A procedure for calculating a consistent set of closure temperatures and cooling rate using Dodson's formula

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Abstract: The cooling rate of a geological material is generally calculated using two isotopic ages with different closure temperatures; however, the closure temperature is itself dependent on the cooling rate. Here we present a simple procedure for calculating a consistent set of closure temperatures and cooling rate using Dodson's mathematical formula.

Keywords: closure temperatures, cooling rates, Dodson's formulation, Tanzawa plutonic complex, Atlantis Bank, ODP Hole 735B

Introduction

Closure temperature can be defined as the temperature of the system at the time corresponding to its apparent age (Dodson, 1973). On this basis, it is possible to calculate the cooling rates of the system using closure temperatures of a variety of minerals combined with their isotopic ages (e.g., Harrison et al., 1979). In such calculations, we often use closure temperatures shown by Dodson and McClelland-Brown (1985), who gave 510 °C for hornblende and 300 °C for biotite in K–Ar system on cooling rate of 30 °C/myr (myr=million years), for example. However, closure temperature is itself dependent on cooling rate, as shown in the analytical model of Dodson (1973). Here we present a simple and convenient procedure for calculating a consistent set of closure temperatures and cooling rates using two isotopic ages with different closure temperatures.

Procedure of calculation

Dodson (1973) gave the following equation for calcu-

lating closure temperatures:

$$E/RT_c = \ln [-AD_0RT_c^2/\{a^2 (dT/dt) E\}], \quad (1)$$

where E is the activation energy for the diffusion process, R is the gas constant, T_c is the closure temperature, A is the numerical constant depending on geometry, D_0 is the diffusion coefficient at infinitely high temperature, dT/dt is the cooling rate (dT and dt are the differences in temperature and time, respectively: $dT=T_{\text{present}}-T_c<0$ and $dt=t_{\text{present}}-t_c (t_c=0)>0$, so dT/dt is negative for cooling) and a is an effective diffusion dimension (i.e., radius in the case of a sphere). Since T_c appears on both sides of equation (1), this equation must be solved iteratively for a given value of cooling rate. Sufficient convergence would be obtained by several iterations.

If it is assumed that dT/dt was constant for an interval between T_c of two different ages, a consistent set of T_c and dT/dt can be obtained. First, a trial value of cooling rate is given by difference in roughly assumed T_c for two different ages, A and B (Fig. 1). Convergence of T_c would be obtained by several iterations of equation (1) for each of A and B (N2 and N3 iterations in Fig. 1, respectively). Using the cooling rate obtained from the converged T_c of A and B, more precise values of T_c of A and B would be obtained by the next iteration of N1. After several iterations of N1, consistent values of T_c of A and B and cooling rate would be obtained. One of two T_c can be held constant by omitting either iteration N2 or N3. A Microsoft Excel spreadsheet for calculating T_c and dT/dt may be available from <http://www.sci.hokudai.ac.jp/~jinmaeda/> or from JM on request (jinmaeda@mail.sci.hokudai.ac.jp).

Applications

The procedure presented above may be particularly useful for rapidly cooled plutonic and metamorphic rocks. We applied the calculation procedure to two plutonic suites, gabbroic lower crust exposed at Atlantis Bank on the Southwest Indian Ridge and Tanzawa granitic pluton in central Japan.

1. Lower oceanic crust exposed at Atlantis Bank on the Southwest Indian Ridge

Atlantis Bank is a wave-cut terrace at a water depth of ~700 m along an eastern transverse ridge of the Atlantis II Fracture Zone, 57° E on the ultraslow-spreading

