Uddyotakara’s Logic and Predicate Logic

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1. Introduction

In Uddyotakara’s Nyāyavārttika (NV), an inference is one in which a conclusion follows from a reason. A valid inference is one in which a conclusion follows from a reason that satisfies three conditions, and an invalid inference is one in which a conclusion follows from a reason that does not satisfy these three conditions. The three conditions are that the reason (1) occurs in the entire pakṣa, (2) occurs in sapakṣa, and (3) does not occur in any vipakṣa.

The NV, as is well known, classifies the reasons that satisfy Condition (1) into sixteen types. Of these, the NV regards the third and the ninth reasons, satisfying Conditions (2) and (3); the tenth and the eleventh reasons, satisfying Condition (2); and the fifteenth reason, satisfying Condition (3), as hetu and regards the rest as hetvābhāsa.

This paper formulates a valid inference in which the conclusion follows from hetu using predicate logic and shows the difference between Uddyotakara’s logic and predicate logic. First, we analyze the 16 reasons that satisfy Condition (1) and characterize the reasons that satisfy Conditions (2) and (3). Then, we formulate an inference in which the conclusion follows from the reasons that satisfy Conditions (1), (2), and (3), that is, Reasons 3, 9, 10, 11, and 15. The analysis and the formulation are based on Okazaki (2005) (2008) and Glashoff (2009). Finally, we interpret the formulation of the inferences and show the difference between Uddyotakara’s logic and predicate logic.

2. NV’s 16 Reasons

The NV on the Nyāyasūtra (NS) 1.2.4 classifies the reasons that satisfy Condition (1) into 16 types as follows.
Uddyotakara’s Logic and Predicate Logic (K. HosoNo)  

<table>
<thead>
<tr>
<th></th>
<th>VP-vyāpaka</th>
<th>VP-ekadeśavṛtti</th>
<th>VP-avṛtti</th>
<th>VP avidyamāna</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP-vyāpaka</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>SP-ekadeśavṛtti</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>11</td>
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<tr>
<td>SP-avṛtti</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>SP avidyamāna</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

SP: sapakṣa  VP: vipakṣa

It is clear that ‘SP-vyāpaka’ and ‘SP-ekadeśavṛtti’ correspond to Condition (2) and that ‘VP-avṛtti’ corresponds to Condition (3).

Reasons 1, 2, 3, 7, 8, 9, 10, and 11 (bold) satisfy Condition (2). Of these, Reasons 3 and 9 satisfy Condition (3). Reasons 10 and 11 are not concerned with Condition (3), because they do not have vipakṣa. Reasons 1, 2, 7, and 8 do not satisfy Condition (3). On the other hand, Reasons 3, 6, 9, and 15 (underline) satisfy Condition (3). Of these, Reasons 3 and 9 satisfy Condition (2). Reason 15 is not concerned with Condition (2), because it does not have sapakṣa. Reason 6 does not satisfy Condition (2).

As a result, Reasons 3, 9, 10, 11, and 15 are hetu. Reasons 3 and 9 satisfy Conditions (1), (2), and (3); Reasons 10 and 11 satisfy Conditions (1) and (2); and Reason 15 satisfies Conditions (1) and (3). And the rest are hetvābhāsa.

3. Analysis of the 16 Reasons

3.1. Analysis

Let us analyze the 16 reasons using predicate logic. First, we divide the domain of discourse into four parts, which are as follows:

(a) The set whose members have both property H and property S.
(b) The set whose members have property H and do not have property S.
(c) The set whose members do not have property H and have property S.
(d) The set whose members have neither property H nor property S.

Then, we apply these sets to the 16 reasons as follows:

**Reason 1:** \(\exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) \land \neg \exists x (\neg Hx \land Sx) \land \neg \exists x (\neg Hx \land \neg Sx)\)

**Reason 2:** \(\exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) \land \neg \exists x (\neg Hx \land Sx) \land \exists x (\neg Hx \land \neg Sx)\)

**Reason 3:** \(\exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) \land \exists x (\neg Hx \land Sx) \land \exists x (\neg Hx \land \neg Sx)\)

**Reason 4:** \(\neg \exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) \land \exists x (\neg Hx \land Sx) \land \neg \exists x (\neg Hx \land \neg Sx)\)

The rest are omitted.
3.2. Four Classifications of the 16 Reasons

These 16 reasons can be classified into four groups according to the criteria \( \exists x (Hx \land Sx) \) and \( \exists x (Hx \land \neg Sx) \) as follows: \(^7\)

\[
\begin{array}{lll}
[\text{I}] & \exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) & : \text{Reasons 1, 2, 7, and 8} \\
[\text{II}] & \exists x (Hx \land Sx) \land \neg \exists x (Hx \land \neg Sx) & : \text{Reasons 3, 9, 10, and 11} \\
[\text{III}] & \neg \exists x (Hx \land Sx) \land \exists x (Hx \land \neg Sx) & : \text{Reasons 4, 5, 13, and 14} \\
[\text{IV}] & \neg \exists x (Hx \land Sx) \land \neg \exists x (Hx \land \neg Sx) & : \text{Reasons 6, 12, 15, and 16}
\end{array}
\]

The Nyāyavārttikatātparyāṭikā (NVTT) classifies the 16 reasons into four groups: sādhāraṇa-hetvābhāsa, hetu, viruddha-hetvābhāsa, and asādhāraṇa-hetvābhāsa. \(^8\) The reasons belonging to Group [I] correspond to sādhāraṇa-hetvābhāsa, the reasons belonging to Group [II] correspond to hetu, the reasons belonging to Group [III] correspond to viruddha-hetvābhāsa, and the reasons belonging to Group [IV], except Reason 15, correspond to asādhāraṇa-hetvābhāsa. Reason 15 is hetu.

