A New Type of Robust Tracking SP-D Control for Manipulators

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This paper is concerned with a new type of robust tracking control scheme for robot manipulators using SP-D (Saturated Proportional and Differential) feedback loops. In this paper, the control scheme based on quasi-natural potential is generalized from set-point regulation to trajectory tracking by exploiting an effective auxiliary variable which consists of a saturated positional error and a velocity error. Without any uncertainties in the manipulator model, the generalized tracking SP-D control scheme ensures globally asymptotic stability. In the presence of parametric model uncertainties, the tracking SP-D control scheme is improved in order to be robust against the uncertainties. The proposed robust tracking SP-D controller is able to guarantee globally uniformly ultimate boundedness theoretically. Experimental results on a two-link direct drive manipulator with the robust tracking controller are in agreement with the theoretical analysis and show the effectiveness of the control scheme.

Key words: saturated proportional and differential control, trajectory tracking, robust control, manipulator

1. Introduction

A robot control scheme based on the principle of energy conservation, such as the Lyapunov-based control and the passivity-based control, has attracted much attention(1). This control scheme tackles robot control problems by exploiting physical structure of manipulators. Contrary to the inverse dynamics control scheme, the control scheme based on the principle of energy conservation is expected to be more robust, since there is no exact cancellation of the nonlinearities.

In recent years, Arimoto(2)(3) has introduced a new concept of quasi-natural potential that derives the SP-D (Saturated Proportional and Differential) feedback loops. The SP-D control scheme causes global asymptotic stability of set-point control, provided the gravity term is carefully compensated. Furthermore, it is effective for a new interpretation of the robot dynamics via the circuit theory(3)(4).

The control scheme based on the quasi-natural potential, however, has not been sufficiently studied in the case of trajectory tracking. Moreover, the exact robot parameters cannot be obtained in general since the model uncertainties inevitably exist in the robot dynamics. Hence, we need to account for tracking control and robustness with respect to the uncertainties.

In this paper, a new type of robust tracking control scheme based on the SP-D feedback is considered. With this new method, globally uniformly ultimate boundedness(1) of the tracking errors is achieved against the model uncertainties. Further, the proposed control scheme has been implemented on a two-link direct drive manipulator(5) in Fig.1 and the effectiveness of the control scheme is evaluated.

First, a new effective auxiliary variable is introduced by refining the former idea(6). Through the use of the new auxiliary variable, we generalize the SP-D control scheme for trajectory tracking problems. Second, the generalized tracking SP-D control scheme is extended to be robust against the uncertainties by exploiting the model representation(7)(8). Finally, the experimental results using the manipulator with payload on the tip show the effectiveness of the proposed robust control scheme.

This paper is organized as follows. In Section 2, the SP-D control scheme based on quasi-natural potential is reviewed in the case of set-point control. Section 3 introduces the effective auxiliary variable and discusses how to generalize the SP-D control scheme from set-point regulation to tracking control. The generalized tracking control scheme is improved to be robust against the model uncertainties in Section 4. Experimental results for confirming the effectiveness and some discussions detected in the experiments are shown in Section 5. Finally, we offer brief concluding remarks in Section 6.

2. Previous Set-Point SP-D Control

Consider the Lagrange's equations of a rigid manipulator with \( n \) (\( n \leq 6 \)) revolitional joints. Without considering the actuator dynamics, it is well known(1) that the manipulator
dynamics can be described as follows:

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + f(q) = \tau = Y(q, \dot{q}, \dot{\theta}) \]  

