Fault Detection and Isolation on the Basis of Dynamic Process Models

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This paper outlines the basic concept of fault detection and isolation (FDI) in dynamic systems based on analytical process models and briefly reviews the most important approaches. Emphasis is placed upon residual generation using state observation techniques. Because of the strong impact of parameter variations on the applicability and efficiency of model-based FDI, the issues of parameter sensitivity as well as the design of robust FDI schemes are addressed in more detail.

Key word : Fault diagnosis, analytical redundancy, model-based fault detection and isolation, robust fault detection filters, unknown-input observers.

1. Introduction

The progress of automation and its extension to more complex systems with the danger of catastrophic consequences in the case of a failure have given rise to an increased demand for fault tolerance of the automated processes. Actually, fault tolerance constitutes a fundamental design feature in any complex automated system. It can be achieved by actively isolating faulty components from further use. This has invoked an according interest in powerful techniques of fault detection, isolation and accomodation (FDIA) in dynamic systems.

The essential part of the FDIA task is the issue of *fault detection and isolation* (FDI). It has become common to distinguish between instrument fault detection (IFD), actuator fault detection (AFD), and component fault detection (CFD), the latter dealing with malfunctions in general functional components of the system.

All FDI schemes need some sort of information about the process under consideration. The *elementary supervision techniques* restricted to simple tests

like limit checking, range checking, trend checking, analysis of characteristic quantities and utilizing available quantities like output signals, frequency, efficiency etc. make only little use of information about the system. It is evident that in most of these cases no detailed fault isolation is possible.

More intriguing results can be obtained if a *functional model* of the system is used for the FDI task. This may be utilized to generate *redundancy* and to *estimate* quantities that are not directly measurable thus allowing a systematic *localization* of faults in the system.

There are two basically different ways to use models for the generation of redundancy. The traditional approach is to use *physical models*, i. e., to parallel the functional units with physical replicas and thus generate so-called *physical* (or *hardware*) redundancy. A typical example is the triplication of a functional device associated with a "two-out-of-three" decision logic. Obviously, the drawback of physical redundancy is that it leads to an enlargement of the physical equipment associated with additional volume, weight, complexity and costs.

An alternative approach known as *analytical* (or

functional or software) redundancy makes use of a mathematical model of the system. In this paper we limit ourselves to the analytical model approach. Clearly, there is also some potential in using "knowledge-based" models in all such cases where analytical models are not available, but much less has been done so far in this field.

The model-based approach makes it possible to estimate nonmeasurable quantities such as state variables or parameters with which faults in the system can be localized. It is most important that such a model-based approach does not need any interaction with the actual system: The complete FDI task can be carried out on a computer in the control room by simply processing the available input and output signals of the system.

However, the price to pay for the model-based FDI approach is expenditure in modeling of the process and an according sensitivity of the FDI scheme to modeling uncertainties and parameter variations of the actual system. Actually, the parameter sensitivity has turned out to be the crucial point in any model-based FDI concept.

The literature provides a great variety of different approaches to the model-based FDI an attempts to overcome the parameter sensitivity problem. Comprehensive surveys are given, for example, by Willsky (1976), Walker (1983), Isermann (1984), Chow and Willsky (1984), Frank (1987), and the latest state of the art is presented in the book of Patton, Clark and Frank (1989).

The goal of this paper is to outline the basic concept and briefly review the different approaches to model-based FDI. Emphasis is given to the process of residual generation. Because of the decisive impact of the parameter variations upon the applicability of the model-based FDI methods, the sensitivity and robustness with respect to parameter variations will be of particular concern in this paper.

2. General principle of model-based FDI

Following Isermann (1984), the general procedure of model-based FDI can be characterized by a structural diagram as shown in Figure 1. For the sake of a realistic description one has to admit that the actual process under consideration with the input

Fig. 1. Unified structural diagram of model-based fault detection and isolation (FDI).

vector $\underline{u}$ and the output vector $\underline{y}$ is subjected to disturbances $\underline{d}_1$, measurement noise $\underline{d}_2$, parameter variations $\underline{d}_3$ (due to modeling uncertainties and parameter changes), and faults that we want to detect, characterized by the vector $\underline{f}$. Assuming the system to be linear (or linearized around a certain operation point), and arranging $\underline{d}_1$, $\underline{d}_2$ and $\underline{d}_3$ to the so-called vector of unknown inputs, $\underline{d} = [\underline{d}_1^T, \underline{d}_2^T, \underline{d}_3^T]^T$, the system state equations take the following form:

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{B} \underline{u} + \underline{E} \underline{d} + \underline{K} \underline{f} \dots \dots \dots (1)$$

$$\underline{y} = \underline{C} \underline{x} + \underline{F} \underline{d} + \underline{G} \underline{f} \dots \dots \dots (2)$$

Here $\underline{x}$ denotes the state vector and $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$, $\underline{E}$, $\underline{F}$, $\underline{K}$ and $\underline{G}$ are known matrices of proper dimensions. In these terms, actuator faults reflected in changes in $\underline{B}$, sensor faults in changes in $\underline{C}$ and component faults in changes in $\underline{A}$ are represented

by $\underline{f}$ with a proper choice of $\underline{K}$ and $\underline{G}$. In general, one has to admit that no assumption can be made on the mode (type and size) of $\underline{d}$ and $\underline{f}$.

