Group Decision-Making Model in Fuzzy AHP
Based on the Variable Axis Method

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Keywords: fuzzy AHP, variable axis method, group optimization, geometric mean

1. Introduction

This paper extends a fuzzy version of Analytic Hierarchy Process (AHP) to reflect human’s subjective judgment. We propose an overall procedure to manage a problem in fuzzy AHP by consolidating optimization method, geometric mean technique, and variable axis method (VAM).

2. Proposed Model

Our model is described as follows. First of all, decision-makers are asked to evaluate the decision criteria and alternatives and to make up a fuzzy pairwise comparison matrix.

In group decision-making process, we need to reach the agreement among a group. Thus, evaluations collected by several decision-makers are to be unified. We practice optimization procedure. This is the way to minimize the total sum of each evaluator’s dissatisfaction against the determination as a group.

Then, we derive the fuzzy weight vectors in pairwise comparison matrix by means of geometric mean method. In AHP, all the elements in a pairwise comparison matrix are given as the ratio of inherent weight in each object. We regard the data as product-type, and then introduce the geometric mean technique to synthesize the fuzzy numbers.

Further, de-fuzzification of the weight vectors are implemented. Fuzzy weight vector is converted into the weight vector in crisp values. New comprehensive technique VAM is developed to decide the total ordering of fuzzy numbers. VAM practiced the following process:

(1) variable axis $\alpha$ is set up. Variable axis depends on both the height of intersection and the modal value of triangular fuzzy numbers to be compared. All the fuzzy numbers are evaluated based on the axis $\alpha$.

(2) mapping $F$ is implemented to manage the obtained values as fuzzy numbers.

(3) a fuzzy ranking method is presented based on the extension principle.

(4) the weight vector in crisp values is generated after normalization.

In the end, we obtain final evaluation. All the weight vectors under each criterion are integrated to achieve the evaluation as a group. Flowchart for obtaining the priority vector is described as follows.

3. Conclusion

We propose a new approach to fuzzy AHP problems in group decision-making environment. The proposed model includes optimization method, geometric mean technique, and VAM. The present work suggests not only the introduction of existing comparable evaluation based on the extension principle in fuzzy numbers, but the adoption of absolute evaluation with the use of the modal values. Then, we can compensate for the shortcomings of only employing comparable evaluation by means of reflecting both the perspectives. VAM will be promising one in the field of fuzzy ranking method and also applicable beyond the border of decision making arena.
Group Decision-Making Model in Fuzzy AHP
Based on the Variable Axis Method

Kaori Ota* Non-member
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Hayao Miyagi* Member

This paper extends a fuzzy version of Analytic Hierarchy Process to reflect human’s vagueness or subjective judgment. New comprehensive technique, variable axis method [VAM], is developed to decide the total ordering of fuzzy numbers. Further, we adopt the fuzzy optimization method in group decision-making, minimizing the total sum of the dissatisfaction of each evaluator. Geometric mean technique is employed to synthesize a fuzzy positive reciprocal matrix. Numerical example demonstrates the effectiveness and the applicability of the proposed decision making model.

Keywords: fuzzy AHP, variable axis method, group optimization, geometric mean

1. Introduction

Evaluations done by decision makers’ may involve qualitative factors because human’s judgments are usually based on their own experiences and intuition. One absolute number may not represent human’s complexity in case of decision making. Introduction of fuzzy theory is, therefore, widespread tendency in the field of decision-making (1)–(6) (9) (10). In the fuzzy decision making, how to decide ordering of several fuzzy numbers is one of the important pending problems.

Further, decision processes in the real world usually involve several participants. How to achieve the agreements among decision makers is another significant concern in the field of evaluation.

Many scholars have addressed the priority theory in fuzzy numbers. Some scholars such as Laarhoven (4), Buckley (2), and Ruoning (5) actually pursued the evaluation in fuzzy numbers. Laarhoven proposed the mathematical model by using logarithmic least squares method [LLSM]. Much effort afterwards has been devoted to the discussion on improving LLSM (2) (4)–(6).