(1)

where \( q := [q_1, \ldots, q_n]^T \) and \( \tau := [\tau_1, \ldots, \tau_n]^T \) denote a joint position vector and a joint torque vector, respectively. The inertia matrix \( M(q) \in \mathbb{R}^{n \times n} \) is symmetric and positive definite, and \( C(q, \dot{q}) \dot{q} \in \mathbb{R}^n \) represents the Coriolis and centrifugal force term. It is worth mentioning that \( M(q) - 2C(q, \dot{q}) \) is skew-symmetric. The terms of \( g(q) \in \mathbb{R}^n \) and \( f(q) \in \mathbb{R}^n \) correspond to the gravitational force term and the frictional force term, respectively. The frictional force term \( f(q) \) is composed of viscous and Coulomb frictions in this paper. It should be noted that, in the robot dynamics (1), the linearity exists between a properly defined parameter vector \( \theta \in \mathbb{R}^m \) and the regressor matrix \( Y(q, \dot{q}, \dot{\theta}) \in \mathbb{R}^{n \times m} \).

Angular position vector \( \theta \) and velocity vector \( \dot{\theta} \) are assumed to be available for measurements in this paper.

The set-point SP-D control scheme proposed by Arimoto(2) is written as

\[ \tau_i := -k_p \sin(q_i - q_{di}) - k_v \dot{q}_i, \quad i = 1, \ldots, n \]  

(2)

where \( q_{di} \) denotes a desired position of the \( i \)-th joint with positive gains \( k_p \) and \( k_v \). For the set-point control problem, each \( q_{di}, i = 1, \ldots, n \) is constant. The function \( \sin(\cdot) \) in the right-hand side of (2) is defined as follows:

\[ \sin(\alpha) := \begin{cases} 1, & \alpha \geq \frac{\pi}{2} \\ \sin(\alpha), & |\alpha| < \frac{\pi}{2} \\ -1, & \alpha \leq -\frac{\pi}{2} \end{cases} \]  

(3)

where \( \alpha \) denotes a scalar. The above \( \sin(\cdot) \) function and the following \( \cos(\cdot) \) function are referred to as saturated trigonometric functions.

\[ \cos(\alpha) := \begin{cases} -\alpha + \frac{\pi}{2}, & \alpha \geq \frac{\pi}{2} \\ \cos(\alpha), & |\alpha| < \frac{\pi}{2} \\ \alpha + \frac{\pi}{2}, & \alpha \leq -\frac{\pi}{2} \end{cases} \]  

(4)

The saturated trigonometric function is continuous and has the following properties

\[ \frac{d}{d\alpha} \cos(\alpha) = -\sin(\alpha), \quad 1 - \cos(\alpha) > 0 \quad (\forall \alpha \neq 0). \]  

(5)

Note that \( 1 - \cos(\alpha) \) is unbounded with respect to \( \alpha \). These properties play a key role for stability analysis in Section 3 and Section 4.

Consider some robotic applications, such as welding, assembling and so on, tracking control is significantly required in practice. The tracking control based on the SP-D feedback, however, has not been studied sufficiently yet. Furthermore, the manipulator model used in the robot controller contains uncertainties in most practical cases. Hence, we will generalize the SP-D control scheme from set-point regulation to tracking control, then improve it to be robust against the model uncertainties.

3. Generalized SP-D Control Scheme for Trajectory Tracking

In the previous section, we reviewed the set-point SP-D control scheme. Exploitation of a new auxiliary variable allows a trajectory tracking control based on the SP-D control method. Throughout this section, we assume that the physical parameters of the manipulator are known completely.

<3.1> Auxiliary Variable

In contrast to the set-point control problem, let \( \dot{q}_d := [\dot{q}_{d1}, \ldots, \dot{q}_{dn}]^T \) stand for time-varying continuous reference trajectories which are twice differentiable functions for \( t \in [0, \infty) \). The positional and velocity errors are defined as \( e := q - q_d \in \mathbb{R}^n \) and \( \dot{e} := \dot{q} - \dot{q}_d \in \mathbb{R}^n \), respectively. Now, we define an effective auxiliary variable \( s \) as

\[ s := \dot{e} + A\varphi(e) \]  

(7)

where \( A \) denotes a diagonal matrix which has positive elements \( \lambda_i, \quad i = 1, \ldots, n \).

