SLIDING CONDITIONS IN WEAK PLANES IN THE CRUST

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1. INTRODUCTION

The shear strength of the rocks constituting the earth's crust, especially its upper layers, is approximately subject to the Coulomb-Navier theory (ANDERSON, 1963; MATUZAWA, 1964; VENING MEINESZ, 1964). In other words the resistance of shear fracture, along any plane, consists of two parts, one is constant, while the other is proportional to the normal (compressive) stress across the plane. The former type of a shearing resistance is called a cohesion, and the latter a frictional resistance.

Under the action of a stress field a double series of fracture planes is theoretically possible in two dimensions.

It is sometimes called complementary set (ANDERSON, 1963). The fracture planes form an angle of $(\pi/4) - (\phi_0/2)$ with the direction of the maximum principal stress, where ϕ_0 is the angle of internal friction of the material.

Several studies have been made to deduce the tectonic stress field in the past geological age in individual regions on the basis of this strength theory (KIMURA, 1966; SUGIMURA, 1963; HIRAYAMA & KAKIMI, 1965, 1966; HIRAYAMA, 1966; KAKIMI, *et al.*, 1966; SUGIMURA & MATSUDA, 1966).

In case of present complicated geological structure of the Japanese Islands, however, it is obvious that the stress field in individual region of the Islands varied both in direction and intensity with the geological time. Then a question arises as to whether the action of a new stress field may cause resliding of the old fracture planes, or a new fracture system across the old one.

The author discussed, in previous papers, the problem of strength of rock with weak planes with the view to applying it to rock mechanics (OKUSA, 1967, 1968). The result obtained in them can be applicable to this question.

This paper deals with, in the first place, the theoretical considerations of the sliding condition in weak planes after failure in two dimensions, and subsequently applies these considerations to a field example.

2. SLIDING CONDITIONS IN WEAK PLANES

Since the problem of the sliding conditions in parallel weak planes was discussed in author's previous papers,

brief reference to the results may be convenient for the reader.

It is assumed that weak planes develop parallel in one direction and that the shear strength of the solid rock and the material in the weak planes can be represented by the following equations respectively:

$$S_0 = c_0 + \sigma \tan \phi_0, \quad (1)$$

$$S = c + \sigma \tan \phi, \quad (2)$$

where S_0 , c_0 , ϕ_0 are respectively the shear strength, cohesion and angle of internal friction of the solid rock; S , c , ϕ are respectively the corresponding parameters of the material in the weak planes; σ is the normal stress across the plane. It may be expected that c_0 will be much larger than c , and ϕ_0 probably slightly larger than ϕ .

If we take σ_1 and σ_2 as the maximum (compressive) and minimum principal stresses, and α as the angle between the σ_1 -axis and the weak plane (Fig. 1), the condition for sliding in the weak plane may be given by one of the next equations:

$$\left. \begin{aligned} \sigma_1 [\sin(2\alpha + \phi) - \sin \phi] - \sigma_2 [\sin(2\alpha + \phi) + \sin \phi] \\ \geq 2c \cos \phi, \end{aligned} \right\} \quad (3)$$

or

$$\left. \begin{aligned} \sigma_1 \cos(\alpha + \phi) \sin \alpha - \sigma_2 \sin(\alpha + \phi) \cos \alpha \geq c \cos \phi, \\ \tau \geq c + \sigma \tan \phi, \end{aligned} \right\} \quad (4)$$

Fig. 1 Stress diagram in a plane

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where

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\alpha, \quad (5)$$

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2) \cos 2\alpha. \quad (6)$$

Eqs. (5) and (6) represent the shear and normal stresses across the plane which makes an angle of α with the σ_1 -axis. It is noted that equation (3) represent the condition for sliding in terms of the principal stresses and equation (4) in terms of σ , τ across the plane. Eqs. (3) and (4) may be applicable as the sliding condition to rocks in which parallel weak planes develop.