Southwest Indian Ridge (e.g., Dick et al., 1999). Detailed observations and samplings conducted by drilling (ODP Holes 735B and 1105A), dredging, manned submersible and ROV revealed that Atlantis Bank is a typical oceanic core complex, that is an uplifted block of lower oceanic crust and upper mantle by detachment faulting (e.g., Dick et al., 2000; John et al., 2004; Baines et al., 2008). John et al. (2004) estimated a cooling rate of $1600^{\circ}\text{C}/\text{myr}$ for temperatures between 850°C and 350°C by thermochronological approach on the Hole 735B rocks. Coogan et al. (2007) determined a cooling rate of the order of $10^3\text{--}10^4^{\circ}\text{C}/\text{myr}$ of the Hole 735B gabbros between 1000°C and 600°C using Ca-in-olivine geospeedometer. Most recently, Schwartz et al. (2009) showed a cooling rate of $1200^{\circ}\text{C}/\text{myr}$ based on U–Pb and (U–Th)/He ages of zircon with assumed T_c of 850°C and 200°C , respectively, and discussed that the core complex cooled rapidly during detachment faulting. We calculated internally consistent sets of T_c and cooling rates for three samples shown by Schwartz et al. (2009). Diffusion parameters of Pb in zircon are from Lee et al. (1997) and Cherniak and Watson (2000) and those of He is from Reiners et al. (2004) (Table 1). Effective grain radii of zircons are assumed to be $50\text{ }\mu\text{m}$ after Schwartz et al. (2009). Our calculation (Table 2) yields higher T_c of both U–Pb and (U–Th)/He ages than those assumed by Schwartz et al. (2009). This means that their T_c are too low to match the proposed cooling rate. As shown in Table 2, T_c of U–Pb ages calculated using parameters of Cherniak and Watson (2000) are $\sim 70\text{--}90^{\circ}\text{C}$ higher than those using parameters of Lee et al. (1997). However, these T_c and cooling rates are consistent within uncertainties.

2. Tanzawa plutonic complex in Izu collision zone, central Japan

The Tanzawa plutonic complex is located in the Izu collision zone in central Japan, where the northern tip of

Fig. 1. (←) Procedure for calculating a consistent set of closure temperatures and cooling rate. This procedure represents a combination of calculations using equation (1) for two isotopic ages with different closure temperatures (A and B). First, a rough value of the cooling rate (dT/dt) is obtained from the isotopic ages and assumed closure temperatures (T_c) of A and B. Using the cooling rate and T_c as trial values, convergence of T_c for A and B is obtained by performing 10 iterations of N2 and N3. Because a difference in the converged T_c of A and B would yield a more precise value of the cooling rate, each additional iteration of N1 yields more precise T_c values for A and B. After 10 iterations of N1, we obtain consistent values of T_c for A and B and the cooling rate. One of the two T_c values (of A and B) can be held constant by omitting iteration N2 or N3.

Table 1. Diffusion parameters of Pb and He in zircon. Geometry of zircon is spherical and numerical constant A in equation (1) is 55 (Dodson, 1973).

Isotopic system	E (cal/mol)	D_0 (cm ² /s)	References
U-Pb	161,000 ($\pm 8,000$)	3.90×10^9 ($+7.76 \times 10^{10}$ – 3.70×10^9)	Lee et al. (1997)
U-Pb	130,000 ($\pm 6,000$)	7.76×10^2 ($+5.04 \times 10^3$ – 6.73×10^2)	Cherniak and Watson (2000)
(U-Th)/He	40,400 (± 900)	4.60×10^{-1} ($+8.70 \times 10^{-1}$ – 3.00×10^{-1})	Reiners et al. (2004)

the Izu–Bonin–Mariana (IBM) arc has been colliding against the Honshu arc from Middle Miocene (e.g., Saito et al., 2007; Yamada and Tagami, 2008). Tani et al. (2010) concluded that the Tanzawa complex was not denuded middle crust of the IBM arc but formed during arc–arc collision. Tani et al. (2010) calculated cooling rates of the Tanzawa pluton using U–Pb and (U–Th)/He ages of zircon. They used Ti-in-zircon temperatures instead of T_c for U–Pb ages, and assumed that T_c for (U–Th)/He ages is 180 °C according to Yamada and Tagami (2008). Using the procedure presented in this paper, we calculated consistent sets of T_c of zircon (U–Th)/He age and cooling rate for two samples. Diffusion parameters of He in zircon are from Reiners et al. (2004). Effective radii of zircons of the dated samples, 60 μ m, are from Yamada and Tagami (2008). Our results shown in Table 2 indicate that slightly higher T_c of (U–Th)/He ages and slightly slower cooling compared to those given by Tani et al. (2010). This means that the assumed T_c of (U–Th)/He age, 180 °C, is low and not suitable for the cooling rates. Smaller intervals between Ti-in-zircon temperatures for U–Pb age and T_c of (U–Th)/He age produced slightly lower cooling rates compared to those given in Tani et al. (2010) as shown in Table 2.

