Abstract: A study was conducted for a granite quarry producing dimensional stone in the Laurentides mountains (Québec, Canada) to forecast the recovery and waste amounts of blocks due to the presence of fractures. In this paper, the term "fracture" refers to any kind of natural structural discontinuities, i.e. fractures, joints, faults, veins, dykes. Equal area stereographic projection of fracture pole density was used to identify four sets of structural discontinuities in the quarry. For each of these sets, four binary indicator functions were defined as different levels in the quality of the rock mass. The value of these binary functions is deduced from two fracture density parameters calculated in two-dimensional unit cell: the number of fracture intersections and the cumulative fractured length. The database underlying the estimation of these parameters was obtained from a systematic field survey of fracture traces observed on the accessible working faces and floors.

A quality index was estimated on a regular grid of elementary blocks according to a systematic production plan. The predicted values of the quality index for each set were combined to estimate a recovery ratio in three target areas. It was found that the geostatistical procedure presented in this paper can be used to optimize both the extent and the orientation of the working faces. The recovery index was estimated at 15% for the first quality blocks and 15% for the second quality blocks in a 40 m x 30 m x 6 m volume of rock. This result indicates very high ratio for the building stone industry, but the selected volume is not sufficient to ensure quarry operation for several years. Other target areas were rejected and an exploration program is currently yield on the same property, but outside the actual limits of the quarry to find additional granite reserves.

Key Words: Dimensional stone, Block quality, Fracture, Recovery, Geostatistics, Québec

INTRODUCTION

Building stone has been extracted for at least 10,000 years in small scale quarries all around the world, to supply local and regional needs, i.e. houses, walls and monuments. Recent technological development through diamond-faced tools and large-capacity gang saws, allows production of precisely calibrated tiles and panels widely used for building fac ing and floor. Unlike bulk industrial minerals, quality is often more important than quantity in the assessment of the potential of a building stone property (Jefferson, 1993).

Variations in the dimensional granite quality (Logan et al., 1993) are related to the esthetical properties (i.e. texture and color homogeneity), the mechanical properties (i.e. porosity, strength) and fracture intensity (i.e. spacing and orientation). Quebec granites are world-wide recognized for their esthetical and durability performance but wastage in most of the quarries exceed 75% and is mainly related to fracture density (Ménard, 1993). The detailed characterization of the spatial distribution of joints and fractures is an important step in an exploration program, to predict block tonnage recovery according to systematic production plan. Data on fracture orientation and density is of considerable significance for optimizing the quarry face orientation and extent, as well as for selecting the dimensions of the extracted blocks and the limits of the excavated area. But the optimization of the generated waste is strongly constrained by the progress of the working benches.

This study proposes an indicator kriging approach to assess the recovery for different production schemes in a granite quarry. Several types of fracture set occurrences are defined from field sampling of fracture parameters. Their presence is coded as binary fracture indicator functions, on the basis of the range of block quality used in the world building stone industry. Estimates of the probability of 4 levels in the rock mass quality are obtained from a case study in the Quebec area. In order to select the optimum production plan, a global quality index is calculated to forecast a recovery ratio in relation with block geometry and boundaries of the excavated area.

SOURCE OF DATA

Data used in this paper come from a granite quarry, located 150 km NW of Quebec City, Canada. The quarry is host by a green charnockitic rock known as the "Vert Laurentides Granite". This stone was used in major building projects in North
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America and Japan for several years. Conventional geological mapping and systematic fracture analysis were conducted on all the accessible working faces and floors of the quarry. A 1 m² orthogonal grid was laid-out over the floors and walls to get spatial reference for the collecting of data. The following analysis focus on three target zones corresponding to three different steps in the production development of the quarry (Figure 1). Zone 3 is the oldest part of the quarry exhibiting a high frequency of "dark strings" (i.e. shear fractures with an infilling mineralogical assemblage dominated by ferro-magnesian minerals) which represent the major factor in the low recovery. Zone 2 has a similar structural setting but the observed structural discontinuities appear to have longer average extent and larger average spacing. Zone 1 is the youngest production area and was recently considered by the producer since only a few dark strings were detected. In the three investigated zones, the mineralogical homogeneity of the rock can be observed on the actual working faces and in a number of excavated blocks in the quarry.