The condition for failure of the solid homogeneous rock is well known (JAEGER, 1962): it is

$$\sigma_1(1 - \sin \phi_0) - \sigma_2(1 + \sin \phi_0) \geq 2c \cos \phi_0, \quad (7)$$

or

$$\tau \geq c_0 + \sigma \tan \phi_0. \quad (8)$$

Equation (7) can be rewritten in the form:

$$\sigma_1 \geq \sigma_2 \tan^2\left(\frac{\pi}{4} + \frac{\phi_0}{2}\right) + 2c \tan\left(\frac{\pi}{4} + \frac{\phi_0}{2}\right), \quad (9)$$

or

$$\sigma_2 \leq \sigma_1 \tan\left(\frac{\pi}{4} - \frac{\phi_0}{2}\right) - 2c \tan\left(\frac{\pi}{4} - \frac{\phi_0}{2}\right). \quad (10)$$

Eqs. (9) and (10) are frequently used in soil mechanics.

As stated above, the fracture planes form an angle of $(\pi/4) - (\phi_0/2)$ and are symmetry to the σ_1 -axis.

The range of α in which sliding will take place in the weak planes before fracturing in the solid rock can be derived from eqs. (3) and (7) as follows:

$$\frac{\sin \phi}{\sin(2\alpha + \phi)} \leq \sin \phi_0 \frac{\sigma_m + c_0 \cot \phi_0}{\sigma_m + c \cot \phi}, \quad (11)$$

or

$$\frac{\sigma_m \sin \phi + c \cos \phi}{\sigma_m \sin \phi_0 + c_0 \cos \phi_0} \leq \sin(2\alpha + \phi), \quad (12)$$

where $\sigma_m = (\sigma_1 + \sigma_2)/2$ represents the mean normal stress.

There are two values of α between zero and $\pi/2$ which satisfy the equality sign of (11) or (12). If we take these values as α_1 and α_2 ($\alpha_1 \leq \alpha_2$), the range of α in which sliding is expected to occur in the weak planes is given by $\alpha_1 \leq \alpha \leq \alpha_2$. Apparently from (11) the following relation between α_1 and α_2 can be made:

$$\alpha_2 = \frac{\pi}{2} - \phi - \alpha_1. \quad (13)$$

If α is out of this range, failure takes place in the solid rock across the weak planes under the condition of (7). The relationship stated above can be made clear by an alternative treatment, based on the Mohr circle. In Fig. 2 the condition for (4) is represented by the straight line BEF and the domain above it, and the condition for (8) by AD and the domain above it. For any inclination of a weak plane to the σ_1 -axis, the value of σ and τ across it will be the coordinates of the point J on the Mohr circle in Fig. 2. If the point J lies in arc IE or HF, i.e. $\alpha < \alpha_1$ or $\alpha_2 < \alpha$, these stresses will not cause failure in the plane, but if J lies in arc EDF, i.e. $\alpha_1 \leq \alpha \leq \alpha_2$ they will cause failure.

Now suppose that σ_2 is held constant and σ_1 is increased. Then the Mohr circle will first come in contact with the fracture line BEF with $\alpha = (\pi/4) - (\phi/2)$. This implies that the weak plane is the most liable to sliding when α is $(\pi/4) - (\phi/2)$. Making the Mohr circle still larger, the critical value α_1 decreases and α_2 increases converging to the extreme values of α determined by (11) or (12). Equations (11) and (12) show that the range of α depends on c_0 , ϕ_0 , c , ϕ and stresses satisfying (7). As σ_m increases, the value α_1 increases and α_2 decreases, converging to the values given by

Fig. 2 Relationship between shear strengths of solid rock and material in weak plane and range of α for resliding along the weak plane in general case

$$\frac{\sin \phi}{\sin (2\alpha + \phi)} = \sin \phi_0 \quad (14)$$

In special cases, the range of (11) or (12) may be simplified and represented as diagrams by suitable parameters. Several examples of such cases are described below.

