Measurements and Electrical Equivalent Model of Polymer PTC as a Function of Joule Heating Energy

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In this paper, experiments were conducted to study the electrical $I - V$ characteristics of the polymer Positive Temperature Coefficient (PTC) resistor as a function of joule heating due to I^2R . More than 80 short-circuit tests were carried out on four samples (rated 60 V/40 A), and the results show that all PTC samples tripped when the threshold input energy nearly equaled 20 J. We propose a new mathematical model for the PTC in the fault current condition, a PTC-TACS (Transient Analysis of Control Systems) model, by using the Electro-Magnetic Transient Program (EMTP), which is a function of the input joule heating energy. A comparison between the experimental results and EMTP simulation results has shown that the PTC-TACS model is valid and very effective to investigate the PTC under fault current conditions in electrical circuits and to design an over-current limiter based on PTC materials for industrial applications. In the discussion about radiation power loss in case that the applied voltage is greater than 50 V or the tripped time is less than 4 ms, it is shown that the radiation power loss can be neglected and the PTC resistance can be expressed only by the input joule heating energy.

Keywords: polymer PTC, joule heating, short circuit test, EMTP, PTC-TACS model, radiation power loss, trip time

1. Introduction

The PTC is one of the important solid state materials designed to interrupt the fault current and to function like a reset-table fuse. The PTC resistors based on polymer composites consist of conducting particles dispersed in a thermoplastic or duromeric matrix⁽¹⁾⁽²⁾. The PTC resistor has the ability to increase its resistance with increasing temperature⁽³⁾.

At normal operation where PTC carries less than hold current I_{hold} , the internal joule heating power is small and balanced with a radiation power loss. If a short circuit or an overload current occurs in the circuit⁽¹⁾, the increased joule heating power will raise the internal temperature of the PTC and results in a non-linear increase in resistance of the PTC over a narrow temperature range.

In the commercial world, the manufacturing techniques allow to design a variation of the PTC materials⁽⁴⁾⁽⁵⁾ (e.g, V_2O_3 , $BaTiO_3$), as well as many new developing areas⁽⁴⁾ of PTC nonlinear effects of temperature dependent resistance $R(T)$ and voltage dependent current $I(V)$ behaviors.

Until today, low current rating and limited voltage do not allow the PTC to be used as an over-current protector in high power devices. The PTC properties have been studied and reported in the literature, where some technical attempts have been made to introduce PTC resistance as a function of temperature under specific

Fig. 1. The PTC resistor element used in this work

applications⁽³⁾⁻⁽⁶⁾. However, the $I - V$ characteristics and PTC resistance in an electrical model for the electric circuit calculations have not been studied in detail.

For a system using several PTC elements to expand rated voltage and current, connected in series and in parallel, an electrical equivalent model of the PTC is very useful to analyze its behavior as a function of joule heating energy of used elements. In this paper, we propose a simple electrical equivalent model of the PTC which is described only with circuit variables and easy to use for circuit calculating program such as SPICE⁽⁷⁾ (Simulation Program with Integrated Circuit Emphasis) and EMTP.

2. Experimental Setup

Figure 1 shows a photograph of the PTC (Type RXE 375) used in this study. Four samples were used and the electrical properties⁽²⁾ are listed in Table 1.

To measure the transient electrical properties of the

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Fig. 2. Experiment circuit for short circuit tests

Table 1. Specifications of polymer PTC resistor (BaTiO₃) rated 60 V/40 A

$V_{max}(V)$	$I_{hold}(A)$	$I_{trip}(A)$	$I_{max}(A)$	$R_{min}(\Omega)$
60	3.75	7.5	40	0.05

PTC under the fault current condition and to compare it with the proposed model, we used the experimental setup in Fig. 2.

After the capacitor C was charged to the specific value $V_{c0} = 60$ V, the thyristor switch Thy1 was triggered and the fault current flowed through L and the PTC where the flowing current was measured by a voltage drop across the shunt resistance R_{sh} . The thyristor switch Thy2 was turned on several 10 ms after Thy1 was turned on to discharge the remaining charge of C in order to prevent the PTC from excessively heating and to minimize the cooling time of the PTC for the next test.

We used three inductances of $L = 4.0, 8.37$ and 11.7 μ H to change the gradient and peak value of fault current.