\[ \varphi(e) := [\sin(e_1), \ldots, \sin(e_n)]^T \]  

(8)

represents a vector of positional errors saturated by the function (3). According to the approach(6), the following reference trajectory \( q_r \) is sequentially defined as

\[ q_r := q_d - A\int_0^t \varphi(e) dt \]  

(9)

Differentiating \( q_r \) with respect to time, we obtain

\[ \dot{q}_r = \dot{q}_d - A\varphi(e). \]  

(10)

It should be noted that the equation (10) has an important relation to the auxiliary variable \( s \); i.e.,

\[ s = \dot{q} - \dot{q}_r. \]  

(11)

Remark 3.1 The auxiliary variable \( s \) is fundamental and important in this paper. It should be pointed out that the variable \( s \) in the definition (7) is simplified to the sliding variable(6) by replacing \( \varphi(e) \) with just \( e \). Similarly, the reference trajectory (9) is reduced to the virtual reference trajectory(6).

<3.2> Tracking SP-D Control Law

We propose a tracking SP-D control law for the manipulator dynamics (1) as

\[ \tau := M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + f(q) - K\dot{s} \]  

(12)

where \( K \) is a diagonal gain matrix with all positive elements \( k_i, i = 1, \ldots, n \). The reference acceleration \( \ddot{q}_r \) includes the time derivations of \( \sin(e_i) \) described as

\[ \frac{d}{dt} \sin(e_i) = \begin{cases} e_i \cos(e_i), & e_i \leq \frac{\pi}{2} \\ 0, & |e_i| > \frac{\pi}{2} \end{cases} \quad i = 1, \ldots, n. \]

Substituting (12) into (1) and taking the auxiliary variable \( s \) into consideration, we obtain the following closed-loop system

\[ M(q)\ddot{s} + C(q, \dot{q})\dot{s} + Ks = 0. \]  

(13)

Theorem 3.1 Consider the robot dynamics (1) in the closed loop with the proposed control law (12). The closed loop system (13) is globally asymptotic stable with respect to \( e \) and \( \dot{e} \), i.e., \( e, \dot{e} \to 0 \) as \( t \to \infty \).
Proof: Consider the following scalar function \( V \) as a Lyapunov function candidate:

\[
V := \frac{1}{2} s^T M(q) s + \sum_{i=1}^{n} k_i \lambda_i (1 - \cos(\epsilon_i)) \quad \ldots \quad (14)
\]

The function \( V \) is continuous, differentiable and positive definite with respect to \( e \) and \( \dot{e} \) via the tracking error relation (7) and the properties (5) and (6) mentioned in Section 2. Furthermore, \( V \) is radially unbounded with respect to \( e \) and \( \dot{e} \).

Differentiating \( V \) along the solution trajectory of (13) gives

\[
\dot{V} = -s^T K s + \frac{1}{2} s^T (M - 2C) s + 2\epsilon^T K \varphi(e).
\]

With the skew symmetry of \( M - 2C \) and substituting (7) for \( s \)

\[
\dot{V} = -e^T K e - \varphi(e)^T (A^T K A) \varphi(e)
\]

where \( A^T K = AK \). The derived function \( \dot{V} \) is negative definite with respect to \( e \) and \( \dot{e} \) since \( K \) and \( A^T K A \) are positive definite. Due to the definition of the saturated function (3), \( \dot{V} \leq 0 \) is satisfied with respect to \( e \) and \( e \). By using the Lyapunov direct method\(^{(1)} \), we see that \( e, \dot{e} \to 0 \) as \( t \to \infty \).

Remark 3.2 The second term in the right-hand side of (14) denotes the quasi-natural potential\(^{(2)} \). This potential term is useful for the tracking SP-D control as well as the set-point SP-D regulation. If \( Y(q, \dot{q}, q_s, \dot{q}_s) \theta \) is dropped from the proposed control law (12), it is corresponding to the set-point SP-D control law\(^{(3)} \). This natural generalization makes the trajectory tracking control possible. The generalized tracking SP-D control scheme is referred to as GSP-D control scheme in this paper.