For the procedure of model-based FDI, three kinds of *models* are required: A model of the *nominal system*, a model of the *faulty system* carrying information about the effect of faults, and a model of the *observed system* defined by the inputs and outputs taken from the actual system. To achieve a high quality of fault detection with a low false alarm rate the nominal model should be tracked with high precision by the observation model.

The first step of the FDI procedure is that of residual generation. This can be accomplished either by *state estimation* or by *parameter identification*. The estimates are compared with the corresponding quantities of the nominal model in order to generate the residuals or parameter error signals, respectively, or they are used to form fault effect textures such as likelihood functions. The basis for the *decision* on a fault is the *fault signature*, i. e., a signal that is obtained from the model of the faulty system defining the effects of faults to be discovered. The final step is the *fault isolation*, i. e., the identification of the fault location and, if desired, its size and cause.

3. Approaches to model-based fault detection

In the widest sense, a fault detection system is defined as a dynamic process which, fed by the input and output of the system under consideration, attenuates all influences except those caused by the faults to be detected.

The great variety of proposed fault detection schemes can be grouped into two major categories:

- (i) Parameter identification methods
- (ii) State estimation methods.

Whilst the *parameter identification* approach requires on-line identification of the parameters of a model of the observed system the *state estimation* approach does with on-line reconstruction of sets or subsets of states or outputs taking the mathematical model as being given. This is done with the aid of observers or Kalman filters, where usually the estimation error (or innovation) is used as the *residual*

that carries the information about the fault.

3.1 Parameter identification approach

This approach makes use of the fact that faults in functional components of a dynamic system are reflected in the physical parameters such as friction, mass, viscosity, resistance, capacitance, inductance, etc. Since the parameters of the mathematical model of the observed system, Figure 1, are dependent on the physical parameters, the faults become also effective in the model parameters. The idea of the parameter identification approach is to detect the faults via estimation of the model parameters as shown in Figure 2 due to the following procedure:

- (1) Choice of a parametric model of the system. Normally one uses linear models with lumped parameters with input/output differential equations of the form

$$a_n y^{(n)}(t) + \dots + a_1 \dot{y}(t) + y = b_0 u(t) + \dots + b_m u^{(m)}(t) \dots \dots \dots (3)$$
- (2) Determination of the relationships between the model parameters θ_i and the physical parameters p_j :

$$\theta = f(p) \dots \dots \dots (4)$$
- (3) Identification of the model parameters θ using the inputs $\underline{u}$ and outputs $\underline{y}$ of the actual system (or of a system component).

Fig. 2. Parameter identification approach to fault detection.

- (4) Determination of the physical parameters due to

$$\underline{p} = \underline{f}^{-1}(\underline{\theta}) \quad \dots\dots\dots (5)$$
- (5) Calculation of their deviations, $\Delta \underline{p}$, from their nominal values taken from the nominal model.
- (6) Decision on a fault by exploiting the relationships between faults and changes in the physical parameters $\Delta \underline{p}$.

A detailed description of the parameter estimation approach is given by Isermann in Patton et al. (1989). Problems may arise from the fact that the inversion $\underline{f}^{-1}$ in Eq. (5) is in general not unique and only little is known on the robustness to model uncertainties, parameter variations and disturbances. Moreover, the identification problem becomes tedious in the case of high-order and/or nonlinear systems.

3.2 State estimation approach

The basic idea of the state estimation approach is to reconstruct the states (and the outputs) of the system with the aid of observers or Kalman filters, using the estimation error or innovation as the *residual* for the detection of the faults. The procedure of fault detection consists then of the following two steps:

- (1) Generation of residuals
- (2) Evaluation of the residuals by a decision-logic.

A state observer or Kalman filter designed particularly for the purpose of fault detection is called a *fault detection* (or *fault diagnosis*) *filter*.

