmes, vol.6, no.74, Moscow, VIEMS Press.
- Fedorchuk, V. P. (1991) Expert geological economical evaluation of ore deposits. Moscow, Nedra Press.
- Ginzburg, A. T., Kuzmin, V. T., and Sidorenko, G. A. (1981) Mineralogical investigations in prospecting works. Moscow, Nedra Press.
- Golubeva, V. A., Smirnov, B. I., and Sirotinskaja, S. V. (1987) Reference book for mathematical methods in geology. Moscow, Nedra Press.
- Grigorian, S. V. (1987) Primary geochemical halos in prospecting for ore deposits. Moscow, Nedra Press.
- Gumbel, E. (1965) Statistics of extremes. , Moscow, Mir Press. (in Russian)
- Kazhdan A. B. (1984) Prospecting and exploration ore deposits. Scientific approach. Moscow, Nedra Press.
- Kendall, M. G. and Stuart, A. (1973) Statistical conditions and ties. Moscow, Nauka Press. (in Russian)
- Kogan, R. I., Belov, Yu. P., and Rodionov, D. A. (1983) Range statistics criteria in geology. Moscow, Nedra Press.
- Kogan, R. I. (1984) Interval estimations in the geological investigations. Proceedings of the XXVII International Geological Congress. Mathematical geology and geological information, vol. 20, VNY Science Press, Uthrecht, The Netherlands.
- Kogan, R. I. (1986) Interval estimations in geology. Moscow, Nedra Press.
- Kogan, R. I. (1989) Interval estimation of mineral prospects. Mathematical Geology, vol. 21, no 6., Plenum Press, new York - London.
- Kogan, R. I. (1989) Robust statistical methods of geological bounding. Proceedings of the XXVIII International Geological Congress, History of Geology, Geological education, Mathematical Geology, Moscow, Nauka Press.
- Kogan, R. I. (1990) Interval robust estimations of geological parameters. Moscow, Razvedka i Okhrana Nedra, no.2.
- Kogan, R. I., and Rodionov, D. A., eds. (1989) The using of mathematical methods for search and prospecting of solid mineral deposit. Methodical guidance, Moscow, SEV.
- Kogan, R. I. Tesakov, S., Garmasch, A., Erdelyi, E., and Somos, L. (1990) Geological - statistic modelling of the spatial interrelations of a mineralization and determination of geological exploration data accuracy. XXII International symposium APCOM, vol. II, Berlin, West Germany.
- Launer R. L., and Wilkinson G. I. (1984) Robustness statistics. Moscow, Mashinostroenie Press. (in Russian)
- Margolin, A. M. (1974) Evaluation of mineral row reserves. Mathematical methods. Moscow, Nedra Press.
- Mardia, K. (1978) Statistical analysis of angle observations. Moscow, Nauka Press. (in Russian)
- Pollard, I. (1982) Reference book for calculation methods of statistics. Moscow, Finansy i Statistika Press (in Russian).
- Rodionov, D. A. (1992) The using of mathematical methods and computer technologies for geochemical and surrounding guarding problem decision. Abstracts of International Simposium in Lvov (Ukraine).
- Rodionov, D. A. (1981) Statistical solutions for geology. Moscow, Nedra Press.
- Rysin, J. (1980) Classification and clustering. Moscow, Mir Press. (in Russian)
- Sachs, L. (1976) Statistical estimation. Moscow, Statistika Press. (in Russian)
- Shuamilin, M. V., and Vikentjev, V. A. (1982) Calculation of uranium deposits reserves. Moscow, Nedra Press.
- Sirotinskaja, S. V. (1986) Bull algebra analysis of geological information. Moscow, Nedra Press.
- Smolyak, S. A., and Titarenko, B. P. (1980) Robust methods in estimation procedure. Moscow, Statistika Press.
- Tesakov, S. N. (1984) Logical structures of geological data bases and problems of data banks compatible. In the book " Computer processing of geological data." Moscow, VIEMS Press.
- Veselovskiy, A. V., and Tesakov, S.N. (1989) Automated database on geological databases. Proceedings of the XXVII International Geological Congress, Moscow, Nauka Press.
- Vuchkov, N., Boyadjieva, E., and Solakov, E. (1987) Applied linear regression analysis. Moscow, Finansy i Statistika Press. (in Russian)