I . Finite Element Applied to EMI/C Problems

Key words

EMC, EMI, Finite Element Method, Crosstalk, Radiation

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1. Introduction

This paper will present a review of the finite element method applied to solutions of problems related to Electromagnetic Interference and Compatibility applications. The general problems related to the Electromagnetic Interference and Compatibility, (EMI/C), are usually divided into susceptibility (or immunity) of the equipment under test (EUT) and electromagnetic radiation from EUT. The numerical techniques, such as finite element method (FEM) can be used to simulate and predict interference parameters before the equipment is in the final design stage, thus minimizing the cost of solving potential emission and susceptibility problems.

The Finite Element Method applied to EMC problems have been an important research topic in the past decade. The necessity of such activities has become increasingly obvious in recent years when the designs were facing more accurate predictions of the EMI/C complex problems for which the analytical techniques could not be applied any more. Numerical methods are chosen on the basis of trade-offs between accuracy, speed and storage requirements and usually allow the transformation of the system of partial differential equations, describing the problem, into a system of linear or differential equations easily solved by computers.

In this review paper a brief description of the method is presented for application to EMI/C problems.

2. Finite Element Approach

In the finite element approach an electromagnetic problem is solved by dividing the studied region into a set of arbitrary shapes, known as finite elements⁽¹⁾. For electrostatic problems which are to be solved in order to evaluate the capacitance matrix of the transmission lines, the general algorithm starts with a functional which has the dimensions of system energy associated with the partial differential equation describing the

two-dimensional potential distribution⁽²⁾⁽³⁾:

$$\frac{\partial^2 \Phi(x, y)}{\partial x^2} + \frac{\partial^2 \Phi(x, y)}{\partial y^2} = 0 \dots \dots \dots (1)$$

with associated Dirichlet and Neumann boundary conditions. The problem is to find the electric potential for an inhomogeneous region, where the permittivity $\epsilon(x, y)$ is a function of position. The appropriate functional to be minimized in this situation is given by⁽¹⁾:

$$F(\Phi) = \frac{1}{2} \int_K \epsilon(x, y) (\nabla \Phi(x, y))^2 dx dy \dots \dots \dots (2)$$

where the finite element region K includes the dielectric substrate and is extended to infinity.

The finite element region K is divided into finite elements of triangular shape. The first-order finite elements used consist of three nodes, 1, 2, 3 with node coordinates (x_i, y_i) , (x_j, y_j) , (x_k, y_k) respectively. Φ_1, Φ_2, Φ_3 represent the associated node potentials. These three nodes define the vertices of the triangle and the potential distribution in each element is approximated by:

$$\Phi(x, y) = \alpha_i(x, y) \Phi_1 + \alpha_j(x, y) \Phi_2 + \alpha_k(x, y) \Phi_3 \dots \dots \dots (3)$$

The shape functions, $\alpha_i(x, y)$, $\alpha_j(x, y)$, $\alpha_k(x, y)$ used in (3) are given by⁽²⁾:

$$\begin{aligned} \alpha_i(x, y) &= \frac{1}{2A} \{ (x_j y_k - x_k y_j) + (y_j - y_k)x \\ &\quad + (x_k - x_j)y \} \\ \alpha_j(x, y) &= \frac{1}{2A} \{ (x_k y_i - x_i y_k) + (y_k - y_i)x \\ &\quad + (x_i - x_k)y \} \\ \alpha_k(x, y) &= \frac{1}{2A} \{ (x_i y_j - x_j y_i) + (y_i - y_j)x + (x_j - x_i)y \} \end{aligned} \dots \dots \dots (4)$$

where $|2A|$ denotes twice the area of the triangle given by:

$$2A = x_j y_k - x_k y_j + x_k y_i - x_i y_k + x_i y_j - x_j y_i \dots \dots \dots (5)$$

The node potentials are used as variational parameters, and after the minimization of the function⁽²⁾, one ends up with a linear system of equations of the form:

$$[S] \underline{\Phi} = \underline{B} \dots \dots \dots (6)$$

where $[S]$ is a symmetrical matrix by virtue of the

selfadjointness of the Laplacian operator describing the potential distribution, and the right-hand side contains information on the boundary conditions.

Once the potential distribution is obtained, the per-unit length capacitance matrix can be calculated.