It is well known from observer theory that for

Fig. 3. Basic concept of fault detection based on state estimation.

state estimation one can use linear or nonlinear full or reduced-order state observers (in the deterministic case) or Kalman filters (in the stochastic case). In either case a mathematical model of the process is required. The basic concept of state estimation using a full order observer is shown in Figure 3. Notice that the observer consists simply of a parallel model of the process with a feedback of the estimation error, $\underline{y} - \hat{\underline{y}}$. The feedback is important for several reasons:

- (1) to compensate for differences in the initial conditions
- (2) to stabilize the parallel model
- (3) to provide freedom for the design of the filter, for example, to decouple the desired effects of faults from the effects of unknown inputs.

Under ideal conditions, the observer will track the system when there are no faults. However, when a fault occurs the model in the observer does no longer accurately model the system, and a nonzero residual results. The objective of the filter design is to choose $\underline{H}$ such that the residual becomes large enough so that one can detect the fault.

In the case of a linear process with the state equations (1), (2), the state $\underline{\hat{x}}$ and output $\underline{\hat{y}}$ of a full-order observer satisfy

$$\dot{\underline{\hat{x}}} = (\underline{A} - \underline{H} \underline{C}) \underline{\hat{x}} + \underline{B} \underline{u} + \underline{H} \underline{y} \quad \dots\dots\dots (6)$$

$$\underline{\hat{y}} = \underline{C} \underline{\hat{x}} \quad \dots\dots\dots (7)$$

where $\underline{H}$ denotes the feedback gain matrix that has to be chosen properly to achieve a desired performance of the observer. With Eqs. (1), (2), (6), (7) the relations for the state estimation error, $\underline{\varepsilon} = \underline{x} - \underline{\hat{x}}$, and the output estimation error, $\underline{e} = \underline{y} - \underline{\hat{y}}$, become

$$\dot{\underline{\varepsilon}} = (\underline{A} - \underline{H} \underline{C}) \underline{\varepsilon} + \underline{E} \underline{d} + \underline{K} \underline{f} \quad \dots\dots\dots (8)$$

$$\underline{e} = \underline{C} \underline{\varepsilon} + \underline{F} \underline{d} + \underline{G} \underline{f} \quad \dots\dots\dots (9)$$

It is seen from Eqs. (8) and (9) that the output estimation error, $\underline{e}$, is a function of $\underline{f}$. Hence, $\underline{e}$ can be used as the *residual*, $\underline{r}$, for FDI. If no fault occurs, i. e., $\underline{f} = 0$, then $\underline{r}$ only depends on the unknown input, $\underline{d}$. If, however, $\underline{f} \neq 0$, $\underline{r}$ will be increased. Hence, a fault can be detected by checking the increment of $\underline{r}$ caused by $\underline{f}$. In the simplest case this can be done by a threshold logic.

In a similar way one can derive residuals using reduced-order observers, nonlinear observers or Kalman filters due to the well-known state estimation theory. To optimize them for the task of fault detection is the basic problem of the design of fault detection filters.

4. Observer schemes for FDI

For the FDI task, the observer (or Kalman filter) is commonly incorporated in a whole system, often containing several observers that may be driven by different sets of inputs or outputs of the process. In particular, different configurations are necessary for AFD, IFD and CFD. The goal of observer schemes is to generate structured sets of residuals that admit the isolation of a fault. The most important types of observer schemes are now briefly reviewed.

4.1 Simplified observer scheme (SOS)

The most simple version of an observer scheme for IFD is the so-called *simplified observer scheme* (SOS) proposed by Clark (1978). It consists of a single (full or reduced-order) observer or Kalman filter driven by only one sensor output from which the whole set of outputs is estimated. A fault in the sensor that is used to drive the observer induces a mismatch between all the outputs and their estimates, while a fault in any other sensor just induces a difference between its output and its estimate. Thus, a unique detection of the fault in one of the sensors is possible.

An advantage of this scheme is that the observability of the full state is required from a single measurement only, and that it is easy to implement. However, the price to pay is little detection capability (only one fault at a time).

4.2 Innovation test

Another simple FDI scheme is that of using a *single Kalman filter* driven by the whole output vector of the process (Mehra and Peschon, 1971). It can be shown that under normal operation the innovation vector, i. e., the difference between the measurement vector and its estimate, is a white noise with zero mean and known covariance. With the occurrence of a fault in the process the character of the noise and the covariance change. Hence, a fault can be detected by statistical tests of

whiteness, mean and covariance of the innovations. Examples are the chi-squared test, the bad data suppression concept and the general likelihood ratio (GLR) test (Willsky, 1976). The latter approach includes a hypothesis test based on different modes of the faults.

Though the generation of the innovations is rather simple, and though simplifications of the GLR test have been proposed, e. g., the CGLR test (Willsky, 1976), the innovation test as a whole turns out to be rather delicate.

