A MATHEMATICAL MODEL FOR HAZARD ZONE MAPPING OF DEBRIS FLOW

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(Received: 8 January 1996 ; Accepted: 31 January 1996)

Abstract: A mathematical model is proposed to determine hazard zone of debris flow and debris deposition. Two layer high concentrated model of solid transport is used in cooperation with continuity equation of solid phase for determination of deposition in the model. Transition from supercritical flow to subcritical flow exist from mountainous stream to flood plain. The proposed model simulated well such transition. Here, the simulated deposition bed are compared with experimental ones to verify the model. Simulated results show fairly good agreement with experimental results. The flow properties and deposition of debris flow in the flood plain can also be calculated by suggested model. The optimum value of the parameter \( a \) (the parameter of the solid transport equation) is found to be around 0.13 for determination of deposition of debris flow. Simulated deposition and debris flow surface can be used to determine hazard zone and simulated flow velocity may be applied to decide hazard risks of different areas. In this respect, the hazard zone of debris flow and its deposition can be determined in the different flow conditions of debris flows.

Keywords: Two dimensional routing, subcritical flow, critical flow, supercritical flow, two layer flow, high concentrated flow.

INTRODUCTION

The debris flow is one of the most severe geological hazard. It can destroy houses, farms and public structures. Also deposited materials change surface of the earth and make the land useless. Deposition of debris flow can change river’s alignment, then debris flow overflows to flood plain and make hazard area broader.

The debris flow is a kind of hyperconcentrated flow which is usually produced in gullies by erosion of steep debris bed. The flow condition in steep channel of gully is usually supercritical but the flow condition can be changed to subcritical when debris flow leaves steep channel of gully and enters into a flood plain with gentle slope. A debris flow may have smaller sediment transport capacity on the flatter slope. Also wider channel of flood plain causes smaller depth and smaller velocity which decrease sediment transport capacity. Therefore, debris deposited on the flood plain.

A mathematical model is required to simulate debris flow properties on the flood plain. The model should be able to determine deposition zone to map the hazard area. Suitable scheme and algorithm should be find to consider different flow conditions in calculation of debris flow properties and deposition.

REQUIRED DIFFERENTIAL EQUATIONS

Since debris flow is an unsteady flow with a steep front, St. Venant equations are not valid in the neighborhood of steep front and conservation form of the differential equations should be used (Banihabib and Hirano, 1995a). The governing equations of the flow can be derived by integrating of the Navier Stokes equations for an incompressible fluid over the flow depth. The continuity equation of flow and momentum equation in \( x \) and \( y \) directions may be written as follows:

The continuity equation of flow:

\[
\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (uh)}{\partial y} = -\frac{\partial z_b}{\partial t}
\]

The momentum equation of flow in \( x \)-direction:

\[
\frac{\partial (uh)}{\partial t} + \frac{\partial (uh^2)}{\partial x} + \frac{\partial (uh)}{\partial y} + gh \frac{\partial (zb + h)}{\partial t} + \frac{\tau_{bx}}{\rho_i} = 0
\]

The momentum equation of flow in \( y \)-direction:

\[
\frac{\partial (uh)}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (uv^2)}{\partial y} + gh \frac{\partial (zb + h)}{\partial y} + \frac{\tau_{by}}{\rho_i} = 0
\]

where \( h \) is flow depth; \( u \) is flow velocity in the \( x \)-direction; \( v \) is flow velocity in the \( y \)-direction; \( g \) is acceleration due to gravity; \( zb \) is bed level; \( \tau_{bx} \) is \( x \)-component of resistance to flow; \( \tau_{by} \) is \( y \)-component of resistance to flow; \( \rho_i \) is specific or bulk density of debris flow.

The continuity equation of the solid phase can be derived by conservation law. Since concentration has
A considerable variation due time and space, it is necessary to include all possible terms of the differential equation. After some process, the equation can be defined as follows:

$$\frac{\partial (ch)}{\partial t} + c \frac{\partial z_h}{\partial t} + \frac{\partial (uhc)}{\partial x} + \frac{\partial (uhc)}{\partial y} = 0$$  (4)

where $c$ is mean volume concentration of solids in entire flow depth; $c^*$ is solid concentration in bed. This equation may be rewritten by using Equation (1) as follows:

$$c^* - c \frac{\partial (zh)}{h \partial t} + \frac{\partial (c)}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = 0$$  (5)

The mean volume concentration of solids can be determined by following equation:

$$c = \frac{qs}{q}$$  (6)

where $q$ is flow discharge per unit width. $qs$ is solid phase discharge per unit width. It may be calculated by following equation for two-layer high-concentrated flow (Hashimoto and Hirano, 1995).

$$\frac{qs}{\sqrt{sgd^3}} = \frac{14}{3} \cdot \tau^2 (1 - \frac{\tau \tau^*}{\tau^*}) \frac{1}{(a - \tan \theta_0) \cos \theta_0}$$  (7)

where $d$ is grain size; $\tau^*$ nondimensional tractive stress; $\tau^*$ nondimensional critical shear stress; $\theta_0$ is bed slope in the flow direction; $a$ is coefficient; $s$ is submerged specific weight of the particle.