When weak planes come into existence in rocks or soils, it is conceivable that effective cohesion in the weak planes becomes zero, or nearly equal to zero, even if there are filling materials of fault clay type in them. Therefore if the assumption of $c=0$ holds good, (11) becomes

$$\frac{\sin \phi}{\sin (2\alpha + \phi)} \leq \sin \phi_0 + k \cos \phi_0 \quad (15)$$

where $k=c_0/\sigma_m$ is the coefficient for estimating the applied stresses in comparison with the cohesion of the solid rock.

Figure 3. indicates the variations of the range of α with ϕ for various ϕ_0 in cases of $k=0.2, 0.4$ and 0.6 .

If the angle α lies between the two critical values

(α_1, α_2) for given k, ϕ, ϕ_0 in Fig. 3, failure takes place in one of the weak planes before failure occurring in the solid rock across the weak planes.

In case of no cohesion both in the rock and in the weak planes, (11) becomes

$$\frac{\sin \phi}{\sin (2\alpha + \phi)} \leq \sin \phi_0. \quad (16)$$

This equation is evidently different from (11) in that this does not contain the stresses. Therefore if ϕ and ϕ_0 are given the two critical values of α determined from (16), i. e. the range for failure in the weak planes, are independent of stresses.

In Fig. 4 the variations of the range of α with ϕ are shown for given values of ϕ_0 in case of $c=c_0=0$.

Next let us consider the case of no internal friction in the rock and weak planes. In this case the conditions for failure in the solid rock and weak planes are

$$\tau \geq c_0 \quad \text{and} \quad \tau \geq c, \quad (17)$$

respectively. These are represented by two straight lines parallel to the σ -axis and the domain above them

Fig. 3-1 Resliding range of α for various ϕ and ϕ_0 in case of $k=0.2$

Fig. 3-2 Resliding range of α for various ϕ and ϕ_0 in case of $k=0.4$

in the σ - τ coordinates (Fig. 5).

In a similar way to the previous cases, the range of α can be obtained from (5), (6) and (17):

$$\frac{c}{c_0} \leq \sin 2\alpha. \quad (18)$$

There will be two critical values of α between zero and $\pi/2$ which satisfy this equation, and if they are α_1 and α_2 ($\alpha_1 \leq \alpha_2$), it is easy to understand from Fig. 5 that α_1 and α_2 are independent of the stresses, i. e. they are constants for each c and c_0 independent of the positions of the Mohr circles. The plot of α against c/c_0 is shown in Fig. 6. It is apparent that

$$\alpha_2 = \frac{\pi}{2} - \alpha_1$$

3. SLIDING CONDITIONS IN WEAK PLANES IN THE CRUST

The problem lies in how to determine the range of the angle in which the stress system having produced

weak planes may rotate in a plane without causing a new fracture system across the weak planes.

As discussed in the preceding chapter, this range of the rotation angle can be determined α_1 and

$$\alpha_2 = \frac{\pi}{2} - \phi_0 - \alpha_1.$$

As is shown in the chapter 1 a complementary set of faults or joints is possible under the action of a stress field, and the angle between the maximum principal stress and these fracture planes is $(\pi/4) - (\phi_0/2)$.

However, field observations often reveal cases in which one-directional set of fracture planes develops without any corresponding complementary set. The facts suggest that two cases, i.e. (1) one-directional fracture planes and (2) complementary set of fracture planes exist, independently.

Fig. 7 shows the case (1) and the range of the angle in which the stress field may rotate without producing a new fracture system cutting the existing fault plane.

Fig. 3-3 Resliding range of α for various ϕ and ϕ_0 in case of $k=0.6$

In this range of the angle, the stress field may produce resliding of the old fault plane (shaded area). From the figure, this range can be represented by

$$-\left[\alpha_2 - \left(\frac{\pi}{4} - \frac{\phi_0}{2}\right)\right] \leq \phi \leq \left(\frac{\pi}{4} - \frac{\phi_0}{2} - \alpha_1\right)$$

$$\text{or } -\left(\frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1\right) \leq \phi \leq \frac{\pi}{4} - \frac{\phi_0}{2} - \alpha_1, \quad (19)$$

and

$$\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 \leq \phi \leq \frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_2$$

$$\text{or } \frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 \leq \phi \leq \frac{3}{4}\pi - \frac{1}{2}\phi_0 - \phi - \alpha_1, \quad (20)$$