4.3 Fault detection filter (FDF)

The fault detection filter (or fault sensitive filter) first suggested by Beard (1971) and Jones (1973) has the configuration of a full order observer or Kalman filter. Its derivation is based on the assumption that the process is characterized by

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{B} \underline{u} + \underline{K} \underline{f} \dots\dots\dots (10)$$

$$\underline{y} = \underline{C} \underline{x} \dots\dots\dots (11)$$

(instead of Eqs. (1), (2), i. e., the unknown inputs are not taken into consideration). Hence, the observer equations become

$$\dot{\hat{\underline{x}}} = (\underline{A} - \underline{H} \underline{C}) \hat{\underline{x}} + \underline{B} \underline{u} + \underline{H} \underline{y} \dots\dots\dots (12)$$

$$\hat{\underline{y}} = \underline{C} \hat{\underline{x}} \dots\dots\dots (13)$$

with the same meanings of the symbols as in Eqs. (6), (7). Defining the state estimation error, $\underline{\varepsilon}$, and the residual, $\underline{r}$, by $\underline{\varepsilon} = \underline{x} - \hat{\underline{x}}$ and $\underline{r} = \underline{y} - \hat{\underline{y}}$, respectively, the response of the filter to a fault is

$$\dot{\underline{\varepsilon}} = (\underline{A} - \underline{H} \underline{C}) \underline{\varepsilon} + \underline{K} \underline{f} \dots\dots\dots (14)$$

$$\underline{r} = \underline{C} \underline{\varepsilon} \dots\dots\dots (15)$$

Typical for the fault detection filter is the choice of $\underline{H}$ so that the residual $\underline{r}$ due to a *particular fault*, f_i (component of $\underline{f}$), is constrained to a single direction or plane in the residual space independent of the mode of f_i . This is often not possible unless the state $\underline{x}$ is accordingly enlarged. Since the important information for the fault detection is in the *direction* of the residual rather than in its time function, the use of a fault detection filter does not require any knowledge on the fault mode. Hence, a fault is detected when one or more of the residual projections along the known fault direction or in the known fault plane are sufficiently large. The conditions under which such a single filter can be configured to detect faults in several components as

well as the design procedures are described in Beard (1971), Jones (1973), and Massoumnia (1986).

It is a most important advantage of the fault detection filter that the residual *direction* for a fault is not affected by the fault *mode* $f_i(t)$ (i. e., the size or time history). Notice, however, that unknown inputs (including parameter variations) have not been considered in the design. Hence, this approach does not account for the effects of disturbances, parameter variations or measurement noise as, for example, the unknown input observer design does that will be described later.

4.4 Parity space approach

The parity space approach originated by Deckert et al. (1977) yields a systematic exploitation of the analytical redundancy provided by the mathematical model of the system. Based on a discrete linear system description equivalent to the continuous version, Eqs. (1), (2), a finite impulse response filter is designed for fault detection. The systematic procedure developed by Chow and Willsky (1984) makes use of the redundancy relations to achieve robust FDI in the presence of model uncertainties. The subspace of $(s+1)q$ dimensional vectors, given by

$$p = \left\{ \underline{v} | \underline{v}^T \begin{bmatrix} \underline{C} \\ \underline{C} \underline{A} \\ \vdots \\ \underline{C} \underline{A}^s \end{bmatrix} = \underline{0} \right\} \dots\dots\dots (16)$$

is called the parity space of order s (q being the dimension of the measurement matrix $\underline{C}$). Every vector in $\underline{v}$ can be associated at any time k with a parity check to $r_{(k)}$:

$$r_{(k)} = \underline{v}^T \left[\begin{bmatrix} y_{(k-s)} \\ \vdots \\ y_{(k)} \end{bmatrix} - \underline{H} \begin{bmatrix} \underline{u}_{(k-s)} \\ \vdots \\ \underline{u}_{(k)} \end{bmatrix} \right] \dots\dots\dots (17)$$

$$\underline{H} = \begin{bmatrix} \underline{0} & & \\ \underline{C} \underline{B} & \underline{0} & \underline{0} \\ \underline{C} \underline{A} \underline{B} & \underline{C} \underline{B} & \underline{0} \\ \vdots & & \\ \underline{C} \underline{A}^{s-1} \underline{B} \dots \underline{C} \underline{A} \underline{B} & \underline{C} \underline{B} & \underline{0} \end{bmatrix} \dots\dots\dots (18)$$

It is evident from Eqs. (17) and (18), that a redundancy relation is simply an input-output model for a part of the dynamic system. Robustness can be

achieved by using primarily the relations of the certain part of the model, which can be found by a minimax optimization process.