Others like Chang (3) or Chian-son (6) derived the priority vectors in crisp values after transferring final fuzzy scores into crisp values. Chang proposed a method over the fuzzy comparing judgment. Adapting fuzzy numbers with triangular membership function, he presented Extent Analysis Method [EAM]. The final priority vector expresses the relative importance among the several fuzzy numbers. The height of intersection of two membership functions is especially signified, drawn as the quantitative ratio in comparing fuzzy numbers.

Following two inconveniences have been found in his work. One point is the normalization of the matrix. Very little valid justification was explained over that normalization.

As to the other point, an evaluation could include serious error when the vertical intersection of the two triangular membership functions approaches zero unlimitedly.

In the present study, we develop a new approach to manage the fuzzy AHP. Variable axis is formulated in order to improve the disadvantages in Chang’s investigation. The axis depends on both the height of intersection and the modal value of triangular fuzzy numbers to be compared. Additionally, we adopt fuzzy optimization method to reach the agreement among a group. This is the way to minimize the total sum of the dissatisfaction of each evaluator. Further, fuzzy geometric mean values are employed to synthesize the matrices in pairwise comparison.

This paper is outlined in the following. First of all, triangular fuzzy numbers are defined as pairwise comparison scale in fuzzy AHP. Then, we adopt optimization procedure when combining the judgments on several evaluators. As to the synthetic matrices of pairwise comparison in fuzzy numbers, we employ geometric mean technique. Further, a new approach involving variable axis $\alpha$ is presented. Example is presented as a fuzzy version of the problem appeared in Laarhoven (4). This paper concludes with the discussion of effectiveness and applicability of the proposed approach.

2. Basic Concept of Fuzzy AHP

2.1 Triangular Fuzzy Numbers  A fuzzy set is characterized by a membership function, which assigns each object a grade of membership ranging between 0 and 1. A fuzzy number is a special fuzzy set, such that $\tilde{M} = \{(x, \mu_M(x)), \ x \in R \}$ where the element $x$ lies on the real space $R$ i.e. $-\infty < x < \infty$, and $\mu_M(x)$ is a continuous mapping
from R to the close interval [0, 1].

A fuzzy number \( \tilde{M} \) on \( R \) can be defined as a triangular fuzzy number if its membership function \( \mu_M(x) : R \rightarrow [0, 1] \) is equal to

\[
\mu_M(x) = \begin{cases} 
\frac{x - l}{m - l}, & x \in [l, m], \\
\frac{u - x}{u - m}, & x \in [m, u], \\
0, & \text{otherwise},
\end{cases}
\]

where \( l \leq m \leq u \). The triangular fuzzy number can be denoted by \((l, m, u)\), when \( l, m, \) and \( u \) stand for lower, modal, and upper values of the fuzzy number \( M \), respectively. If \( l = m = u \), it is a nonfuzzy number.

The operational laws of two triangular fuzzy numbers \( \tilde{M}_i = (l_i, m_i, u_i) \) and \( \tilde{M}_j = (l_j, m_j, u_j) \) are defined as follows:

\[
\gamma \times \tilde{M}_i = (\gamma l_i, \gamma m_i, \gamma u_i), \quad \gamma \in R,
\]

\[
\tilde{M}_i + \tilde{M}_j = (l_i + l_j, m_i + m_j, u_i + u_j),
\]

\[
\tilde{M}_i \times \tilde{M}_j = (l_i l_j, m_i m_j, u_i u_j),
\]

\[
\tilde{M}_i / \tilde{M}_j = (l_i / u_j, m_i / m_j, u_i / l_j).
\]

2.2 AHP in Fuzzy Numbers

The implementation of AHP includes the following steps. Triangular fuzzy numbers are employed to present vagueness of the evaluation.