In the fault current condition, the PTC resistance R_{PTC} changes its value mainly due to its resistivity which is a function of its temperature, because the change of its radius and the width is very small. A thermal resistance within the PTC is much less than that between the PTC and an ambient space, and it is considered that the temperature is uniform within the PTC and the PTC itself is in the thermal equilibrium. So, under the atmospheric constant pressure ($P = 1$ atm), PTC resistance can be expressed by its internal energy⁽⁸⁾ U which is also a function of temperature.

From the energy conservation's law, the time derivative of U is expressed as follows,

$$dU/dt = R_{PTC}I^2 - P_{out} \dots\dots\dots (1)$$

where $R_{PTC}I^2$ is an input joule heating power (P_{in}), and P_{out} is radiation power loss to an ambient space. Because of $R_{PTC}I^2 \gg P_{out}$, in the fault current condition, U is simply calculated from the input joule heating energy $E_{in} = \int_0^t R_{PTC}I^2 dt$,

$$U = U_0 + \int_0^t R_{PTC}I^2 dt = U_0 + E_{in}$$

where U_0 is a U at room temperature. And in the fault

Table 2. Average values of threshold input energy E_{th} [J] for four PTC samples to trip where $V_{c0} = 60$ V and $L = 4.0$ μ H

Interval time (min)	2	5	10	20
PTC1	21.22	21.57	20.89	20.11
PTC2	20.97	19.69	19.27	17.92
PTC3	17.20	20.33	18.53	17.74
PTC4	21.48	20.72	20.49	20.01

Table 3. Average values of threshold input energy E_{th} [J] for two PTC samples to trip, used as a typical case, where $L = 8.37$ μ H and $L = 11.7$ μ H

L (μ H)	Applied Voltage : V_{c0} (V)		
	50	60	70
8.37	19.85	19.18	21.11
11.7	18.10	19.06	20.74

current condition, the function of $R_{PTC} = f(U)$ is obtained from an experimental result of the relationship between R_{PTC} with E_{in} . Furthermore, we carried out many short circuit tests for several PTC samples and determined the function of $R_{PTC} = f(U)$ with specific parameters for characterizing individual PTC elements.

3. Experimental Results

In Fig. 3, we show typical waveforms of (a) the PTC voltage V_{PTC} and (b) the current I_{PTC} of four short circuit tests in case of $V_{c0} = 60$ V. After Thy1 is turned on, the current increased to the peak value of 1 kA at 0.3 ms and suddenly decreased less than 93 A within short period of 0.1 ms. The voltage spike at $t = 0.4 \sim 0.5$ ms is due to the large increase of R_{PTC} and $(-Ldi/dt)$ during the transient. The dotted curve in Fig. 4 is R_{PTC} as the function of E_{in} where $R_{PTC} = V_{PTC}/I_{PTC}$ and $E_{in} = \int_0^t V_{PTC}I_{PTC}dt$. This curve shows that R_{PTC} increases exponentially in $E_{in} > 20$ J. So, in this paper, we define E_{th} as a threshold value of E_{in} where R_{PTC} crosses 0.5Ω .

With 4 PTC samples, we measured the effect of interval time between short circuit tests in case of $V_{c0} = 60$ V. This result is shown in Table 2, where the interval time is 2, 5, 10 and 20 min. If an interval time is not long enough, the value of E_{th} may become small in comparison with its previous measured one because PTC can not be restored to the same initial condition. And from Table 2 there is no significant difference between E_{th} of interval time 2 ms and 20 ms. Thus, the interval time of 2 ms is sufficient for these tests. Furthermore, the scattering of E_{th} is less than 3 J for the same sample. Without the auto-discharge circuit of Thy2 and 1.6Ω , because of an over-heating of PTC for a long period, the interval time and the scattering of E_{th} become large.

Also further short circuit tests were done on eight PTC identical samples with $L = 8.37 \mu$ H and $L = 11.7 \mu$ H where the applied voltages V_{c0} were 50, 60, and 70 V. In Table 3, we show the average values of E_{th} for two PTC samples, used as typical case. In all the tests, the current and voltage waveforms were similar to the results discussed before, where the peak current and tripped

Fig. 3. (a) The PTC voltage waveform, and (b) the current waveform during short circuit tests from the experiments where PTC sample No.4 used as a typical case, and four tests are plotted

time change depending on L and V_{c0} . These experimental results demonstrate that the threshold input energy E_{th} into each PTC resistor required to trip does not depend on experimental conditions such as an inductance value and an applied voltage, and that the R_{PTC} can be expressed as a function of E_{in} .