Several parameters were recorded for each fracture trace observed on the survey plane. These parameters include: the location of the center of the trace in the survey plane, the true orientation of the fracture plane (strike and dip), the length, the width, the number and the nature of visible terminations, the nature of the fracture itself and the infilling or alteration mineralogical assemblages when present. A visibility index was quantified for each 1 m² cell ranging from 0 to 100 % (Figure 2). This index is a useful indicator of the confidence that can be assigned to further predictions. Orientation data were plotted using equal area stereographic projection to discriminate fracture sets (Figure 3). The statistical description of field observations was used to describe the genetic and geometric properties of each fracture set.

Fig. 1 Location of the 3 investigated zones in the Vert Laurentides quarry.

Fig. 2 Example of fracture trace and visibility index measured on survey plane.

Fig. 3 Iso-contour diagram of fracture pole density (A) and average set orientation (B) in the Vert Laurentides quarry.
DESCRIPTION OF FRACTURE PATTERNS

Identification of fracture sets

Four sets were identified from orientation data collected in the Vert Laurentides quarry (Figure 3). The first set is constituted of 124 sub-horizontal fractures. Set 2 to 4 have a subvertical average plane striking respectively N048° (102 fractures), N105° (152 fractures) and N165° (24 fractures). Fractures of set 4 have only been observed in zone 1. The boundaries of each set are fitted along each survey plane to optimize the number of fractures involved in the following geostatistical analysis. In the Vert Laurentides quarry, 96% of the 429 sampled fractures are grouped into one of the four sets. The strike and dip tolerance apart the average plane are generally set at 30° for vertical fracture set. For the horizontal set, the range of the strike is very wide but the maximum dip is set at 30°.

In order to optimize the quarry operations, fractures should occur in an orthogonal pattern characterized by two perpendicular sets with vertical dips and one set with horizontal dip. Since three vertical sets cut across the rock mass in zone 1, this particular fracture network geometry may affect the potential of the quarry block size. In the two other zones, the fracture pattern is favorable even if the average planes of sets 2 and 3 are not exactly orthogonal.

Fracture trace length and width

The fracture length does not vary greatly in the three studied zones (Figure 4). The four sets have

![Fracture length distribution](image-url)

Fig.4 Frequency distribution of fracture trace length.
similar frequency distribution with a lognormal shape. They are highly skewed with a tail to the highest values (3 to 11 m). The skewness is similar in the different zones while the average length of sets 2 and 3 are higher in zones 2 and 3. These latter observations can be explained by the extend of the surveyed planes. Both terminations of more than 75% of the sampled fracture traces were observed on the sampling plane. There is no significant correlation between the length and the number of visible terminations of the fractures, indicating that the truncation bias is similar for short and long fracture traces.

Fractures of sets 1 and 3 have generally a larger width than those of sets 2 and 4. But for all these sets, the width is lower than 0.03 m meaning the absence of any major deformation zone (i.e. shear zone, vein, dyke) in the quarry. The width of the fracture is therefore a negligible parameter for the prediction of block quality.

Filling and occurrence of fractures

The filling material in the sampled fractures can be classified according to two types, shear joints and pegmatitic veinlets. The former is the most common type representing the dark strings filled with chlorite, biotite and quartz. The pegmatite veins are often a source of block rejection in many granite quarries. They are referred in the building stone industry as "white" or "pink string" depending upon the nature of the feldspar (plagioclase or K-feldspar). In the Vert Laurentides quarry, less than 10% of the fractures can be classified in this group. The infilling minerals are mostly microcrystalline without fibrous growth evidence. Apart these two types of string, a large proportion of each fracture set is constituted of joints without significant thickness or aperture.