In the first step, a hierarchical structure is established to present a problem.

The next step derives the priority based on the pairwise comparison. It is calculated on two levels separately: decision criteria and alternatives under each criterion. By using triangular fuzzy numbers, a positive reciprocal matrix \( M \) is constructed:

\[
M = [\tilde{M}_{ij}] = \begin{pmatrix} 
\tilde{M}_{11} & \tilde{M}_{12} & \cdots & \tilde{M}_{1n} \\
\tilde{M}_{21} & \tilde{M}_{22} & \cdots & \tilde{M}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{M}_{n1} & \tilde{M}_{n2} & \cdots & \tilde{M}_{nn}
\end{pmatrix}
\]

where fuzzy numbers hold the following nature.

\[
\tilde{M}_{ii} = (1, 1, 1),
\]

\[
\tilde{M}_{ij} = (l, m, u),
\]

\[
\tilde{M}_{ji} = \tilde{M}_{ij}^{-1} = (1/u, 1/m, 1/l).
\]

Linguistic scale is employed to make comparisons, containing imprecise judgments due to either uncertainty inherent in individual judgments or variations in participants’ perception. Therefore, we relate response in a linguistic scale to a triangular fuzzy scale as in Table 1. Decision criteria are compared in pairs to assign weights. Further, alternatives under each criterion are assessed, and the weights are also allocated.

In the last step, final evaluation is derived by integrating the weights over criteria and alternatives.

3. Calculation of the Priority Vector in Fuzzy Numbers

3.1 Group Decision-Making Based on Optimization Procedure

Optimization method is adopted in order to combine some viewpoints into one, gaining the unified evaluation among a group. We consider an optimization procedure as an approach which minimizes the total sum of each evaluator’s dissatisfaction against the final determination as a group. This measure is also instituted by Nakanisi (8) in the context of group decision-making although he utilized the procedure in crisp values. By taking advantage of the process, all the viewpoints are graded rationally.

Optimal solution is acquired by minimizing the summation of dissatisfaction \( \tilde{D} \) under the constraint \( \tilde{g}_i \). \( \tilde{D} \) and \( \tilde{g}_i \) are defined as follows:

\[
\tilde{D} = \sum_{i=1}^{n} (\tilde{g}_i \cdot \tilde{x}_i - \tilde{c})^2, \quad \sum_{i=1}^{n} \tilde{g}_i = \tilde{n},
\]

where

\[
\tilde{c} = \frac{1}{n} \sum_{i=1}^{n} (\tilde{g}_i \cdot \tilde{x}_i),
\]

and \( \tilde{g}_i \) is the grade on \( i \)th-evaluation, \( \tilde{x}_i \) represents \( i \)th fuzzy evaluation \((i = 1, 2, \ldots, n)\), \( \tilde{c} \) denotes average evaluation as a group. Since \( n \) is the number of evaluation \( \tilde{g}_i \), we describe \( \tilde{n} = (n, n, n) \).

To solve the problem, Lagrange function \( \tilde{L} \) is defined as follows:

\[
\tilde{L}(\tilde{g}, \lambda) = \tilde{D} + \lambda(\tilde{n} - \sum_{i=1}^{n} \tilde{g}_i)
\]

\[
= \sum_{i=1}^{n} (\tilde{g}_i \cdot \tilde{x}_i - \tilde{c})^2 + \lambda(\tilde{n} - \sum_{i=1}^{n} \tilde{g}_i), \quad \lambda \geq 0,
\]

where \( \lambda \) is Lagrange’s undetermined multiplier. Then the function is partially differentiated with respect to \( \tilde{g}_i \) and \( \lambda \). Thus, we have

\[
\frac{\partial \tilde{L}(\tilde{g}, \lambda)}{\partial \tilde{g}_i} = 0, \quad \frac{\partial \tilde{L}(\tilde{g}, \lambda)}{\partial \lambda} = 0.
\]

By solving the above equations, we obtain \( \tilde{g}_i \) which minimizes dissatisfaction \( \tilde{D} \).