Another way of explaining the redundancy relations is presented by Frank and Wünnenberg in Patton et al. (1989). It is easily seen that the filter in Eq. (17) has all poles in the origin and can thus be interpreted as a dead-beat observer where the residual is defined as its estimation error.

Various approaches were presented to find the vector $\underline{v}^T$ that is optimally robust with respect to model uncertainties while being sensitive to faults. The latest papers on this subject are by Lou et al. (1986) and Wünnenberg and Frank (1988).

4.5 Unknown input observer approach

Unknown input observers (UIO) (Park and Stein, 1988) are gaining increasing importance in FDI schemes. The freedom in the choice of the observer feedback matrix $\underline{H}$ can systematically be used to make the estimation error invariant to the unknown input signal $\underline{d}$ and sensitive to the fault $\underline{f}$.

System description

It is assumed that the system under consideration can be described by a linear discrete state space model according to Eqs. (19) and (20):

$$\underline{x}_{k+1} = \underline{A} \underline{x}_k + \underline{B} \underline{u}_k + \underline{E} \underline{d}_k + \underline{K} \underline{f}_k, \quad \underline{x}_{k=0} = \underline{x}_0 \dots\dots\dots (19)$$

$$\underline{y}_k = \underline{C} \underline{x}_k + \underline{D} \underline{u}_k + \underline{F} \underline{d}_k + \underline{G} \underline{f}_k \dots\dots\dots (20)$$

$\underline{x}_k$ represents the n -dimensional system state, $\underline{u}_k$ the r -dimensional known input of the system and $\underline{y}_k$ the m -dimensional measurement vector. In addition to the signals $\underline{u}_k$ and $\underline{y}_k$, the matrices $\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{D}$ are assumed to be known. The term $\underline{E} \underline{d}_k$ models uncertainties in the difference equation (19) of the system. These uncertainties may either be parameter uncertainties or parameter variations of the system, neglected nonlinearities, or unknown input signals acting on the system, such as noise or unmeasurable couplings to other parts of a complex dynamic system. It could as well represent a fault that we do not want to affect the residual. The term $\underline{F} \underline{d}_k$ represents uncertainties in the measurements. The input distribution matrices $\underline{E}$ and $\underline{F}$ are assumed to be known. The term $\underline{K} \underline{f}_k$ represents faults that appear in the system, the so-called

component faults including actuator faults. Instrument faults are modelled by the term $\underline{G} \underline{f}_k$. Again, the matrices $\underline{K}$ and $\underline{G}$ are assumed to be known. Notice that no assumption about the appearance or type of the p -dimensional signal $\underline{d}_k$ or the q -dimensional signal $\underline{f}_k$ is made.

The state equations (19) and (20) can be rewritten as

$$\begin{bmatrix} \underline{y}_{k-s} \\ \underline{y}_{k-s+1} \\ \vdots \\ \underline{y}_k \end{bmatrix} = \begin{bmatrix} \underline{C} \\ \underline{C} \underline{A} \\ \vdots \\ \underline{C} \underline{A}^s \end{bmatrix} \underline{x}_{k-s} + \underline{H}_1 \begin{bmatrix} \underline{u}_{k-s} \\ \underline{u}_{k-s+1} \\ \vdots \\ \underline{u}_k \end{bmatrix} + \underline{H}_2 \begin{bmatrix} \underline{d}_{k-s} \\ \underline{d}_{k-s+1} \\ \vdots \\ \underline{d}_k \end{bmatrix} + \underline{H}_3 \begin{bmatrix} \underline{f}_{k-s} \\ \underline{f}_{k-s+1} \\ \vdots \\ \underline{f}_k \end{bmatrix} \quad (21)$$

where s is the considered time horizon, and

$$\underline{H}_1 = \begin{bmatrix} \underline{D} & \underline{0} & & & \\ \underline{C} \underline{B} & \underline{D} & \underline{0} & & \\ \underline{C} \underline{A} \underline{B} & \underline{C} \underline{B} & \underline{D} & \underline{0} & \\ \vdots & & & & \underline{0} \\ \underline{C} \underline{A}^{s-1} \underline{B} & \dots & & \underline{C} \underline{B} & \underline{D} \end{bmatrix} \quad (22)$$

$$\underline{H}_2 = \begin{bmatrix} \underline{F} & \underline{0} & & & \\ \underline{C} \underline{E} & \underline{F} & \underline{0} & & \\ \underline{C} \underline{A} \underline{E} & \underline{C} \underline{E} & \underline{F} & \underline{0} & \\ \vdots & & & & \underline{Q} \\ \underline{C} \underline{A}^{s-1} \underline{E} & \dots & & \underline{C} \underline{E} & \underline{F} \end{bmatrix} \quad (23)$$

and

$$\underline{H}_3 = \begin{bmatrix} \underline{G} & \underline{0} & & & \\ \underline{C} \underline{K} & \underline{G} & & \underline{0} & \underline{0} \\ \underline{C} \underline{A} \underline{K} & \underline{C} \underline{K} \underline{G} & & \underline{0} & \\ \vdots & & & & \underline{0} \\ \underline{C} \underline{A}^{s-1} \underline{K} & \dots & \underline{C} \underline{K} \underline{G} & & \end{bmatrix} \quad (24)$$