For the 4 sets, fracture often occur in a parallel mode (80%), sometimes isolated (15%) and sparsely en-echelon (5%). The predominance of the parallel mode is a clear indicator of the clustering of fractures for all the sets. Even if fractures of sets 2 and 4 are more often intersected, there is no consistent cutting/abutting relationship between the different sets that can be interpreted as a chronological evidence in a local structural model. All these sets are probably due to the same tectonic event and they developed at the same time. The presence or absence of the fourth set may be due to local perturbation in the stress field.

FRAC TURE DENSITY

Definition

Fractures can be considered as discrete planes with variable orientation and spacing (Rouleau and Gale, 1985). Frequency of occurrence of different fracture spacings can be compiled to average or simulate the shape of blocks in the rock mass (Gervais et al., 1992; Martinsen and Wingen, 1991). This procedure considers every pair of fractures and gives global result based on the statistical distribution of fracture parameters. But this method is of poor interest to predict recovery in a particular volume of the rock mass. A different approach is to use the density of fracture in a reference unit interval, area or volume (Dershowitz and Herda, 1992). Definition of the density parameter depends on the dimension of the survey and on the geometric representation of the fractures. The density approach constitutes a transformation of a discontinuous phenomena into a continuous parameter amenable to spatial estimation using linear interpolating techniques. The size of the unit cell must be selected to avoid excess in the smoothing of the results.

In order to calculate their density field, the fractures are discriminated from orientation data. For each set (Figure 3), two fracture density parameters were calculated in each 1 m² cell according to the following relationships:

\[ D_1 = \frac{NK}{I} \]  
\[ D_2 = \frac{\sum l_{i,k}}{I} \]

where \( N_k \) is the number of fracture intersecting cell \( k \), \( l_{i,k} \) is the length of fracture \( i \) in \( k \), \( D_1 \) is density expressed as the number of fractures in \( k \) and \( D_2 \) is density expressed as the cumulative length of fractures in \( k \). The density values were normalized with respect to a virtual sampling direction taken as a plane perpendicular to the average orientation of each fracture set using director cosinus (Tavchandjian, 1992; Terzaghi, 1965).

The resulting fracture density fields (i.e. number of fractures and cumulative fracture length) have highly skewed frequency distribution with a large proportion (> 70%) of nil and very low values (Figure 5). This particular shape of the histogram is observed for all the sets, reflecting the clustering of their fractures. A better continuity in the frequency distribution may result in a better confidence in the prediction of fracture density. This is observed for the sets having longer fractures with a larger spacing (sets 2 and 3). The four sets identified in the Vert Laurentides quarry, present a high correlation between the two fracture density parameters defined in [1] and [2]. Therefore the spatial distribution of one of these parameter can be used to predict the other.

Geostatistical analysis

To study the spatial variability of fracture density, geostatistical methods based on the theory of regionalized variables (Matheron, 1963) have been selected as the best suited technics (Tavchandjian, 1992). For more detailed on the mathematical concepts underlying the fundamentals of geostatistics, the reader is referred to classical publications (Isack and Srivastava, 1990; David, 1987; Journal and Hjourbrgts, 1983). The geostatistical approach uses the variogram as a mathematical parameter which quantify the spatial continuity of the data. This func
tion \( \gamma(h) \) can be defined as the average semiquadratic difference between two data points separated by a distance \( h \). For the variable \( z \), \( \gamma(h) \) is given by the following relationship where \( N(h) \) is the number of data pairs separated by a distance \( h \):

\[
\gamma(h) = \frac{1}{2N} \sum_{i=1}^{N} [z(x) - z(x+h)]^2
\]

In practice, the experimental variogram estimated from all the sampled points is fitted to an analytical function (Figure 6). The most commonly used model is a linear combination of a nugget and a spherical function. The nugget model represents a pure random distribution of the variable \( z \) and is defined by \( \gamma(h) = c \), where \( c \) is approximately equal to the measured variance of the samples. The spherical model is related to an increasing function from the origin of the graph to a sill (=c), reaches at a distance called the range (a). The spherical variogram is given by:

\[
\gamma(h) = \frac{3}{2} \left[ \frac{h}{a} - \frac{1}{2} \left( \frac{h}{a} \right)^3 \right] \quad \text{if } h < a
\]

\[
\gamma(h) = C \quad \text{if } h \geq a
\]