The grade \( \tilde{g}_i \) consists of lower \( g_{il} \), modal \( g_{im} \), and upper \( g_{iu} \) because of a triangular fuzzy number. However, we deal with only modal value \( g_{im} \) when obtaining \( \tilde{c} \) and \( \tilde{D} \) to avoid the complexity in calculation as well as to consider the importance of modal value.
3.2 Geometric Mean Technique to Synthesize a Fuzzy Positive Reciprocal Matrix Additive-type and product-type procedures are usually involved in synthesizing some numbers. Both the procedures are easily extended to fuzzy positive reciprocal matrix $M$ for deriving the fuzzy weights. Additive-type procedure has been employed in some works (9)(10). They give fuzzy synthetic values as follows:

$$\hat{w}_i = \frac{\sum_{j=1}^{n} \hat{M}_{ij}}{\left( \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{M}_{ij} \right)^{\frac{1}{n}}}, \quad \forall i, j = 1, 2, \ldots, n.$$  

(8)

In AHP, however, all the elements in a pairwise comparison matrix are given as the ratio of inherent weight in each object. We regard the data as product-type, and then introduce the geometric mean technique to synthesize the fuzzy numbers. We employ the geometric mean in each row to obtain fuzzy weights. That is, the procedure includes multiplying every element each other in a row, taking the $n$th root, and then, as in the typical AHP, normalizing the numbers obtained. Total sum of the weights is usually assigned as one in AHP. Fuzzy synthetic values based on product-type procedure are defined as

$$\hat{w}_i = \frac{\left( \prod_{j=1}^{n} \hat{M}_{ij} \right)^{\frac{1}{n}}}{\sum_{j=1}^{n} \left( \prod_{i=1}^{n} \hat{M}_{ij} \right)^{\frac{1}{n}}}, \quad \forall i, j = 1, 2, \ldots, n.$$  

(9)

4. Setting Variable Axis: Calculation of the Priority Vector in Fuzzy AHP

4.1 Extent Analysis Method Chang presented a method over the ordering of several fuzzy numbers in AHP by utilizing triangular fuzzy numbers. He has departed from Laarhoven’s (4) result on LLSM, and achieved EAM by applying the principle of the comparison of fuzzy numbers. EAM is subject to two main criticisms. One point is normalization in case of non-overlap. When combining a fuzzy pairwise comparison matrix based on fuzzy synthetic extent technique, he classified positional relationship in fuzzy numbers into two categories: overlap and non-overlap. Overlay indicates $i \leq u_j$ as shown in Fig.1. Non-overlap expresses $i > u_j$. Without mentioning these two categories, Chang actually dealt with these two classifications in his example. In the former case, comparison was performed based on the principle of the comparison of fuzzy numbers. In the latter case, Chang normalized each element of the original matrix, from which synthetic evaluation vectors was obtained. Very little valid justification was explained over that normalization.

Subsequent studies by Kahraman (9) and Buyukozkan (10) have dealt with fuzzy Analytic Network Process [ANP] based on EAM. They achieved final results without adopting normalization.

As to the other point, the outcome could be questioned even though calculation was justifiably performed based on the principle of the comparison of fuzzy numbers. When the ordinate of the highest intersection gets close to zero unlimtedly, the ratio of two fuzzy numbers $M_1$ and $M_2$, would be infinity to one. Introducing these figures as an evaluation of fuzzy numbers should be quite disputable.

4.2 Variable Axis $\alpha$ We propose the variable axis method [VAM] as a new comprehensive procedure for de-fuzzification when obtaining the ordering of several fuzzy numbers. We believe that our approach of setting up the variable axis $\alpha$ leads to the unified procedure without classification of overlap and non-overlap in obtaining the priority vector. Further, we can overcome the problematic situation in which the ratio of two fuzzy numbers would be infinity to one.

