A Labeling of Projected Patterns in Associative Mappings — Image Retrieval as an Example —

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An associative retrieval for an image database using orthogonal projection operation is studied with respect to our biological knowledge base. The square norm of the residual vector is utilized for the orthogonal projection operation in this trial. Each sequential number is needed to be superimposed over the part of input pattern vector at the memorizing process, which leads to the parabolic trajectory of the residual square norm at the recalling process. Thus one of the reference patterns can be pointed out by the vertex of this parabola. Some experimental simulations are made for confirmation of this system performance using randomized pattern vectors for input images.

Keywords : Associative memory, Image retrieval, Orthogonal projection

1. Introduction

A similarity retrieval in monochrome images can be classified into the type of character-to-image or image-to-image⁽¹⁾. One is represented by keywords, and the other by image itself. The former⁽²⁾ seems to be unpreferable regarding the system flexibility or the nature of visual human interface. Some associative searching algorithms have been reported^{(3)~(6)} in the latter case but the feature extractions were essential to them for indexing the target images. Since extracted features are limited in use, it is effective to the specific image retrieval, but not to the ordinary one⁽⁷⁾. This paper is related to the latter case, and belongs to the type of single layer neural networks with interconnections. It is therefore different from the conventional similarity retrieval as follows: the retrieval output must be unique, the similarity measure as well as the feature extractions can be neglected, and a set of the stored images in database is made of the orthogonally projected matrix data.

Incidentally, a qualitative nature of the associative memory has also been studied and

analyzed by S. Amari⁽⁸⁾ and Y. Uesaka⁽⁹⁾. Some models of auto-associative identification has been introduced as a combined vectors of pictorial patterns and symbolic patterns at the orthogonalization process⁽¹⁰⁾⁽¹¹⁾. However, an application of associative memory towards image retrieval have apparently not been reported to date besides this author⁽¹²⁾.

With these points as background, this study has focussed attention to how the merits of associative encoding can be applied to an image retrieval system. Here, the square norm of the residual vector is utilized under orthogonal projection operation in a linear vector space. The target image among the reference patterns can be estimated by the imperfect key-pattern vector for the vertex lying on the parabolic trajectory of the residual norm. The main point of this paper lies on the functional expansions of the associative memory based on orthogonal projections, which is the labeling scheme in recalling process to indicate the specific location of the target image. Thus we apply this function into the image retrieval system as an example, and examine this scheme in simulations for its validity.

2. Associative mappings

2.1 Orthogonal projection

The set of all linear combinations is called subspace and denoted by L . Let $x^{(k)}$ ($k=1, 2, \dots, m$) be the m -input pattern vectors consisted of n -dimensional column vector: $x^{(k)} = [x_1^{(k)}, x_2^{(k)}, \dots, x_n^{(k)}]^t$, where t represents transpose matrix. Then an arbitrary column vector $x^{(k)}$ can be uniquely decomposed as⁽¹³⁾ $x^{(k)} = \tilde{x}^{(k)} + \hat{x}^{(k)}$, where $\tilde{x}^{(k)} \in L$, $\hat{x}^{(k)} \in L^\perp$, and $L \perp L^\perp$, here, vector $\tilde{x}^{(k)}$ is called orthogonal projection of $x^{(k)}$ on L , and $\hat{x}^{(k)}$ is the residual vector and orthogonal to subspace L . Upon arranging this expression by introducing Q as an orthogonal projection, we have $\tilde{x}^{(k)} = x^{(k)} - \hat{x}^{(k)} = Qx^{(k)}$. Generally, an orthogonal projection Q is recursively created by Gram-Shmidt asymptotic equations⁽¹⁴⁾ as $Q^{(0)} = I$, $Q^{(k)} = Q^{(k-1)} - (\tilde{x}^{(k)} \cdot \tilde{x}^{(k)t}) / \|\tilde{x}^{(k)}\|^2$, where $\|\cdot\|$ and I represent the norm of vector and unit matrix of size $n \times n$, respectively.

Let y , at the recalling process, be an imperfect key vector: $y = [y_1, y_2, \dots, y_n]^t$, the associative recall as an orthogonal projection of y is given by $\hat{y} = (I - Q)y$, where $\hat{y} \in L$. Therefore $\hat{y}$ approximates $\tilde{x}^{(k)}$ in the sense of least square errors, while the residual vector on the subspace of the orthogonal complement is given by $\tilde{y} = Qy$. The square norm of this residual vector is utilized as a similarity measure between key-vector and reference vector.