Close to the origin, a discontinuity is often observed in the variography. This discontinuity originates from measurement errors or more often, from the existence of a microstructure that cannot be detected by the scale of the study. This discontinuity is called the nugget effect (\( c_o \)). The mathematical expression of the model fitted to the experimental variogram is used to solve linear kriging. Kriging is a local estimation procedure using a Lagrange multiplier to minimize the variance of the estimation error and to provide the best weighting of the available samples.

In the Vert Laurentides quarry, the experimental variograms of raw fracture density (Figure 6) present a succession of bumps and holes that can be expressed as a cumulative fractured length.

Fig. 5 Frequency distribution of fracture density expressed as a cumulative fractured length.

Fig. 6 Directional experimental variograms and omnidirectional modeling of \( F_z \) in zone 1.
plained by the presence or the absence of few extreme values, i.e. the outliers, in the calculation of the variogram (Isack and Srivastava, 1990). Since modeling these variograms is very difficult, some transformation of the raw data is required to avoid the complexity arising from the presence of the outliers (Isack and Srivastava, 1990).

**QUALITY INDEX**

**Definition**

An indicator coding approach (Sullivan, 1985; Isack, 1984) was applied to transform density values into new binary functions quantifying the probability to find a particular type of structural discontinuities. For instance, in the Vert Laurentides quarry, 4 indicator functions have been defined for each set j: the absence of j ($F_{jI1}$), the presence of a single fracture plane with a length less than 0.6 m ($F_{jI2}$), the presence of a single fracture with length greater or equal than 0.6 m ($F_{jI3}$) and the presence of several fractures in k ($F_{jI4}$). These functions can be interpreted as 4 decreasing levels in the quality of the rock mass for the building stone industry, i.e. 4 increasing discount ratios applied to the full commercial value of a six sided block without any fracture.

This type of information is more relevant for recovery prediction than the fracture density parameters defined using relationships [1] and [2]. These levels of quality can be extended to the 3-D extension of the cell, i.e. the thickness assigned to the 2-D cell. In the present study, the assumption is made that fracture density calculated on a 1 m² cell can be extended on each side in the rock mass for at least 0.5 m. This hypothesis seems reasonable since the average length of fracture trace is in the order of 2 m for the four sets. The indicator of quality is therefore assigned to 1 m³ unit blocks.

To evaluate the benefits generated by the extracted rocks, the quality index must be expressed in terms of the 4 commercial levels corresponding to specific ratio applied to the highest price for a particular stone. Commercial value of the untransformed dimensional granites of Québec range between US$ 400 and US$ 800 for a cubic meter (Ménard, 1993). These prices do not include the cost of transport. $F_{1I}$ represent the probability to have a first quality block with a full commercial value. The value of second quality blocks depends on several factors including: market needs, stone utilization (e.g. building, funeral) and volume purchased. Third and fourth quality are considered as waste but some of these blocks can be distributed through several mansions works at cost prices for example in bridges, sidewalk, monuments and houses. The quality index $I_q$ is defined as a linear combination of fracture indicators and is given by:

$$ I_q = E \left[ \sum_{i=1}^{4} (u_i p_i) \right] $$

where $u_i$ is the minimum boundary of indicator $i$ and $p_i$ is the kriged value of indicator $i$ for the block. $I_q$ is an integer value ranging from 1, i.e. absence of fracture (best quality) to 4, i.e. presence of several fractures (worst quality). $I_q$ is calculated for each fracture set separately to take into account the orientation of the structural discontinuities that can be found in each block. A global index $I_g$ is deduced from the $I_q$ of each set using the following relationships:

$$ I_g = 1 \text{ if } I_q = 1 \text{ for all the sets.} $$
$$ I_g = 2 \text{ or } 3 \text{ if } I_q = 2 \text{ or } 3 \text{ for only one set.} \quad [6] $$
$$ I_g = 4 \text{ if } I_q = 4 \text{ for one or more set, or } I_q = 2 \text{ or } 3 \text{ for more than one set.} $$