4. The Formulation of an Inference in Which the Conclusion Follows from *hetu* and Its Interpretation

4.1. The Formulation of an Inference in Which the Conclusion Follows from the Reasons Belonging to Group [I]

If Condition (1) is formulated as \( Hp \), an inference in which the conclusion follows from the reasons belonging to Group [I] can be formulated as follows: \(^9\)

\[
Hp, \exists x (Hx \land Sx), \neg \exists x (Hx \land \neg Sx) \vdash Sp
\]

\( \neg \exists x (Hx \land \neg Sx) \) is equivalent to \( \forall x (Hx \rightarrow Sx) \), then the formulation is

\[
Hp, \exists x (Hx \land Sx), \forall x (Hx \rightarrow Sx) \vdash Sp
\]

This inference is valid. Of these premises, however, \( \exists x (Hx \land Sx) \) is unnecessary. This may show the difference between this inference and the inference in which the conclusion follows from the reasons belonging to Group [IV].

4.2. The Formulation of an Inference in Which the Conclusion Follows from Reason 15
Uddyotakara’s Logic and Predicate Logic (K. Hosono) (95)

An inference in which the conclusion follows from reasons belonging to Group [IV] can be formulated as follows:

\[ \text{H}_p, \; \neg \exists x \,(H_x \land S_x), \; \neg \exists x \,(H_x \land \neg S_x) \vdash \]
\[ \neg \exists x \,(H_x \land S_x) \] is equivalent to \( \forall x \,(H_x \rightarrow \neg S_x) \), and \( \neg \exists x \,(H_x \land \neg S_x) \) is equivalent to \( \forall x \,(H_x \rightarrow S_x) \), then the formulation is

\[ \text{H}_p, \; \forall x \,(H_x \rightarrow S_x), \; \forall x \,(H_x \rightarrow \neg S_x) \vdash \]

In this formulation, premises \( \text{H}_p, \; \forall x \,(H_x \rightarrow S_x), \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) are syntactically inconsistent,\(^{10}\) that is, both \( S_x \) and \( \neg S_x \) follow from the premises. And the premises are semantically inconsistent, that is, the premises cannot be true together. If \( \exists x \text{H}_x \) is true, then either \( \forall x \,(H_x \rightarrow S_x) \) or \( \forall x \,(H_x \rightarrow \neg S_x) \) is true, or both \( \forall x \,(H_x \rightarrow S_x) \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) are false. If \( \exists x \text{H}_x \) is false, then both \( \forall x \,(H_x \rightarrow S_x) \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) are true. Two problems now arise.

The inference ‘\( \text{H}_p, \; \forall x \,(H_x \rightarrow S_x), \; \forall x \,(H_x \rightarrow \neg S_x) \vdash \beta \) (\( \beta \) is arbitrary)’ is valid\(^{11}\) because an inference in which the premises are inconsistent is valid no matter what the conclusion is. However, according to the NV’s criterion of validity, inferences in which the conclusion follows from Reasons 6, 12, and 16 are invalid, and the inference in which the conclusion follows from Reason 15 is valid. It is difficult to explain this disagreement.\(^{12}\) We shall not inquire further into this.

Furthermore, in the inference ‘\( \text{H}_p, \; \forall x \,(H_x \rightarrow S_x), \; \forall x \,(H_x \rightarrow \neg S_x) \vdash \),’ premises \( \text{H}_p, \; \forall x \,(H_x \rightarrow S_x), \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) are inconsistent in the domain of discourse. The NV, however, holds that these are not inconsistent. How is this possible? One possible explanation is as follows. If \( \text{H}_p \) holds in one domain, and \( \forall x \,(H_x \rightarrow S_x) \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) hold in another domain, these are not inconsistent. Let us introduce Hayes’s notion of an ‘induction domain’ that excludes the subject \( \text{pakṣa} \).\(^{13}\) If \( \text{H}_p \) holds in the subject domain, and \( \forall x \,(H_x \rightarrow S_x) \) and \( \forall x \,(H_x \rightarrow \neg S_x) \) hold in the induction domain, these are not inconsistent. The inference may be analogical one.

5. Conclusion

The NV states that a valid inference is one in which the conclusion follows from \textit{hetu}, that is, Reasons 3, 9, 10, 11, and 15. We formulated these inferences using predicate logic, and considered the difference between Uddyotakara’s logic and predicate logic. The difference is found when we formulate the inference in which the conclusion follows from Rea-
sons 6, 12, 15, and 16 as ‘Hp, ∀ x (Hx→Sx), ∀ x (Hx→¬Sx) ⊨.’ First, while this inference is valid in predicate logic,\(^{14}\) only the inference from Reason 15 is valid in Uddyotakara’s logic. We cannot explain this disagreement. This is an issue for the future.

Second, while the premises are inconsistent in predicate logic, these are consistent in Uddyotakara’s logic. This may indicate that an inference is performed in one domain in predicate logic and within two domains in Uddyotakara’s logic.

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1) NV on NS 1.2.4, 156.3–4.
2) The NV also uses the term ‘taajaśya’ and ‘saajaśya’ for ‘sapakṣa.’
3) NV on NS 1.1.5, 43.7–15; NV on NS 1.1.34, 112.19–113.5; NV on NS 1.1.35, 116.2–4. See Okazaki (2005: 53–56).
4) NV on 1.2.4, 156.12–157.17.
5) NV on NS 1.2.4, 157.17–19.
8) NVTT on NS 1.2.4, 286.16–287.4.
9) The formulation depends on Okazaki (2008).
11) However, this inference is not sound.
12) Okazaki (2008) constructs a non-classical derivation system in which the inference from Reason 15 is valid.
14) Actually, predicate logic does not allow an inference whose domain of discourse does not have members.

Abbreviations and References


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