2.2 Associative identification

The n -dimensional pattern vector is formed for a combination of two subpatterns: the pictorial pattern consisted of $(n-1)$ components and the symbolic pattern of the single component (integer $a^{(k)}$). This symbolic component is set to be a consecutive number to each reference patterns, and is superimposed into the vector $x^{(k)}$ at the memorizing process, expressed as

$$x^{(k)} = [x_1^{(k)}, x_2^{(k)}, \dots, x_{n-1}^{(k)}, a^{(k)}]^t. \quad (1)$$

The memorizing scheme with the component $a^{(k)}$ in a pattern vector $x^{(k)}$ is shown in Fig. 1. Here, the n -th row and the n -th column in matrix Q serve as a submatrix for providing informations of the

$$Q^{(k-1)} - (\tilde{x}^{(k)} \cdot \tilde{x}^{(k)t}) / \|\tilde{x}^{(k)}\|^2$$

Fig. 1. Memorizing scheme.

Fig. 2. Flow of memorizing process.

indexing $a^{(k)}$. This $a^{(k)}$ is numbered consecutively according to the memorizing order, which gives the effective location in data-base at the recalling process. The flow of the memorizing process is shown in Fig. 2.

Likewise, an appropriate integer (β) is superimposed into the key-vector y at the recalling process as

$$y = [y_1, y_2, \dots, y_{n-1}, \beta]^t. \quad (2)$$

Let Q be an orthogonal projection in a matrix notation: $\{q_{ij}\}$, $i, j=1, \dots, n$, and then the residual vector appears as

$$\begin{aligned}
\tilde{y} &= Qy \\
&= \begin{bmatrix} q_{1,1} & \cdots & q_{1,n} \\ \vdots & \cdots & \vdots \\ q_{n-1,1} & \cdots & q_{n-1,n} \\ q_{n,1} & \cdots & q_{n,n} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \\ \beta \end{bmatrix} \\
&= \begin{bmatrix} h_1 + q_{1,n}\beta \\ \vdots \\ h_{n-1} + q_{n-1,n}\beta \\ h_n + q_{n,n}\beta \end{bmatrix} \cdots \cdots \cdots (3)
\end{aligned}$$

where

$$h_i = \sum_{j=1}^{n-1} q_{i,j} y_j. \cdots \cdots \cdots (4)$$

Upon arranging this expression, we get the trajectory of parabola with respect to $\alpha^{(k)}$, and expressed as a square norm :

$$\|\tilde{y}\|^2 = \sum_{i=1}^n (q_{i,n}^2 \beta^2 + 2h_i q_{i,n} \beta + h_i^2). \cdots \cdots \cdots (5)$$

Simply, this square norm can be written in a different expression introducing notations u and v to

$$\|\tilde{y}\|^2 = r(u - \beta)^2 + v, \cdots \cdots \cdots (6)$$

where r indicates its curvature (Appendix 1 gives a definite view on the essential qualities of the curvature), and (u, v) is the coordinate of its vertex. These r, u and v are easily obtained in the followings. The curvature r is

$$r = \sum_{i=1}^n q_{i,n}^2, \cdots \cdots \cdots (7)$$

and the vertex u and v are

$$u = - \sum_{i=1}^n (h_i / q_{i,n}), \cdots \cdots \cdots (8)$$

$$v = \sum_{i=1}^n h_i^2 - \left(\sum_{i=1}^n h_i q_{i,n} \right)^2 / r, \cdots \cdots \cdots (9)$$

respectively. The flow of the recalling process described as above is shown in Fig. 3. Because the curvature r in Eq. (7) is always positive, the parabola in Eq. (6) takes minimum value at the vertex (u, v) . It is clear that $\|\tilde{y}\|^2$ and the value v become numerically equivalent when the superimposed number β falls into the location of u . Calculating r, u , and v according to Eqs(7)~(9) together with any appropriate integer β ($\beta=0$ for instance), yields $\|\tilde{y}\|^2$ in Eq. (6). Which means nothing to pay attention to find some optimal value

Fig. 3. Flow of recalling process.

for β . Incidentally, the vertex of this parabola automatically approximates one of the memorized patterns in the sense of minimum error fittings (Appendix 2).