Residual generation

In order to generate the scalar residual r_k one has to check if the above state equations hold for the available input and output data. This is done by calculating Eq. (25) on-line at each sampling time k .

$$r_k = \underline{v}^T \left[\begin{bmatrix} \underline{y}_{k-s} \\ \underline{y}_{k-s+1} \\ \vdots \\ \underline{y}_k \end{bmatrix} - \underline{H}_1 \begin{bmatrix} \underline{u}_{k-s} \\ \underline{u}_{k-s+1} \\ \vdots \\ \underline{u}_k \end{bmatrix} \right] \quad (25)$$

The vector $\underline{v}^T$ that has to be found is called the

residual generator. It has to meet the following requirements. In order to make the residual r_k independent of any initial condition $\underline{x}_{k-s}$, the vector $\underline{v}^T$ has to be chosen so that the following condition is fulfilled:

$$\underline{v}^T \begin{bmatrix} \underline{C} \\ \underline{C} \underline{A} \\ \vdots \\ \underline{C} \underline{A}^s \end{bmatrix} = \underline{0} \quad (26)$$

Furthermore, because the residual has to be affected by the fault, the condition

$$\underline{v}^T \underline{H}_3 \neq \underline{0} \quad (27)$$

must hold. The "ideal" case would be that the residual is not affected by any unknown input vector $\underline{d}_k$. Therefore,

$$\underline{v}^T \underline{H}_2 = \underline{0} \quad (28)$$

would be the "ideal" solution.

Lou et al. (1986) call the space, spanned by the solutions of Eq. (26) the parity space of order s . The choice of s clearly influences the solvability of Eqs. (26)~(28). For the "ideal" solution an upper bound is generally given by

$$s \leq n \quad (29)$$

and for the special case that the measurements are not disturbed ($\underline{F} = \underline{0}$), we have

$$s \leq n - m + 1 \quad (30)$$

as shown in Frank and Wünnenberg (1989), where also a necessary and sufficient condition for the existence of the "ideal" FDF is given. Increasing the order s beyond these bounds does not provide additional freedom in the design of the "ideal" filter.

Optimal time domain approximation

If the "ideal" FDF does not exist, the best that can be done is to find an "optimal" solution in the sense of a certain performance index that contains a measure of the effect of the unknown input vector $\underline{d}_k$ and the effect of the fault $\underline{f}_k$. Because no assumptions or restrictions are made about the history of the disturbance and the fault, the performance index can only take the distribution of the signals into the system into account.

As a proper choice of the performance index we therefore define

$$P = \frac{\|\underline{v}^T \underline{H}_2\|}{\|\underline{v}^T \underline{H}_3\|} \quad (31)$$

$\|\cdot\|$ denotes the Euclidean vector norm. With this definition condition Eq. (26) remains unchanged so that the problem can be reformulated as follows: Find a vector $\underline{w}^T$ such that the performance index

$$P = \frac{\|\underline{w}^T \underline{v}_0 \underline{H}_2\|}{\|\underline{w}^T \underline{v}_0 \underline{H}_3\|} \dots\dots\dots (32)$$

becomes minimal. Here, $\underline{v}_0$ is a base for all possible solutions to Eq. (26).

Hence $\underline{w}^T$ singles out the best vector $\underline{v}^T$ of all possible solutions represented by $\underline{v}_0$. The condition from Eq. (26) guarantees that the residual is neither affected by the initial condition $\underline{x}_{k-s}$ nor by the input signal $\underline{u}_k$. Clearly, the choice of the filter degree s has an influence on an "optimal" solution. The least square character of the index shows that an increase of s generally reduces the value of the performance index. An appropriate value for s has to be selected by the designer. Note that the performance index used here is different from the one used in Lou et al. (1986), which does not guarantee independence of the residual of each system input $\underline{u}_k$. With Eq. (26) it is ensured that the residual generator is an observer, as shown in Frank and Wünnenberg (1989). The only restriction is that the observer poles are equal to zero. The residual generator according to Eq. (25) is therefore a so-called dead-beat observer, and the residual its output estimation error. As also shown in Frank and Wünnenberg (1989) the residual of any observer with arbitrary poles can be generated by low-pass filtering of the residual of a dead-beat observer.