Fig. 8 shows the case (2) and the range of the angle in which the stress field may rotate in the same manner as in the case (1). In this range of the angle too the stress field cause resliding of the rocks along one of the the old planes (shaded area). In Fig. 8 the range for the fault I can be represented by (19) and (20). But

the range for the fault II has the property of symmetry to the σ_1 -axis, and can be represented by the following inequalities:

$$-\left(\frac{\pi}{4} - \frac{\phi_0}{2} - \alpha_1\right) \leq \phi \leq \frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1, \quad (21)$$

and

$$-\left(\frac{3}{4}\pi - \frac{\phi_0}{2} - \phi - \alpha_1\right) \leq \phi \leq -\left(\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1\right),$$

or

$$\frac{\pi}{4} + \frac{\phi_0}{2} + \phi + \alpha_1 \leq \phi \leq \frac{3}{4}\pi + \frac{\phi_0}{2} - \alpha_1, \quad (22)$$

Therefore, the range for the case (2) may be taken as the "join" of (19), (20), (21) and (22). In these equations there are following relations:

$$\left(\frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1\right) - \left(\frac{\pi}{4} - \frac{\phi_0}{2} - \alpha_1\right) = \phi_0 - \phi > 0$$

$$\left(\frac{3}{4}\pi + \frac{\phi_0}{2} - \alpha_1\right) - \left(\frac{3}{4}\pi - \frac{\phi_0}{2} - \phi - \alpha_1\right) = \phi_0 + \phi > 0$$

Fig. 4 Resliding range of α for various ϕ and ϕ_0 in case of $c=c_0=0$

Fig. 5 Relationship between shear strengths of solid rock and material in weak plane, and range of α for resliding along the weak plane in case of $\phi=\phi_0=0$

If $\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 > \frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1$, i. e. $\phi_0 - \phi - 2\alpha_1 < 0$

$$\text{and } \frac{3}{4}\pi - \frac{1}{2}\phi_0 - \phi - \alpha_1 < \frac{\pi}{4} + \frac{\phi_0}{2} + \phi + \alpha_1,$$

$$\text{i. e. } \frac{\pi}{2} - \phi_0 - 2\phi - 2\alpha_1 < 0,$$

then this range is represented by the three inequalities:

$$-\left(\frac{\pi}{4} + \frac{1}{2}\phi_0 - \phi - \alpha_1\right) \leq \phi \leq \frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1, \quad (23)$$

$$\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 \leq \phi \leq \frac{3}{4}\pi - \frac{1}{2}\phi_0 - \phi - \alpha_1, \quad (24)$$

and

$$\frac{\pi}{4} + \frac{1}{2}\phi_0 + \phi + \alpha_1 \leq \phi \leq \frac{3}{4}\pi + \frac{1}{2}\phi_0 - \alpha_1. \quad (25)$$

If $\phi_0 - \phi - 2\alpha_1 < 0$ and $\frac{\pi}{2} - \phi_0 - 2\phi - 2\alpha_1 > 0$, then this range is represented by the two inequalities:

$$-\left(\frac{\pi}{4} + \frac{1}{2}\phi_0 - \phi - \alpha_1\right) \leq \phi \leq \frac{\pi}{4} + \frac{1}{2}\phi_0 - \phi - \alpha_1, \quad (26)$$

Fig. 6 Resliding range of α for various c/c_0 in case of $\phi = \phi_0 = 0$

and

$$\frac{\pi}{4} - \frac{1}{2}\phi_0 + \alpha_1 \leq \psi \leq \frac{3}{4}\pi + \frac{\phi_0}{2} - \alpha_1. \quad (27)$$