4. Robust Tracking SP-D Control

In Section 3, we have assumed that there are no uncertainties in the manipulator dynamics (1). However, the model uncertainties inevitably exist in practice. In this section, we improve the GSP-D control scheme proposed in the previous section in order to be robust against the parametric uncertainty. Then, we show its globally uniformly ultimate boundedness\(^{(1)} \).

<4.1> Manipulator Model with Uncertainty

Though there are some types of uncertainties in the manipulator dynamics, we focus on the parametric uncertainty among them. With respect to this type of the uncertainty, a nominal manipulator model is defined as follows\(^{(7)} \):

\[
\dot{q} = C(q, \dot{q}) \dot{q} + \dot{f}(q) = Y(q, \dot{q}, \dot{q}) \dot{\theta} \quad \ldots \quad (15)
\]

where the symbol \( \dot{\theta} \) denotes the nominal value, and the symbol \( \dot{\theta} \) denotes the uncertainty, i.e.,

\[
\dot{\theta} = \delta \dot{\theta}, \quad \delta \dot{\theta} = \delta \dot{q} + \dot{\delta}(q, \dot{q}) \quad \ldots \quad (16)
\]

where \( \delta \dot{\theta} \) denotes the uncertainty of the real parameter \( \theta \).

In practical control, the representation of the model uncertainty is crucial. This description has the advantage that the bound of the model uncertainty is represented simply and tractably as follows:

\[
|\delta \dot{\theta}| \leq \rho \quad \ldots \quad (17)
\]

where \( \rho \) is a positive constant.

<4.2> Robust Tracking SP-D Control Law

Assume that none of the robot parameters are exactly known though the bounds of the model uncertainty \( \rho \) are known. Under the above assumptions, we consider the following RSP-D (Robust SP-D) control law

\[
\tau := M(q) \dot{q} + \dot{C}(q, \dot{q}) \dot{q} + \dot{f}(q) + Y(q, \dot{q}, \dot{q}) \dot{\theta} - K s + Y(q, \dot{q}, \dot{q}) \dot{\theta} \quad \ldots \quad (18)
\]

Without \( Y(q, \dot{q}, \dot{q}) \dot{\theta} \) from (18), the above control law is corresponding to the nominal control law (12) proposed in Section 3. The i-th element of the additional control input vector \( u \) is described as

\[
u_i := \begin{cases} -\rho \dot{\theta}_i, & |\dot{\theta}_i| > \epsilon_i \\ \frac{\rho}{\epsilon_i} \dot{\theta}_i, & |\dot{\theta}_i| \leq \epsilon_i \end{cases} \quad \ldots \quad (19)
\]

where \( \dot{\theta}_i \) denotes the i-th component of the vector \( Y(q, \dot{q}, \dot{q}) \dot{\theta} \) and \( \epsilon_i (>0) \) are suitably chosen parameters. (See Remark 4.1.) Fig 2 shows the block diagram of the proposed RSP-D control scheme. If the additional input vector \( u \) in Fig 2 is eliminated, the RSP-D control law (18) and (19) is reduced to the previous GSP-D control law (12).

Substituting (18) and (19) into the manipulator dynamics (1), the closed-loop system is derived as follows:

\[
M(q) \ddot{q} + (C(q, \dot{q}) + K) s = Y(q, \dot{q}, \dot{q}) (\dot{\theta} + u) \quad \ldots \quad (20)
\]

In contrast to the closed loop system (13) in the nominal case, the above closed loop system has the term \( Y(q, \dot{q}, \dot{q}) (\dot{\theta} + u) \) in the right-hand side. It should be noted that \( Y(q, \dot{q}, \dot{q}, \dot{\theta}) \dot{\theta} \) stands for the disturbance term caused by the model uncertainty, which generates an undesirable perturbation. The additional input vector \( u \) can play a
The key role of reducing this undesirable perturbation. Here, we show the following main result of this paper.