Forecasting recovery ratio may not only consider the relative proportion of each quality level, but also production as well as esthetical constraints. For instance, selecting the best target zones must take into account the progress of the working faces, i.e. when blasting occur, extremeties of bench must be free. Also, orientation of the quarry walls must be chosen according to the average plane of the rift, i.e. the magmatic or tectonic foliation. Finally, if a qualitative or quantitative estimation of the esthetical properties is available, this information can be added in the selection of the recoverable blocks. A dilution factor can also be used in a well known quarry to incorporate well established production problems such as microfractures related to blasting or residual strength.

**Geostatistical analysis**

Experimental variograms of fracture indicators were modeled using a linear combination of a nugget and a spherical model (Figure 6 and Table 1). The modeling results were used to perform a 3-D block kriging estimation (Figure 7 and 8) for each fracture indicator. The resulting maps and level diagrams can be directly used to identify potential zones of very bad recovery. The actual and future production plans would be optimized by eliminating these zones from

![Fig.7 Kriging map of $I_q$ at elevation 2.5m from the floor of zone 1.](image-url)
The experimental variograms of each fracture indicator were computed for 6 different orientations covering two quadrants (90°) in the X-Y, X-Z and Y-Z planes (Figure 1). All the experimental variograms have the same shape but the magnitude of the nugget effect is inversely proportional to the number of unit cells in each quadrant (Figure 6). Directional variograms can be fitted to the linear combination of a nugget effect and a spherical model (Figure 6 and Table 1). The apparent isotropic spatial distribution of fracture indicators is rather surprising since they reflect the presence of fractures belonging to the same set, defined from orientation data. This result may be explained by the mode of occurrence of these discontinuous and elongated objects. Fractures are not evenly spaced, but grouped into clusters occurring parallel or en echelon. As a consequence, the spatial continuity of unit cells affected by fractures of a particular set is similar in many directions.

Block kriging of the 4 fracture indicators was combined in equations [5] and [6] to predict I_q and then derive I_g in a 10 m x 8 m x 8 m volume of the rock mass. The size of block was 3 m along the X-axis, 2m along Y-axis and 2m along Z-axis. Each block was estimated using 12 points on a regular 3-D 1 m³ discretization grid. The prediction maps of I_q and I_g can be used both to exclude the worst blocks if it is technically possible and to predict the commercial value of the extracted stone (Figure 7). For instance, sector A is excluded because of the high probability of occurrence of fracture set 3. For an analogous reason, sector B must be avoided since recovery will be potentially very bad due to the presence of clusters of small fractures of set 1. The remaining stone seems to be of good quality but the volume is not sufficient to justify its extraction.

zones 2 and 3

The experimental variograms and univariate statistics of the fracture indicators in zones 2 and 3, and in zone 1 are not much different (Table 1). For instance, the same analytical model can be adjusted to the experimental variograms of each fracture indicator in the Vert Laurentides quarry suggesting that the spatial structure of the fracture pattern is uniform. This was verified by cross-validation of fracture indicator data in the three zones. Therefore, when the variogram modeling is well established for a quarry, additional data provided by new rock outcrops or faces can be directly used to provide more robust kriging estimates of the indicators.

Using indicator kriging, estimates of I_q were obtained on a regular grid of blocks overlaying zones 2 and 3. The unit blocks have the same size than in zone 1 (i.e. 3 m x 2 m x 2 m) but the orientation of their longest edge was tested along the X-axis, Y-axis and Z-axis and compared to select the optimum production plane. Figure 8 shows the I_g map of I_g at elevation 4m from the floor of zone 3.
tained with the best recovery rate and the statistics of block quality are detailed for various block geometry in Table 2. Results presented on these map and table are restricted to a selected area of 40 m x 30 m x 6 m, representing the highest potential for short-term production. On each border of the selected area, the quality of the rock mass rapidly decreases. With depth, recovery remains very good, i.e. higher than 40% of quality 1 and 2, for at least the first 6 meters (3 benches). Estimates of Ig clearly indicate that the longest edge of blocks must be oriented along the X-axis to maximize recovery (Table 2).