According to the extension principle (11), the degree to which $M_i$ is ranked as the greatest fuzzy number is defined as

$$P(M_i) = \sup_{k \in N_n} \tilde{M}_k(r_k), \quad \text{for all } i = 1, 2, \ldots, n.$$  

(10)

where $i \in N_n$, $N_n = \{1, 2, \ldots, n\}$. The supremum is taken over all vectors $(r_1, r_2, \ldots, r_n) \in R^n$ such that $r_i \geq r_j$ for all $j \in N_n$.

We need to consider their positional relation in order to compare two fuzzy numbers $M_i$ and $M_j$ ($M_i \geq M_j$). Thus, we define both the values of $P_i(M_j)$ and $P_j(M_i)$.

We describe $P_i(M_j)$ as presenting that the degree to which $M_i$ is ranked as greater fuzzy number compared to $M_j$. When the modal value $m_i$ of $M_i$ lies to the right side of the modal value $m_j$ of $M_j$ as shown in Fig.1, we obtain $P_i(M_j) = 1$ as shown in formula (11). On the other hand, we obtain $P_j(M_i)$ as the height of their intersection as shown in formula (12).

$$P_i(M_j) = \sup_{r_i \geq r_j} [\tilde{M}_i(r_i), \tilde{M}_j(r_j)] = 1, \quad \text{for all } i, j = 1, 2, \ldots, n.$$  

(11)

$$P_j(M_i) = \sup_{r_j \geq r_i} [\tilde{M}_i(r_i), \tilde{M}_j(r_j)] = \frac{l_i - u_j}{(m_j - u_j) - (m_i - l_i)}. \quad \text{for all } i, j = 1, 2, \ldots, n.$$  

(12)

Then, we discuss how to set up variable axis $\alpha$. We suppose that triangular fuzzy number would be isosceles triangle in an ideal state. On that condition, the ratio of quantity in two fuzzy numbers could be their respective
Further, in Fig. 3, \( \alpha > \alpha_F \) exceeds 1. Thus, we consider the following mapping awkward because the maximum membership grade exceeds the minimum membership grade.

Based on variable axis \( M \) respectively. Based on variable axis \( \alpha \), all the fuzzy numbers are also evaluated based on the axis \( \alpha \), which is determined by the maximum and the minimum fuzzy numbers.

According to the extension principle in formula (10), the ordering of fuzzy numbers \( \tilde{M}_1' \) to \( \tilde{M}_n' \) is given as in Table 2.

As in EAM practiced, all the elements are compared in each row. Then, the minimal \( P_{ij}(\tilde{M}_i') \) is selected.

We define

\[
P_{ij}(\tilde{M}_i') = \frac{l_i - u_j'}{(m_j - u_j') - (m_i - l_i')}, \quad i = 1, \ldots, n
\]

where new priority fuzzy set \( P_{ij}(\tilde{M}_i') \) stands for the degree to which \( \tilde{M}_i' \) is ranked as the greatest number, proposed in our method.

In the case of involving several fuzzy numbers, all the fuzzy numbers are also evaluated based on the axis \( \alpha \), which is determined by the maximum and the minimum fuzzy numbers.

According to the extension principle in formula (10), the ordering of fuzzy numbers \( \tilde{M}_1' \) to \( \tilde{M}_n' \) is given as in Table 2.

As in EAM practiced, all the elements are compared in each row. Then, the minimal \( P_{ij}(\tilde{M}_i') \) is selected.

We define

\[
W' = (d'(\tilde{M}_1'), d'(\tilde{M}_2'), \ldots, d'(\tilde{M}_n'))^T \quad \cdots (22)
\]

We reach the final weight vector after normalization such that

\[
W = (d(\tilde{M}_1'), d(\tilde{M}_2'), \ldots, d(\tilde{M}_n'))^T \quad \cdots (23)
\]

where \( W \) is a priority vector made up of crisp values, showing the ordering of \( \tilde{M}_1', \tilde{M}_2', \ldots, \tilde{M}_n' \).