Calculation of the per-unit length inductance matrix is obtained from the capacitance matrix calculated with the dielectric removed:

$$[L] = \epsilon_0 \mu_0 [C_0]^{-1} \quad (7)$$

In the above expression ϵ_0 and μ_0 are the permittivity and permeability of free space respectively, and $[C_0]$ is the per-unit length capacitance matrix in the absence of the dielectric.

Once the capacitance and inductance matrices are calculated the EMC crosstalk problem can be solved for a system of parallel tracks on Printed Circuit Boards (PCB's) or multiconductor lines⁽²⁾.

The method described above has the disadvantage that it can be applied only for parallel tracks for frequency below 1 GHz (for PCB's, for instance). At higher frequencies, where higher order modes of propagation should be considered the solution by FEM can be obtained by solving the full wave equation associated to the field problem⁽⁴⁾⁻⁽⁶⁾.

The full-wave numerical solution for the crosstalk problems can be formulated by using the above approach based on the nodal finite element. It has been found out that an extension of the nodal FEM approach to a full-wave three-dimensional analysis produces spurious modes. For this reason, the method of the edge-based FEM is preferred as this approach does not generate spurious modes and models very well the sharp edges of the PCB's tracks without the need for any singular trial functions⁽⁷⁾⁽⁸⁾.

In the following, the edge-based FEM is presented briefly for a PCB with a dielectric constant ϵ_r , shielded by a metallic enclosure of infinite conductivity, where the tangential component of the electric field $\bar{E}$ vanishes. The electric field satisfies the inhomogeneous wave equation⁽⁴⁾.

$$\nabla_x \frac{\nabla_x \bar{E}}{\mu_r} = k_0^2 \epsilon_r \bar{E} \quad (8)$$

in which k_0 the free space. Alternatively, the field is the solution of the variational equation

$$\delta F(\bar{E}^*) = 0$$

where

$$F(\bar{E}^*) = \int_v \left[\frac{(\nabla_x \bar{E}) \cdot (\nabla_x \bar{E}^*)}{\mu_r} - k_0^2 \epsilon_r \bar{E} \cdot \bar{E}^* \right] dV \quad (9)$$

In the above equations ϵ_r , μ_r are relative dielectric

constant and magnetic permeability of the dielectric enclosed in the volume v and k_0 is the propagation constant in free space. The tracks inside v can have any arbitrary geometrical configuration. The boundary conditions related to the electric field are the tangential component of the electric field vanishes at any point located on the metallic track or enclosure. The field is generated by voltages applied between tracks and ground planes.

The functional (9) is discretized into tetrahedral elements where the unknowns are the tangential components of the electric field along each edge. The field in each element is expressed as:

$$\bar{E} = \sum_{i,j} e_{ij} \bar{W}_{ij} \quad (10)$$

where, $\bar{W}_{ij}$ is the Whitney function defined by⁽⁸⁾ and e_{ij} are proportional to the tangential components of the electric field along edges E_i of the tetrahedral finite element. The edge elements discretization described converts the functional into the matrix form⁽⁴⁾:

$$F = [E^*]^T ([S] - k_0^2 [T]) [E_t] \quad (11)$$

and finally this can be reduced to a solution of a system of linear equations where the unknowns are the tangential components of the electric field on the edges of tetrahedral finite elements. The field and other related parameters such as voltages developed between tracks and ground to evaluate the crosstalk can be now calculated from (10).

The method described above has been successfully applied to evaluate crosstalk in shielded printed circuit boards and the theoretical predictions have been compared favorably with measurements.

It should be noticed that the application of FEM to some complicated electromagnetic problems follows a similar approach, but the computations are more complex. Applications of FEM to radiation from unintentional antennas are described in⁽¹⁰⁾⁽¹¹⁾.

The finite element can also be applied to study the electromagnetic radiation from a VLSI package heat-sink. A heatsink fastened to the IC package can be effective in eliminating performance degrading thermal effects, however its presence will also alter the pattern of the electromagnetic radiation. The finite element algorithm solves the magnetic field distribution about an axisymmetric model of an integrated circuit mounted onto a heatsink⁽¹¹⁾. The configurations are greatly simplified and an equivalent voltage source is used to excite the antenna consisting of the chip and the heat-sink. A radiation boundary condition allows the mesh to be truncated close to the heatsink.