**Theorem 4.1** Consider the closed loop system (20). Assume that \( p_i \) are known, and let \( e_i > 0 \). If \( u_i \) are chosen according to (19), then the RSP-D control law (18) with (19) makes the closed loop system **globally uniformly ultimately bounded**.

**Proof:** Consider a Lyapunov function candidate for the closed-loop system (20) as follows:

\[
W := \frac{1}{2} x^T M(q) x + 2 \sum_{i=1}^{n} k_i \lambda_i \{1 - \cos(e_i)\}. \tag{21}
\]

The above function \( W \) is the same as \( V \) defined in (14). Recalling stability analysis in the subsection 3.2, \( W \) is continuous, differentiable, positive definite and radially unbounded with respect to \( e \) and \( e \). The time derivation of \( W \) along the solution trajectory of the closed-loop system (20) yields

\[
\dot{W} \leq -x^T Q x + \sum_{i=1}^{n} \frac{\xi_i^2}{\rho_i} + \sum_{i=1}^{m} \xi_i u_i.
\]

If \( |\xi_i| > \epsilon_i \) are satisfied for all \( i \), the additional control inputs \( u_i \) of (19) yield

\[
W \leq -x^T Q x. \tag{22}
\]

The system error vector \( x \) is composed of \( e \) and \( \varphi(e) \). Hence, \( W \) is negative definite with respect to \( e \). Suppose that \( \xi_i (i = 1, \ldots, r, r \leq m) \) satisfy \( |\xi_i| \leq \epsilon_i \). By using the additional inputs (19) with the uncertainty bounds \( p_i \), we obtain

\[
\dot{W} \leq -x^T Q x + \sum_{i=1}^{n} \frac{\xi_i^2}{\rho_i} + \sum_{i=1}^{m} \xi_i u_i.
\]

If \( |\xi_i| > \epsilon_i \) are satisfied for all \( i \), the additional control inputs \( u_i \) of (19) yield

\[
W \leq -x^T Q x. \tag{22}
\]

The physical parameters \( B_i \) are estimated experimentally by using a practical identification method. Table 1 shows the values of the estimated parameters with no load and with payload (6 kg) attached to the manipulator. The values of

<table>
<thead>
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<th>Table 1. Physical Parameters</th>
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<tr>
<td>( \epsilon_1 )</td>
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<tr>
<td>0.0000E+0</td>
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5. **Experiments**

In this section, we present experimental results on the two-link planar direct drive manipulator shown in Fig.1. The purpose of the experiments is to verify

- **globally uniformly ultimately boundedness**
- **robustness against the parametric uncertainty of the manipulator system with the proposed RSP-D controller.**

In order to verify the effectiveness of the proposed RSP-D control scheme, experimental results are evaluated by the tracking error \( E := [e^T, e^T]^T \), the root-mean-squared error RMS(\( E \)) and the Euclidean norm of \( E \).
the estimated parameters with no load are utilized for the nominal physical parameters $\hat{\theta}$. The other values of the parameters with payload are used as the real physical parameters $\theta$. Thus, the parametric uncertainties $\hat{\theta}$ are calculated according to the equation (16), and the uncertainty bounds $\rho_i$ satisfying (17) are defined in Table 2.

The reference trajectory affects the effectiveness of the proposed RSP-D controller. We choose a typical reference trajectory shown in Fig.3 for each joint. To be positive and diagonal, the control gains $K = \text{diag}[2.0, 2.0]$, $A = \text{diag}[50.0, 30.0]$ are selected, and all uncertainty boundary layer parameters $\epsilon_i$ are set 0.05. Therefore, the uniformly ultimately bound $\omega = 1.6710e^{-2}$ is calculated by these related parameters.