### 5. Procedure in Fuzzy AHP

We propose an overall procedure to manage a problem in fuzzy AHP by consolidating optimization method, geometric mean technique, and VAM. Flowchart for obtaining the priority vector is described as follows.

In Step 1, decision-makers are asked to evaluate the decision criteria and alternatives and to make up a pairwise comparison matrix in fuzzy numbers. Various manners have been proposed for making up a fuzzy pairwise comparison matrix. It is not our intention to get involved in the debate over which procedure is the best. We follow the method presented by much investigation (2)–(6).

In group decision-making process, we acquire several evaluations to be unified. In Step 2, viewpoints collected by several decision-makers are combined into one, i.e. \( \tilde{c} \)

### Table 2. Ordering of fuzzy numbers

<table>
<thead>
<tr>
<th>( \tilde{M}_1' )</th>
<th>( \tilde{M}_2' )</th>
<th>( \ldots )</th>
<th>( \tilde{M}_n' )</th>
<th>( \min P_{ij}(\tilde{M}_i') )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{M}_1' )</td>
<td>1</td>
<td>( P_{21}(\tilde{M}_1') )</td>
<td>( \ldots )</td>
<td>( P_{n1}(\tilde{M}_1') )</td>
</tr>
<tr>
<td>( \tilde{M}_2' )</td>
<td>( P_{21}(\tilde{M}_2') )</td>
<td>1</td>
<td>( \ldots )</td>
<td>( P_{n2}(\tilde{M}_2') )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( \tilde{M}_n' )</td>
<td>( P_{n1}(\tilde{M}_n') )</td>
<td>( P_{n2}(\tilde{M}_n') )</td>
<td>( \ldots )</td>
<td>1</td>
</tr>
</tbody>
</table>

where new priority fuzzy set \( P_{ij}(\tilde{M}_i') \) stands for the degree to which \( \tilde{M}_i' \) is ranked as the greatest number, proposed in our method.
through optimization procedure.

We derive the fuzzy weight vectors in pairwise comparison matrix in Step 3. Synthetic priority vector \( \tilde{w} \) in fuzzy numbers is attained by means of geometric mean method.

Step 4 includes defuzzification of the weight vectors in which fuzzy weight vector \( \tilde{w} \) is converted into the weight vector \( W \) in crisp values. VAM practiced the following process. At first, variable axis \( \alpha \) is set up. Then, mapping \( F \) is implemented. Further, a fuzzy ranking method is presented based on the extension principle. Finally, the weight vector \( W \) in crisp values is generated after normalization.

Final evaluation is obtained in Step 5. All the weight vectors under each criterion are integrated to achieve the evaluation as a group.

6. Application

The following example is taken from the paper presented by Laarhoven (4). Suppose that at a university the post of professor in Operations Research is vacant. After a first selection, three candidates remain. We shall call them A1, A2, and A3. A member of committee has authorized to decide which applicant is best qualified for the position. Three members are in the committee and have identified the following decision criteria:

- mathematical creativity (\( C_1 \))
- creativity implementation (\( C_2 \))
- administrative capability (\( C_3 \))
- human mutuality (\( C_4 \))

The committee constructs a fuzzy evaluation matrix with respect to decision criteria. It is performed through a pairwise comparison shown in Table 3. One numerical value appeared on the element in the comparison matrices shows that only one result is available, and two numerical values on the element stand for two results available.

We apply our procedure to the above mentioned example, following the steps in flowchart in Fig.4. Construction of fuzzy evaluation matrix corresponds to Step 1. By working with optimization procedure in Step 2, we unify the evaluations by members of the committee. These calculations are based on the formula (3) to (7). The result is described in Table 4.