3. Experiments

Some experimental simulations are made for this system performance using random pattern vectors of 16×16 [pixels/frame]. The amount of pattern vectors are $m=20$ kinds in this trial. Thus

$$\begin{aligned}
n &= 256, \quad k = 1 \sim 20, \\
x^{(k)} &= \{(x_1^{(k)}, \dots, x_{255}^{(k)}, \alpha^{(k)})\} \\
x_1^{(k)}, \dots, x_{255}^{(k)} &= 0 \sim 9, \quad \alpha^{(k)} = k, \\
y &= \{(y_1, \dots, y_{255}, \beta)\} \quad y_1, \dots, y_{255} = 0 \sim 9, \\
&\quad \beta = \text{appropriate integer}.
\end{aligned}$$

The key-patterns used in this experiment are the input patterns with some random noise overridden. The variation of the curvature r in relation to the amount of memorizations is shown in Fig. 4. In this figure, as the amount of memorizing patterns increase the curvature gradually decrease. This tendency had been estimated in the related points between the amount of memorizations and the dimensional size of pattern vectors by T. Kohonen⁽¹⁵⁾. The trajectories of the triggering index β versus square norm of the residual vectors are shown in Fig. 5, where (a), (b) are in the case of noise 0% and 40%, respectively. Without noise and with noise of 40% in Fig. 5, β as a triggering

Fig. 4. Variation of curvature versus amount of memorizations.

Fig. 6. Coordinate (u, v) of vertex versus amount of noise.

(a) without noise, (b) with noise of 40%

Fig. 5. Square norm of residual vector.

index in both cases is initially set to be five. The randomly generated or located noise used here is trimmed off by one-half of the signal level. As we can see in this figure (a), the coordinate of the vertex correctly indicates the location of five with the residual norm of zero. However, the coordinate of the vertex is incorrectly shifted into its location of four with the residual norm of more than twenty in figure (b). The trajectories of the coordinate (u, v) versus the amount of noise are shown in Fig. 6. The curve u is waved locally but globally shifted down from the exact level as the amount of noise

increases, the curve v as well. The reason for the unsmoothly varied of these curves is not obvious in this experiment, but assumed to be the randomness of the noise, the crosstalk among the patterns, and so on.

As a result, the curves as seen in this experiments imply that the residual norm and the curvature give a clue to the similarity measure and the reliability measure, respectively to a certain extent.

4. Results

This study discussed the labeling scheme in recalling process to indicate the specific location of the target image, which is based on the parabolic trajectory of the residual square norm in a linear vector space. The major merits of the proposed algorithm discussed here are (i) no feature extractions, (ii) a simple matrix computation, and (iii) a simple computational recalling, whereas the subjects for the future study include, (iv) reducing the projection matrix size, (v) crosstalk of patterns, (vi) allowable noise margin, and so on.

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Appendix

1. Curvature

An Eq. (6) is simply expressed as $y = r(u - \beta)^2 + v$, where y represent $\|\tilde{y}\|^2$, and the differentiation of y become $y' = 2r(\beta - u)$, $y'' = 2r$. We get

curvature $= y'' / (1 + y'^2)^{3/2} = 2r / \{1 + 4r^2(\beta^2 - 2u\beta + u^2)\}^{3/2}$. Therefore the curvature become $2r$ at the vertex $\beta = u$.

2. Minimum error fittings⁽¹⁵⁾

Let M be an unknown matrix operator, then $y^{(k)} = Mx^{(k)}$, ($i=1, 2, \dots, m$) in a linear mapping operation from $x^{(k)}$ to $y^{(k)}$. Denoting X and Y as:

$$X = [x^{(1)}, x^{(2)}, \dots, x^{(m)}], \text{ and}$$

$$Y = [y^{(1)}, y^{(2)}, \dots, y^{(m)}],$$

we can write $Y = MX$. By employing the pseudo-inverse matrix⁽⁺⁾ of Penrose, the unknown operator $\bar{M}$ is obtained as: $\bar{M} = YX^+$, in the sense of the least square approximation. The sufficient condition to exist an exact solution for appropriate value Y is $X^+X = I$, which means $x^{(k)}$ must be linearly independent.

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