The economic evaluation of the estimated reserve in the selected area must consider only blocks of quality 1 and 2 since the use of waste material is not always possible and often realized at cost prices. Furthermore, dressing loss will increase the quantity of waste. This factor is related to the irregular geometry of the excavated blocks. A variable quantity of rock had to be removed on each border to produce precise six-sided sawn blocks. In the Vert Lauren-tides quarry this factor is estimated at 25%. The economic value of the selected area can be deduced from an estimated ratio of 15% of first quality blocks and 15% of second quality blocks. The overall volume of this area will guaranty benefits on short term production plan but quarry operation for several years require an exploration program outside the actual limits of the quarry. A fourth interesting zone has been identified on the northern border of the property and will be shortly tested.

CONCLUSIONS

Systematic sampling of 429 fractures in a granite quarry was used to discriminate up to four fracture sets. Three dominant orientation of fractures, observed in the three investigated area of the Vert Lauren-tides property, cut the rock mass with a favourable pattern for the production of cubic blocks (6 sides). A fourth set interfer with this pattern in zone 1 to affect the size and shape of the recovered blocks.

Fracture length, width, infilling and occurrence vary slightly from one zone to another and for the different sets. Trace length show similar distribution for the four sets. The fracture width is negligible with respect to the prediction of block quality since 60% of the fracture are interpreted as joints without any mineralogical infilling or opening. The remaining fractures are mainly shear joints with width lower than 3 cm. Fractures are mainly clustered in a parallel mode. All the fractures originate from the same tectonic event. Local perturbation in the stress field have influenced both the number of sets and the spatial structure of the fracture network.

For each set, a density parameter was calculated according to two different definitions: the number and the cumulative length of fractures in a reference unit cell. Frequency distribution of these parameters are highly skewed reflecting the clustering of fractures. The geostatistical approach was applied to the density data. Clusters of fractures result in chaotic experimental variogram that are difficult to model without transforming the raw density data. Binary indicator functions were defined for each fracture set from the frequency distribution of the density parameters. They can be interpreted as different levels in the quality of the rock mass for building stone production. These binary functions are more relevant for recovery prediction in dimensional stone quarry since they can be directly used to evaluate the commercial value of the rock mass at the exploration stage. The experimental indicator variogram were modeled using an isotropic linear combination of a nugget effect and a spherical model. The same analytical model can be used in in the quarry suggesting that the spatial structure of the fracture pattern is uniform.

Kriging estimations of the fracture indicators on a regular 1 m$^2$ 3D grid were combined to predict a recovery index for each fracture set in 12 m$^3$ blocks over-laying the three sampled areas. This index was used to evaluate the potential of each zone for producing large blocks of building stone. Results lead to the rejection of zone 1 since the spatial distribution of good quality blocks is too sparse and the volume is not sufficient. Prediction of the quality index in zones 2 and 3 provide selection of a high potential sector. The recovery ratio of the first and second quality blocks reaches 30% in a 40 m x 30 m x 6 m volume and these reserves may be extended with depth. Kriging estimates were compared for several schemes and the best results are obtained with the longest edge of block oriented parallel to the X-axis. The stone volume of this target is not enough to ensure quarry operations for several years and the exploration program is currently extended on the same property out of the quarried area.

ACKNOWLEDGEMENTS

This work is based upon work supported by the Ministère des Resources Naturelles du Québec. Mr. Louis Bienvenu is gratefully acknowledged for his scientific and technical advising. The authors would like to thank the technical staff of Granilac Inc for their assistance in the field works.

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