The paper by Lee et al⁽⁷⁾ addressed the effects of a

heatsink mounted over a chip package on the electromagnetic emissions. Simulations done by Lee et al with finite difference time domain (FD-TD) technique to compute the radiated power showed the occurrence of resonances in the low gigahertz range for heatsinks of typical dimensions.

The finite element method can be applied in the frequency domain to solve the magnetic field distribution about a configuration where the heatsink penetrates the substrate and the VLSI chip is mounted on top. The finite element formulation assumes a problem with cylindrical symmetry and perfect conductors, however it can include losses in the dielectric. A radiation boundary condition is used to bound the solution domain, yet still preserve the effects of the far-field wave phenomenon.

VLSI packages are typically rectangular, however to simplify the simulation, the package is assumed to have cylindrical symmetry about the z axis.

The VLSI package is modeled as a small slab of dielectric material with the top surface being perfectly conducting. The substrate is modeled as a dielectric and has a ground plane which can either be on the bottom or on the top side. The ground plane and the dielectric layer both extend infinitely in the radial direction. The effects of any signal tracks or other nearby packages are neglected. In the absence of any heatsink, the chip sits directly on the substrate. The problem is excited with a voltage source at the center of the chip in the gap between the top surface or the chip and the ground plane.

The finite element formulation including the radiation boundary condition is presented in (10) (11) and is briefly described below.

The equation governing the magnetic field is

$$-\frac{\partial}{\partial r}\left(\frac{1}{\sigma+j\omega\epsilon}\frac{\partial rH_\phi}{\partial r}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\sigma+j\omega\epsilon}\frac{\partial rH_\phi}{\partial z}\right)+\frac{j\omega\mu}{r}rH_\phi=0. \quad (12)$$

The radiation boundary condition is based on the assumption that

$$E_{tan}=-\frac{\eta}{r}rH_\phi \quad (13)$$

on a surface sufficiently far removed from sources, where η is the complex intrinsic impedance of the medium.

Using the finite element and Galerkin formulation, equation (12) can be discretized by first assuming the trial solution for H_ϕ for the k th element of the domain of the problem as,

$$H_\phi^{(k)} \approx \sum_{j=1}^n u_j N_j^{(k)}(r, z) = U^{(k)}(r, z) \quad (14)$$

where u_j are unknown complex constants, N_j are realvalued FEM interpolation functions and n is the number of nodes defining the k th element.

By Galerkin formulation, expansion and testing of equation (12) using (14) yields,

$$\iint_k \left\{ -\frac{\partial}{\partial r} \left(\frac{Y}{r} \frac{\partial}{\partial r} (rU^{(k)}) \right) - Y \frac{\partial}{\partial z} \left(\frac{\partial U^{(k)}}{\partial z} \right) + ZU^{(k)} \right\} N_i^{(k)} dr dz = 0 \quad (15)$$

for $i=1, 2, \dots, n$, $Y=(\sigma+j\omega\epsilon)$ and $Z=j\omega\mu$.

As a result of implementing the radiation boundary condition in (15), a final system of linear complex equations is obtained. Solving the system of equations with respect to the unknown u_j we can predict the near and far-field radiated by heatsinks mounted on printed circuit boards.

3. Hybrid Methods

The finite element method can be used to predict field distributions related to the EMC/I problems. However, where the stucture to be analyzed becomes too complex, a direct application of nodal or edge-based finite element will generate a very large number of unknowns which cannot be handled by average-size computers. In this case, the finite element method can be combined with analytical techniques. The hybrid method which is a combination of numerical techniques with analytical techniques can simplify the mathematical formulation and reduce the time computation. The reference ⁽⁴⁾ illustrates the EMC problems modeled by hybrid methods.

4. Conclusions

This review paper has provided a brief description of the Finite Element Method applied to EMI/C. The finite element method requires considerable computation time, large storage and is very versatile, in comparison with the method of moments, which requires less storage and computer times, but is less versatile.

It seems that the future in numerical techniques belongs to the hybrid techniques which are able to accommodate complex geometry and reasonable computation time.

An excellent reference for other applications of the numerical method applied to EMC is ⁽¹²⁾.

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