Equations (1) to (3) and Equation (5) are a system of nonlinear differential equations which have no analytical solution. Therefore, a numerical simulation is suggested to determine the flow properties and deposition of debris flow in the discrete points of flood plain.

**CALCULATION METHOD**

The calculation is done in two stages to determine velocities in two directions, flow depth, and deposition thickness of debris in flood plain. In the first stage, flow properties have been determined by an implicit scheme (Banihabib and Hirano, 1995b). In the second stage, new bed levels have been calculated by discretization of Equation (5) and using Equation (7). The stages are repeated by new values of flow properties and bed levels.

To determine velocities in two directions and flow depth, Equations (1) to (3) can be rewritten in matrix form as

$$U_x + E_x + F_y + S = 0$$  (9)

Each vector of the $U$, $E$, $F$ and $S$ can be split into two components (Banihabib and Hirano, 1995b). Therefore Equation (16) may be written as follows:

$$U_z + E_{z} + F_{z} + S = 0$$  (10)

Flow properties are calculated by using Equations (9) and (10) in first stage.

To determine bed levels in discrete points, Equation (5) is integrated by using Equation (7) in second stage. The calculated velocities and flow depth are used in calculation of new deposition. New bed levels are determined by consideration of deposited sediment.

**DISCUSSION ON THE SEDIMENT TRANSPORT EQUATION**

Equation (7) is derived by using two layer high concentrated flow model. Fully developed debris flow can transport large amount of solids in upstream steep channel of mountain rivers, but it starts to deposit the solids in gentle slope of flood plain. In this case debris flow may be considered as two layer high concentrated flow. However, the equation is derived for determining of the solid phase discharge in steady state flow. In this respect, it can define steady state transport phenomena, but it may not explain unsteady deposition mechanism very well.

To determine deposition, the continuity equation of solids, Equation (5), is applied. The deposition of unsteady high concentrated flow is determined by considering Equation (7). The optimum value of parameter $a$ is found to determine deposition. In this respect the experimental results of Arai (1991) is used. Experimental setup is shown in Figure 1. Figure 2 shows comparison between calculated bed profile and experimental bed levels on the axis of flood plain. It shows that large amounts of solids deposit in upstream for larger value of $a$, but the amount of deposition is generally smaller for smaller value of $a$. The optimum value of $a$ is 0.13.
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DISCUSSION ON THE FLOW CONDITION
The existence of subcritical, critical, and supercritical flows are studied in two dimensional routing of debris flow. The configuration of flood plain is the same as Figure 1. The slope of the steep channel is 18° and its width is 15 cm. The flow of steep channel is supercritical and its concentration is 0.35. The suggested mathematical model is used to simulate velocities and flow depths in the flood plain. The simulated velocities and Froude Numbers are shown in Figure 3. This figure shows that subcritical, critical, and supercritical flows may exist in two dimensional routing of high concentrated flow. In this example, the upstream part is supercritical. A big part of flow zone has transition zone from supercritical to subcritical. This example shows that suggested mathematical model can simulate a high concentrated flow with the such transition.

VERIFICATION
The calculated cross sections are compared with experimental data to verify the simulation. Two cross sections are chosen. The positions of cross sections are shown in Figure 1. The first one is at 35 cm downstream of the steep channel. The calculated cross section is compared with experimental bed levels in Figure 4. The second one is at 100 cm downstream of steep channel. The simulated cross section is compared with experimental results in Figure 5. The calculated cross sections have fairly good agreement with experimental data in both figures.

RESULTS
Calculated bed surface, velocities, and water surface are presented to demonstrate ability of suggested model in simulation of debris flow hazard zone. The bed surface before deposition and after deposition are shown in Figures 6 and 7, respectively. Figure 7 shows that a large amount of solids may deposit on the axis of flood plain in the upstream. However, deposition may spread widely and makes its hazard zone broader in downstream of flood plain.

Figure 8 shows the simulated contour lines of water surface, flow velocities, and bed levels after deposition of debris flow. It further clarifies that hazard zone of inundation is wider than the zone of deposition. The high speed velocity’s zone is also shown by the model. The simulated velocity may be used in determination of hazard risk of area.

CONCLUSION
Debris flow and its deposition are studied by numerical method to determine hazard zone. Following results are concluded from this study:

Fig.2 Comparison between calculated bed levels and experimental ones for different $\alpha$.

Fig.3 Froude Numbers and velocity vectors.

Fig.4 Comparison of the calculated cross section and experimental bed levels at x = 35 cm.

Fig.5 Comparison of the calculated cross section and experimental bed levels at x = 100 cm.
1. Two layer model of high concentrated flow is used in cooperation with the continuity equation of solid phase to determine deposition of two dimensional routing of debris flow. The optimum value of the parameter $\alpha$ is found to be around 0.13 for above condition.

2. Transition from supercritical to subcritical flow may exist in two dimensional routing of debris flow. It should be considered in simulation. The suggested mathematical model simulated such transition.

3. Simulated depositions are compared with experimental results and show fairly good agreement.

4. The bed surface after deposition, flow surface, and velocities are simulated by the suggested model. The deposited-zone, flow-zone, and the velocities are useful to decide hazard risks of different areas.

REFERENCES