3. Uncertainties in the results

Uncertainties in the calculated cooling rates may be mainly stemming from errors in ages and T_c . We assume that uncertainty in T_c could be produced only by experimental errors in diffusion parameters, E and D_0 . Furthermore, we define uncertainty on cooling rate as (maximum cooling rate–minimum cooling rate)/2, where the maximum cooling rate is maximum dT /minimum dt and the minimum cooling rate is minimum dT /maximum dt (here dT is absolute value of $T_{\text{present}} - T_c$). Let us consider now two ages A and B : $A \pm a$ with T_c of $X \pm x$ and $B \pm b$ with T_c of $Y \pm y$ ($A > B$, $X > Y$). The lower case letters refer to errors. Minimum and maximum cooling rates are expressed as $\{(X-x)-(Y+y)\}/\{(A+a)-(B-b)\}$ and $\{(X+x)-(Y-y)\}/\{(A-a)-(B+b)\}$, respectively. Since minimum dt may be negative in some case, we redefine

uncertainty of cooling rate as nominal cooling rate–minimum cooling rate, i.e., $\{(X-Y)/(A-B)\}-\{(X-x)-(Y+y)\}/\{(A+a)-(B-b)\}$. Values of x and y are determined by calculation of T_c using combination of maximum and/or minimum values of E and D_0 . Thus, cooling rate is expressed as $(X-Y)/(A-B) \pm [\{(X-Y)/(A-B)\}-\{(X-x)-(Y+y)\}/\{(A+a)-(B-b)\}]$. Experimental errors in the diffusion parameters of Pb and He in zircon are given in Table 1. Calculated uncertainties in cooling rates and T_c of the Atlantis Bank gabbros and Tanzawa granites are shown in Table 2. Detail of the estimation of uncertainties is given in the Microsoft Excel spreadsheet.

4. Utility of the proposed procedure

Using the procedure we proposed here, we calculated internally consistent sets of cooling rates and T_c for the abovementioned two suites. However, it is noted that this procedure does not increase the accuracy of the calculations of cooling rate and T_c . Closure temperatures, T_c , often used in calculation of cooling rates are those standardized to a cooling rate of 30 °C/myr by Dodson and McClelland-Brown (1985). If cooling rate calculated using such T_c is of the order of $\geq 10^2$ – 10^3 °C/myr, inconsistency between T_c and cooling rate would be serious. The procedure presented in this paper is useful in such cases to determine an internally consistent set of T_c and cooling rate.

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Table 2. Ages, closure temperatures and cooling rates of the Atlantis Bank gabbros and Tanzawa granites. Diffusion parameters are from Lee et al. (1997), Cherniak and Watson (2000) and Reiners et al. (2004). Uncertainties in closure temperatures and cooling rates are estimated from errors in experimentally determined diffusion parameters and in isotopic ages. Numerals in parentheses are those estimated only from errors in experimental diffusion parameters.

Atlantis Bank gabbros									
Sample#	Age			Schwartz et al. (2009)			This study		
	Zircon U-Pb (Ma)	Zircon (U-Th)/He (Ma)	Zircon U-Pb T_c (°C)	Zircon (U-Th)/He T_c (°C)	Zircon (U-Th)/He T_c (°C)	Cooling rate (°C/myr)	Diffusion parameters	Grain radius (μm)	Zircon (U-Th)/He T_c (°C)
460-15	12.01 ± 0.20	11.89 ± 1.07	~850	~200 ± 15	252 ± 54 (27)	~5417	Lee Cherniak	50	6513 ± 6064 (1309)
647-5/6	12.15 ± 0.22	11.44 ± 1.03	~850	~200 ± 15	254 ± 54 (27)	~915	Lee Cherniak	50	7256 ± 6753 (1281)
651-1	12.09 ± 0.10	10.96 ± 0.99	~850	~200 ± 15	230 ± 35 (26)	~575	Lee Cherniak	50	1082 ± 764 (211)
Tanzawa granites									
Sample#	Age			Tani et al. (2010)			This study		
	Zircon U-Pb (Ma)	Zircon (U-Th)/He (Ma)	Zircon U-Pb T_c (°C)	Zircon (U-Th)/He T_c (°C)	Zircon (U-Th)/He T_c (°C)	Cooling rate (°C/myr)	Diffusion parameters	Grain radius (μm)	Zircon (U-Th)/He T_c (°C)
K1035	4.62 ± 0.17	3.00 ± 0.10	753 ± 56	~180	220 ± 25 (24)	~354	--	60	329 ± 87 (48)
K1037	4.25 ± 0.10	3.40 ± 0.10	739 ± 53	~180	227 ± 26 (25)	~658	--	60	602 ± 184 (88)

* Ti-in-zircon temperatures from Tani et al. (2010)

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