### Experimental Results

In order to evaluate the effectiveness of the proposed RSP-D control scheme, experiments are performed with payload (6 kg) on the tip of the manipulator. By evaluating the value of the root-mean-squared error $\text{RMS}(E)$ and by comparing Euclidean norms of each positional and velocity error signal, we shall confirm the effectiveness of the RSP-D controller.

Fig.4 shows the $\text{RMS}(E)$ of the RSP-D controller at each time when the payload is attached on the tip of the manipulator. A solid curve denotes the $\text{RMS}(E)$ at each time. A dotted line represents the uniformly ultimately bound $\omega$. The experimental result in Fig.4 shows that the root-mean-squared error is decreasing, then remains within the uniformly ultimately bound $\omega$ after 1.2 s. This result is in agreement with globally uniformly ultimately boundedness of the manipulator system based on the RSP-D control scheme.

Fig.5 and Fig.6 show the positional and velocity errors using the RSP-D and GSP-D control schemes, respectively. A solid curve and a dotted curve denote the results using the RSP-D and GSP-D control scheme, respectively. The experimental results, especially the positional errors in Fig.5, show that the tracking error using the RSP-D controller is smaller than that using the GSP-D controller. Euclidean norms of each positional and velocity error signal are shown in Table 3. Thus, the RSP-D control scheme is verified to be more robust against the parametric uncertainty caused by the payload than the GSP-D control scheme.

### Table 2. Uncertainty Bounds

<table>
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<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
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<tr>
<td>1.7014E-4</td>
<td>5.2237E-4</td>
<td>1.9960E-4</td>
<td>9.1938E-4</td>
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<tr>
<td>8.4516E-3</td>
<td>2.2991E-4</td>
<td>1.0942E-3</td>
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### Table 3. Euclidean Norm of Tracking Errors

<table>
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<th>Control Scheme</th>
<th>RSP-D</th>
<th>GSP-D</th>
</tr>
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<tbody>
<tr>
<td>Positional Error</td>
<td>2.9239E-1</td>
<td>2.4463E-1</td>
</tr>
<tr>
<td>Velocity Error</td>
<td>1.9977</td>
<td>2.8413</td>
</tr>
</tbody>
</table>
parameters $\varepsilon_i$, that is, the bound $\omega$ could be sufficiently small. In other words, the tracking errors could be sufficiently small theoretically. In these experiments, we see that the proposed RSP-D controller with the suitable gains $K$, $A$ and the suitable uncertainty boundary layer parameters $\varepsilon_i$ could achieve the satisfactory tracking errors. If the higher tracking performance is required, the following problems should be taken into consideration.

- The proposed controllers are all designed in continuous time based on the ideal Lagrangian model (1) and the theoretical proofs of convergence are valid only under these assumptions. In practice, the controllers are implemented on digital controllers with certain values of sampling periods. Furthermore, the velocity signals obtained by difference of the positional signals with the above sampling periods contain the measurement noise. Thus, the maximum value of the feedback gains is limited in practice since the high feedback gain causes a chattering phenomenon.

- In this paper, we deal with only the parametric uncertainty, and consider the robust control scheme against the uncertainty. However, there exist another type uncertainties in the real manipulator system, such as unmodeled dynamics and disturbance. For instance, the dynamics model described the equation (1) is short of an actuator dynamics. We should consider these uncertainties in order to reduce the tracking error.

6. Conclusions

By exploiting the effective auxiliary variable $s$, the SP-D control scheme was generalized from set-point regulation to tracking control. Furthermore, the generalized tracking SP-D control scheme was improved to be robust against the parametric uncertainty of the manipulator dynamics, and was proved to ensure globally uniformly ultimate boundedness. The effectiveness of the proposed RSP-D control scheme was confirmed by the experiments on the SICE standard manipulator.

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