The solution to the optimization problem according to Eq. (31) is derived by a differentiation of the performance index with respect to $\underline{w}^T$. The necessary condition for a solution,

$$\underline{w}^T (\underline{v}_0 \underline{H}_2 \underline{H}_2^T \underline{v}_0^T - P \underline{v}_0 \underline{H}_3 \underline{H}_3^T \underline{v}_0^T) = \underline{0} \dots\dots\dots (33)$$

shows that the problem reduces to a generalized eigenvalue-eigenvector problem. The minimal eigenvalue is the optimal value of the performance index and the corresponding eigenvector is the selector for the "optimal" residual generator $\underline{v}^T$

Example

To illustrate the above idea consider the following academic example. An optimal FDF for the system

$$\underline{x}_{k+1} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \underline{x}_k + \begin{bmatrix} 1 & 0 \\ 0 & c \end{bmatrix} \underline{d}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \underline{f}_k \dots\dots (34)$$

$$\underline{y}_k = \underline{x}_k \dots\dots\dots (35)$$

is to be designed. A base $\underline{v}_0$ for all solutions to Eq. (8) is

$$\underline{v}_0 = \begin{bmatrix} a & 0 & -1 & 0 \\ 0 & b & 0 & -1 \end{bmatrix} \dots\dots\dots (36)$$

and the eigenvalue-eigenvector problem is then described by

$$\underline{w}^T \left[\begin{bmatrix} 1 & 0 \\ 0 & c^2 \end{bmatrix} - P \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right] = \underline{0} \dots\dots\dots (37)$$

The resulting eigenvalues to the problem are

$$P_1 = \frac{c^2}{c^2 + 1} \quad \text{and} \quad P_2 = \infty \dots\dots\dots (38)$$

The first eigenvalue shows that the filter can only have a limited performance. The eigenvalue P_2 shows that there exists a residual generator for the system that has a finite sensitivity with respect to disturbances $\underline{d}_k$ but zero sensitivity with respect to faults. The eigenvector corresponding to P_1 is

$$\underline{W}_1^T = [c^2, 1] \dots\dots\dots (39)$$

and the residual generation is described by the equation

$$\underline{r}_k = [ac^2, b, -c^2, -1] \begin{bmatrix} \underline{y}_{k-1} \\ \underline{y}_k \end{bmatrix} \dots\dots\dots (40)$$

This shows that the FDF checks the validity of the dynamic state equations. The variable c acts like a weighting factor. For $c=0$, the filter uses only the second state equation and for large c mainly the first one, which is in agreement with the results of Lou et al. (1986).

If the filter is rewritten in state space form for $c=0$,

$$\underline{z}_{k+1} = [0, b] \underline{y}_k \dots\dots\dots (41)$$

$$\underline{r}_k = \underline{z}_k + [0, -1] \underline{y}_k \dots\dots\dots (42)$$

it becomes clear that this is what is known as a disturbance decoupled dead-beat observer (Frank and Wünnenberg, 1989). $\underline{r}_k$ is the measurable part of the estimation error that is robust with respect to disturbances.

A different approach to an observer design that yields a decoupling of the effects of $\underline{d}$ and $\underline{f}$ was pursued by Patton et al. (1987). This concept uses an observability criterion to directly assign the eigenvectors and eigenvalues of the observer so that

the residual becomes least sensitive to model uncertainties while remaining sensitive to faults.

It is in the nature of the UIO that it is particularly suited to form robust observer schemes that allow the detection of multiple faults. For this purpose one has to partition the faults into different sets denoted by vectors f_i with corresponding matrices K_i and G_i . Each UIO of the observer scheme is designed to be sensitive to a particular set of faults and invariant to the rest of faults as well as to unknown inputs. This is done to such an extension that a sufficiently structured set of residuals is obtained so that a unique decision on the appearance and isolation of a fault can be made.

This design philosophy of an FDI scheme using the "ideal" UIO turns out to be a unified approach covering a number of special cases that have been treated so far in the literature.

- When the system is considered undisturbed, and simultaneous faults in all sensors are to be detected, the design procedure leads to the dedicated observer scheme (DOS) proposed by Clark (1978).

- When disturbances are considered and only a single sensor fault at a time is to be detected, the design procedure leads to the generalized observer scheme (GOS) as described by Frank (1987).

- When simultaneous component and/or actuator faults are to be detected, one arrives at the FDF as described above.

The general cross connections between the various approaches are explained more closely in Patton et al. (1989) and Frank (1989). The latter reference also outlines an alternative frequency domain approach.