In the classical AHP, Consistency Index (C.I.) is widely utilized to address the issue of consistency (5). We borrow the numerical example appeared in Laarhoven (4), as mentioned above, after confirming whether C.I. \( \leq 0.1 \) is satisfied. The modal values of triangular fuzzy numbers are employed to calculate C.I. There is still no explicit articulation on how consistency should be determined within fuzzy AHP context, indeed.

Then, geometric mean technique in Step 3 is utilized to synthesize the fuzzy reciprocal matrix. By applying formula (9), we acquire a fuzzy evaluation vector \( \tilde{w} \) (in Table 5) regarding the decision criteria \( C \).

Further in Step 4, a fuzzy weight vector \( \tilde{w} \) is transferred into \( W \). VAM is instituted to generate a priority vector in crisp values. Based on the prospective minimum and maximum fuzzy numbers, we decide the variable axis \( \alpha \). In this example, \( \tilde{w}_3 \) and \( \tilde{w}_4 \) are the candidate values. Height of the intersection \( D \) between \( \tilde{w}_3 \) and \( \tilde{w}_4 \) is 0.035. Based on formulas (14) and (15), \( \delta \) and \( \alpha_0 \) are given as 0.964 and 1.722 respectively. Therefore, the variable axis \( \alpha \) is obtained as \(-0.722\), derived from formula (16). All the values are examined and compared based on the axis \( \alpha \).

Then, we implement mapping of

\[
F : [-0.722, 1] \rightarrow [0, 1].
\]

By adopting the extension principle, we obtain the degree to which \( \tilde{w}_i \) is ranked as the greatest fuzzy number and the ordering of fuzzy weight in Table 6. \( W' \) is composed of the elements of min \( P_{ij}(\tilde{w}_i) \), given as

\[
W' = \begin{bmatrix}
0.559 & 0.990 & 0.440 & 1.000
\end{bmatrix}^T.
\]

In the end, the weight vector \( W \),

\[
W = \begin{bmatrix}
0.187 & 0.331 & 0.147 & 0.334
\end{bmatrix}^T
\]

is obtained in crisp number by means of normalization of \( W' \). This is the weight vector over \( C_1, C_2, C_3, \) and
committee further compare the candidates.

Table 5. Fuzzy weight vector under criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$\tilde{w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>(0.115, 0.176, 0.225)</td>
</tr>
<tr>
<td>$C_2$</td>
<td>(0.220, 0.335, 0.507)</td>
</tr>
<tr>
<td>$C_3$</td>
<td>(0.100, 0.149, 0.230)</td>
</tr>
<tr>
<td>$C_4$</td>
<td>(0.223, 0.340, 0.509)</td>
</tr>
</tbody>
</table>

Table 6. Ordering of fuzzy weight

<table>
<thead>
<tr>
<th>$\tilde{w}_1$</th>
<th>$\tilde{w}_2$</th>
<th>$\tilde{w}_3$</th>
<th>$\tilde{w}_4$</th>
<th>$\min P_r(\tilde{w}_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{w}_2$</td>
<td>1</td>
<td>0.559</td>
<td>1</td>
<td>0.559</td>
</tr>
<tr>
<td>$\tilde{w}_3$</td>
<td>1</td>
<td>1</td>
<td>0.590</td>
<td>0.990</td>
</tr>
<tr>
<td>$\tilde{w}_4$</td>
<td>0.891</td>
<td>0.450</td>
<td>1</td>
<td>0.440</td>
</tr>
<tr>
<td>$\tilde{w}_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7. Weight vector on criteria

<table>
<thead>
<tr>
<th>$W'$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.550</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.990</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.440</td>
</tr>
<tr>
<td>$C_4$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8. Pairwise comparison based on $C_1$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(1, 1, 1)</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(2/3, 1/3, 2/3)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(1, 1, 1)</td>
<td>(2/3, 1/3, 2/3)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(1, 1, 1)</td>
</tr>
</tbody>
</table>