If $\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 < \frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1$, i. e. $\phi_0 - \phi - 2\alpha_1 > 0$,

and $\frac{3}{4}\pi - \frac{1}{2}\phi_0 - \phi - \alpha_1 < \frac{\pi}{4} + \frac{1}{2}\phi_0 + \phi + \alpha_1$,

i. e. $\frac{\pi}{2} - \phi_0 - 2\phi - 2\alpha_1 < 0$,

then this range is represented by the two inequalities:

$$-\left(\frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1\right) \leq \psi \leq \frac{3}{4}\pi - \frac{1}{2}\phi_0 - \phi - \alpha_1, \quad (28)$$

and

$$\frac{\pi}{4} + \frac{1}{2}\phi_0 + \phi + \alpha_1 \leq \psi \leq \frac{3}{4}\pi + \frac{1}{2}\phi_0 - \alpha_1. \quad (29)$$

If $\phi_0 - \phi - 2\alpha_1 > 0$ and $\frac{\pi}{2} - \phi_0 - 2\phi - \alpha_1 > 0$, then this range is represented by one inequality:

$$-\left(\frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1\right) \leq \psi \leq \frac{3}{4}\pi + \frac{1}{2}\phi_0 - \alpha_1. \quad (30)$$

Fig. 7 The range of the rotation angle in which the stress field may produce resliding along the old fault plane (shaded area).

Fig. 8 The range of the rotation angle in which the stress field may produce resliding along one of the old complementary faulting (shaded area).

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一度できた地殻の破壊面がふたたびすべる条件

大 草 重 康

(要 旨)

本論は、破壊面が発達する地殻に応力が加えられた時、その面にそってすべりがおこるか、それともその面を切って新しい破壊面が発生するかを、2次元の場合について理論的に明らかにし、その結果を一つのフィールドの例にあてはめたものである。地殻の強さは、みかけの内部摩擦角と粘着力であらわされるクーロン・ナビアの理論にしたがうものとする。地殻本来の内部摩擦角、粘着力をそれぞれ ϕ_0 , c_0 , 破壊面のものをそれぞれ ϕ , c とし、 $\phi_0 > \phi$, $c_0 > c$ と仮定した。この場合、破壊面と最大圧縮主応力 σ_1 の軸との角を α とすれば、 α が

$$\frac{\sin \phi}{\sin (2\alpha + \phi)} \leq \sin \phi_0 \frac{\sigma_m + c_0 \cot \phi_0}{\sigma_m + c \cot \phi}, \quad (1)$$

を満足すると、破壊面にそってふたたびすべりがおこりうる。ここに $\sigma_m = (\sigma_1 + \sigma_2)/2$ であり、 σ_2 は最小圧縮主応力である。式(1)の等号を満足する α は、 $0 \sim (\pi/2)$ に二つ存在し、これを $\alpha_1, \alpha_2 (\alpha_1 \leq \alpha_2)$ とすれば、式(1)を満足する α の範囲は $\alpha_1 \leq \alpha \leq \alpha_2$ である。 α がこの範囲外であると、この応力場は、新しい破壊系を発生させる。

ある応力場が無傷の地殻に一方向の破壊系を発生させたとすれば、この面は σ_1 軸と $(\pi/4) - (\phi_0/2)$ の角をなす。この応力場が2次元的に ϕ だけ回転しても、この破壊系にそってふたたびすべる可能性のある範囲は、 σ_1 軸から破壊系への回転方向を正として

$$-\left(\frac{\pi}{4} + \frac{\phi_0}{2} - \phi - \alpha_1\right) \leq \phi \leq \frac{\pi}{4} - \frac{\phi_0}{2} - \alpha_1, \quad (2)$$

および

$$\frac{\pi}{4} - \frac{\phi_0}{2} + \alpha_1 \leq \phi \leq \frac{3}{4}\pi - \frac{\phi_0}{2} - \phi - \alpha_1. \quad (3)$$

共役の破壊系がある場合には、 σ_1 軸に関して(2), (3)式と対称なもう二つの式が得られる。共役な破壊系のいずれかにそってふたたびすべる可能性のある ϕ の範囲は、以上の四つの式の“結び”としてあらわされる。

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