4. PTC-TACS Model Description and Numerical Comparison

Using TACS elements in EMTP⁽⁹⁾⁽¹⁰⁾, special elements of nonlinear resistances as a function of circuit variables can be treated in circuit transient analysis.

From the discussion results about the function of $R_{PTC} = f(U)$ in Sec. 2, and the experimental results of the $R_{PTC} = f(E_{in})$ in Sec. 3, by expressing R_{PTC} as a function of E_{in} , it is possible to analyze circuits including PTC elements.

From the dotted curve in Fig.4, although R_{PTC} may be expressed by three partitioned functions in the range of (i) $0 < E_{in} < 15$, (ii) $15 < E_{in} < 20$ and (iii) $20 < E_{in}$, we simply express it by a single function as follows,

$$R_{PTC} = R_{min} \times \left(1 + \exp \frac{E_{in} - E_{th}}{E_N} \right) \dots \dots (2)$$

Fig. 4. PTC resistance as a function of input heating energy (joule heating) from the experiments and the PTC-TACS model

Fig. 5. A PTC-TACS model image created by ATPDraw

where R_{min} is the PTC resistance at normal temperature, E_{in} is the input energy to the PTC, E_{th} is the threshold energy value, and E_N is the forming factor related to the gradient of dR_{PTC}/dE_{in} .

Comparing the experimental data of $R_{PTC}(E_{in})$, we adopt $R_{min}=0.05 \Omega$, $E_{th}=20 \text{ J}$, and $E_N = 1 \text{ J}$ and show its function curve in Fig. 4 as a solid line.

To confirm this model availability (for PTC-TACS model validation), transient circuit of $I - V$ characteristics of the PTC is calculated using Eq.(2). In Fig.5, we show a PTC-TACS model image created by ATPDraw⁽¹¹⁾ where: (1) The voltage V_{PTC} and the current I_{PTC} of the PTC are probed by the TACS probes (passing EMTP information to TACS) T1 and T2 respectively, P1 is the probe branch voltage request across the PTC, P2 and P3 are the branches current output request. (2) The input heating power $P_{in} = I_{PTC} \times V_{PTC}$ is calculated by TACS function FORTRAN block 1, (3)

Fig. 6. EMTP simulation results of the PTC - TACS model: (a) PTC voltage waveform, and (b) current waveform

the input heating energy $E_{in} = \int_0^t V_{PTC} I_{PTC} dt$ is calculated by TACS controlled integrator block 2 and (4) the value of the R_{PTC} is adjusted according to Eq.(2) by using TACS FORTRAN block 3. In Fig.6, we show the analyzed results of (a) the voltage V_{PTC} and (b) the current I_{PTC} waveforms of the PTC under the condition of $V_{c0} = 60V$. From these results, the maximum current value of 1kA and the time interval of 0.45 ms for the PTC to trip coincide with the experimental values.

The small difference shown in the two curves in Fig.4 of R_{PTC} (Experiment) and R_{PTC} (PTC-TACS) does not have large effect on the voltage and current waveforms in Fig.6, but may have an effect on the peak spike voltage or tripped time. These results show that our PTC model as a function of input joule heating energy is valid for the analysis of PTC during the fault current condition.

5. Effects of Radiation Power Loss

In case of small current or small joule heating power, the radiation power loss P_{out} can not be neglected. In this section, we discuss P_{out} effects on E_{th} and trip time.

Fig. 7. Calculated results of threshold input energy E_{th} and ΔU_{th} versus applied voltage V_{c0} where ($\tau = 4ms$)

Fig. 8. The relationship between applied voltage V_{c0} and trip time T_{trip} under the conditions of $\tau = 2, 4, 8$, and ∞ ms

Using Fourier's law for P_{out} , Eq.(1) is modified as follows,

$$\frac{d\Delta U}{dt} = P_{in} - \frac{\Delta U}{\tau} \dots \dots \dots (3)$$

$$\Delta U = U - U_0 \dots \dots \dots (4)$$

where τ is the time constant relating to geometry and the ambient condition of the PTC. Moreover, ΔU differs from E_{in} by an amount of $\int_0^t P_{loss} dt$.