Table 9. Pairwise comparison based on $C_2$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(1, 1, 1)</td>
<td>(5/2, 3/2, 7/2)</td>
<td>(5/2, 3/2, 7/2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(2/7, 1/3, 2/5)</td>
<td>(1, 1, 1)</td>
<td>(2/3, 1/3, 2/3)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(1, 1, 1)</td>
</tr>
</tbody>
</table>

Table 10. Pairwise comparison based on $C_3$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(1, 1, 1)</td>
<td>(5/2, 3/2, 7/2)</td>
<td>(5/2, 3/2, 7/2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(2/7, 1/3, 2/5)</td>
<td>(1, 1, 1)</td>
<td>(2/3, 1/3, 2/3)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(2/3, 1/3, 2/3)</td>
<td>(1, 1, 1)</td>
</tr>
</tbody>
</table>

Table 11. Pairwise comparison based on $C_4$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(1, 1, 1)</td>
<td>(3/2, 2/3, 5/2)</td>
<td>(3/2, 2/3, 5/2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(3/2, 2/3, 5/2)</td>
<td>(3/2, 2/3, 5/2)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(3/2, 2/3, 5/2)</td>
<td>(3/2, 2/3, 5/2)</td>
<td>(1, 1, 1)</td>
</tr>
</tbody>
</table>

Table 12. Evaluation vectors of alternatives under each criterion

<table>
<thead>
<tr>
<th>Criterion</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.290</td>
<td>0.252</td>
<td>0.458</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.523</td>
<td>0.186</td>
<td>0.291</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.587</td>
<td>0.217</td>
<td>0.196</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.246</td>
<td>0.453</td>
<td>0.301</td>
</tr>
</tbody>
</table>

Table 13. Final evaluation

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FinalScore</td>
<td>0.396</td>
<td>0.292</td>
<td>0.312</td>
</tr>
</tbody>
</table>

$C_4$ shown in Table 7.

As in the procedure mentioned above, members of the committee further compare the candidates $A_1$, $A_2$, and $A_3$ based on each criterion respectively, then again construct fuzzy evaluation matrices, shown in Table 8 to Table 11.

Description “—” in Table 9 and Table 11, explains the situation where no numerical results on the element are available, or all the results are missing for some reasons. There are several arguments regarding how to treat incomplete pairwise comparison matrices. Two-stage method (12) is one of the approaches to address the issue. It allows us to ignore the lacking results in comparison matrices and to calculate tentative weights based on the geometric mean method. Then lacking results are presumptively supplemented with the ratio of the tentative weights. The ultimate priority vectors are finally acquired by the recalculation of the matrices with the compensated data. This technique is employed to synthesize Table 9 and Table 11.

Table 12 indicates the weight of each candidate under each criterion separately. As in Step 5, the final evaluation is demonstrated in Table 13, which is obtained from $W$ in Table 7 and $A_1$, $A_2$, and $A_3$ in Table 12.

7. Conclusion

This paper suggested a new approach to construct a fuzzy AHP model in group decision-making environment.

we adopted the optimization method in group
decision-making, minimizing the total sum of the dissatisfaction of each evaluator toward the group evaluation. Further, the product-type procedure is employed to obtain fuzzy weights. Then, VAM is practiced to decide the total ordering of fuzzy numbers. VAM led the unified procedure effectively without classification of overlap and non-overlap when obtaining final priority vector. It also overcame the problematic situation in which the ratio of two fuzzy numbers would be infinity to one.

The present work proposed not only the introduction of existing comparable evaluation based on the extension principle in fuzzy numbers, but the adoption of absolute evaluation with the use of the modal values. Then, we could compensate for the shortcomings of only employing comparable evaluation by means of reflecting both the perspectives. VAM will be promising one in the field of fuzzy ranking method and also applicable beyond the border of decision making arena.

Our approach enables decision-makers to achieve rational result with more simple procedure among the alternatives, while bringing ambiguity to the scientific decision-making analysis.

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