Whereas the applicability of the "ideal" UIO is limited to severe existence conditions, the "optimal" UIO design is always applicable if only the fault has an influence on the system output.

4.6 Multiple hypothesis testing

The idea of FDI by multiple hypothesis test, first addressed by Montgomery et al. (16), is to define a hypothesis for each fault, i. e., H_0 : "no fault", H_1 : "fault in sensor 1", ..., H_q : "fault in sensor q ", and test these hypotheses by using Bayesian decision

Fig. 4. Multiple hypothesis test.

Fig. 5. Duplex-sensor-system with analytical redundancy.

theory. Each hypothesis requires a Kalman filter and a likelihood computation from the innovations, see Figure 4. Hence, a bank of parallel Kalman filters is needed. The erroneous sensor is detected with the aid of m -array hypothesis testing. A moving window of the innovation of each Kalman filter drives a detector that calculates the likelihood ratio for each hypothesis to a possible failure mode.

This FDI scheme can be used for IFD, AFD and CFD. The disadvantage is that for each fault type a Kalman filter is needed and that despite great efforts made for simplification the computational expenditure remains large and assumptions on the fault modes have to be made.

4.7 Combined hardware-software redundancy

Instead of pure software or hardware redundancy, one can also combine both of them in order to achieve fail safe operation after the occurrence of faults. This concept has gained some importance for IFD in aircrafts.

In the most simple case of a *duplex sensor system*, Figure 5, one uses two sets of sensors, each set being supervised by an analytical redundancy FDI scheme. For the latter, any of the analytical IFD

schemes described above can be used. Once a fault in one of the sensor sets is detected, the decision logic provides that the system is further operated with the unfaulted sensor. Using duplicated sensor sets complemented by dedicated observer schemes leads to the quality of a triplex hardware redundancy system (Onken and Stuckenberg, 1979).

4.8 Hierarchical observer schemes for CFD (HOS)

The isolation of component faults requires more insight in the *structure* of the system than the isolation of sensor or actuator faults. Therefore, though well suitable for the alarm task in CFD, IFD schemes are in general not capable of isolating arbitrary faulty components. If no assumption on the fault mode can be made, the logical approach is to decompose the system according to Figure 6 and apply a hierarchical scheme of local observers or Kalman filters as proposed by Janssen and Frank (1984).

The problems of local state observation arises from the couplings among the components. If the couplings are zero or sufficiently small or measurable, then malfunctions in the components affect only the estimates of the corresponding local observers. A unique fault isolation is then possible. However, if there are substantial non-measurable interactions, then the effect of a faults in a compo-

nent propagates to observers of other components, and the observer scheme fails to isolate the fault. What could be isolated is a fault in a complex of coupled components, which has no unknown couplings to other such complexes.

The hierarchical observer scheme provides to divide the whole system into an upper level of complexes of components the couplings of which are all measurable, and a lower level of components with unavailable and therefore unknown couplings. For each of the resulting configurations in both levels a local observer scheme is applied.

The crucial point in the design of a hierarchical observer scheme is to find the coupling network for the observers on the lower level. Several approaches have been suggested in Janssen and Frank (1984) and Hengy and Frank (1986). The best solution is that of using unknown input observers with residuals that have been made invariant to the non-measurable couplings, because then the coupling network in the lower level can be totally avoided. An advantage of HOS is that the order of the local observers is low even when the order of the system is high. Moreover, only local observability is required.

5. Monitoring

The second important step in any fault detection scheme is that of *decision* and *monitoring*. The final goal is to maximize the detection probability while minimizing the false alarm rate. The decision logic operates on decision functions formed from the error signals (residuals) that are generated by the fault detection filters. The decisions can be made either by direct threshold logic or by statistical tests using likelihood functions. Most popular is the general likelihood (GLR) test (Willsky, 1976). While it is always necessary to monitor the time and location of a fault, it is sufficient in many cases to just discover that the fault effect has surpassed a tolerable boundary. In general, however, the decision procedure will include sophisticated deterministic or probabilistic tests with an increasing tendency towards the inclusion of heuristic knowledge of the system (knowledge-based models, see Patton et al., 1989).

Fig. 6. Local observer scheme to detect component faults.

6. Conclusions

This paper has outlined the state of the art in the residual generation for FDI based on analytical models of the process. Emphasis was given to the observer approach with due regard to the robustness to parameter variations of the process. There are a number of encouraging results in the application of model-based FDI schemes to mechanical systems as, for example, in aircrafts or advanced transport systems, and some of the methods have reached a rather high degree of maturity. Yet in those cases where only poor models are available, as, for example in chemical plants, the model-based FDI approach has not yet advanced beyond the stage of research.

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