In Fig.7, we show E_{th} and ΔU_{th} as a function of the capacitor charging voltage V_{c0} . We define ΔU_{th} ($\tau = 4ms$) as the threshold value of ΔU where R_{PTC} crosses 0.5Ω .

E_{th} remains nearly 20J in the range of $V_{c0} \geq 50V$, whereas it becomes large as V_{c0} decreases due to the radiation power loss. But estimating the radiation power loss using $\tau = 4ms$, ΔU_{th} remains constant value of 20J over a wide range of V_{c0} .

Furthermore, to indicate the adequacy of $\tau = 4\text{ms}$, in Fig.8, we compared the trip time measured by experiments when I_{PTC} decreases to 90% of its peak value with calculated one at which ΔU becomes 20J for the conditions of $\tau = 2, 4, 6, 8\text{ms}$ and $R_{PTC} = 0.05\Omega$. From these results, calculated values of $\tau = 4\text{ms}$ are closest to the experimental ones.

From these analysis, it is shown that the radiation power loss can be treated as $\Delta U/\tau$ where in this case τ is 4ms. Especially, this radiation effect can be neglected in case that the trip time $\ll 4\text{ms}$. And R_{PTC} is expressed by ΔU with threshold value of 20J.

6. Conclusions

In this paper, more than 80 short circuit tests were carried out in experiments on four samples of PTC resistors and the results show that all PTC samples tripped where the input energy nearly equals 20 J. Therefore, this demonstrates that the PTC resistance is a function of joule heating. A new PTC-TACS model equation is proposed to determine useful electrical information of the PTC under the fault current condition. The results obtained from the experimental technique used in this paper are compared with the results obtained by using EMTP simulation, which validated the model. To our knowledge, it has been shown here for the first time that the EMTP with the PTC-TACS model is used as a function of input joule heating energy and tripped PTC resistance in a simple model. Furthermore, we discussed the effect of the radiation power loss on the trip time of the PTC and showed that the time constant related to the radiation power loss is nearly 4 ms and the radiation power loss can be neglected in the case of $V_{c0} > 50\text{V}$. Detailed research on a new over-current limiter based on PTC materials in our laboratory is under experiment.

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References

- (1) R. Strümpfer, J. Skindhøj, J. Glatz-Reichenbach, J.H.W. Kuhlefeldt, and F. Perdoncin: "Novel Medium Voltage Fault Current Limiter Based on Polymer PTC Resistors", *IEEE Trans. Power Delivery*, Vol.14, No.2, pp.425-430 (1999-4)
- (2) "Product's Catalog of PolySwitch of Raychem Company Japan (K06-2699.6.15000)"
- (3) J. Skindhøj, J. Glatz, and R. Strümpfer: "Repetitive Current Limiter based on Polymer PTC Resistor", *IEEE Trans. Power Delivery*, Vol.13, No.2, pp.489-494 (1998-4)
- (4) L. Wang and Z. Zhou: "A New Smart Material-Multilayer PTC Thermistor", *IEEE Trans. Components, Packaging, & Manufacturing Tech-part A*, Vol.18, No.2, pp.249-251(1995-6)
- (5) F.A. Doljack: "PolySwitch PTC Devices-A New Low-Resistance Conductive Polymer-Based PTC Device for Over-current Protection", *IEEE Trans. Components, Hybrids, & Manufacturing Tech*, Vol.Chmt-4, No.4, pp.372-378 (1981-12)
- (6) R.S. Perkins, A. Rüegg, M. Fischer, P. Streit, and A.Menth: "A New PTC Resistor for Power Applications", *IEEE Trans. Components, Hybrids, & Manufacturing Technology*, Vol.Chmt-5, No.2, pp.225-230 (1982-6)
- (7) L.W. Nagel and D.O. Pederson: SPICE-Simulation Program with Integrated Circuit Emphasis, Memo No.382, Electronics Research Laboratory, Univ. of California, Berkeley (1973-4)
- (8) Y.A. Çengel: Introduction to Thermodynamics and Heat Transfer, International Edition, The McGraw-Hill Companies, Inc. (1997)
- (9) ATP User Groups and Distribution Asia-Japanese ATP User Group (JAUG)
- (10) Alternative Transient Program (ATP) Rule Book (1987-99 by Canadian/American EMTP User Group)
- (11) H.K. Høidalen: ATPdraw SINTEF Energy Research, Norway

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