In this paper, experiments were conducted to study the electrical $I - V$ characteristics of the polymer Positive Temperature Coefficient (PTC) resistor as a function of joule heating due to $I^2R$. More than 80 short-circuit tests were carried out on four samples (rated 60 V/40 A), and the results show that all PTC samples tripped when the threshold input energy nearly equaled 20 J. We propose a new mathematical model for the PTC in the fault current condition, a PTC-TACS (Transient Analysis of Control Systems) model, by using the Electro-Magnetic Transient Program (EMTP), which is a function of the input joule heating energy. A comparison between the experimental results and EMTP simulation results has shown that the PTC-TACS model is valid and very effective to investigate the PTC under fault current conditions in electrical circuits and to design an over-current limiter based on PTC materials for industrial applications. In the discussion about radiation power loss in case that the applied voltage is greater than 50 V or the tripped time is less than 4 ms, it is shown that the radiation power loss can be neglected and the PTC resistance can be expressed only by the input joule heating energy.

**Keywords:** polymer PTC, joule heating, short circuit test, EMTP, PTC-TACS model, radiation power loss, trip time

1. **Introduction**

The PTC is one of the important solid state materials designed to interrupt the fault current and to function like a reset-table fuse. The PTC resistors based on polymer composites consist of conducting particles dispersed in a thermoplastic or duromeric matrix\(^1\)(\(^2\)). The PTC resistor has the ability to increase its resistance with increasing temperature\(^3\).

At normal operation where PTC carries less than hold current $I_{\text{hold}}$, the internal joule heating power is small and balanced with a radiation power loss. If a short circuit or an overload current occurs in the circuit\(^1\), the increased joule heating power will raise the internal temperature of the PTC and results in a non-linear increase in resistance of the PTC over a narrow temperature range.

In the commercial world, the manufacturing techniques allow to design a variation of the PTC materials\(^4\)(\(^5\)) (e.g., $V_2O_5$, $BaTiO_3$), as well as many new developing areas\(^5\) of PTC nonlinear effects of temperature dependent resistance $R(T)$ and voltage dependent current $I(V)$ behaviors.

Until today, low current rating and limited voltage do not allow the PTC to be used as an over-current protector in high power devices. The PTC properties have been studied and reported in the literature, where some technical attempts have been made to introduce PTC resistance as a function of temperature under specific applications\(^3\)(\(^6\)). However, the $I - V$ characteristics and PTC resistance in an electrical model for the electric circuit calculations have not been studied in detail.

For a system using several PTC elements to expand rated voltage and current, connected in series and in parallel, an electrical equivalent model of the PTC is very useful to analyze its behavior as a function of joule heating energy of used elements. In this paper, we propose a simple electrical equivalent model of the PTC which is described only with circuit variables and easy to use for circuit calculating program such as SPICE\(^7\) (Simulation Program with Integrated Circuit Emphasis) and EMTP.

2. **Experimental Setup**

Figure 1 shows a photograph of the PTC (Type RXE 375) used in this study. Four samples were used and the electrical properties\(^2\) are listed in Table 1.

To measure the transient electrical properties of the
PTC under the fault current condition and to compare it with the proposed model, we used the experimental setup in Fig. 2.

After the capacitor C was charged to the specific value \( V_{c0} = 60 \text{ V} \), the thyristor switch Thy1 was triggered and the fault current flowed through \( L \) and the PTC where the flowing current was measured by a voltage drop across the shunt resistance \( R_{sh} \). The thyristor switch Thy2 was turned on several 10 ms after Thy1 was turned on to discharge the remaining charge of C in order to prevent the PTC from excessively heating and to minimize the cooling time of the PTC for the next test.

We used three inductances of \( L = 4.0, 8.37 \) and \( 11.7 \) \( \mu \text{H} \) to change the gradient and peak value of fault current.

In the fault current condition, the PTC resistance \( R_{PTC} \) changes its value mainly due to its resistivity which is a function of its temperature, because the change of its radius and the width is very small. A thermal resistance within the PTC is much less than that between the PTC and an ambient space, and it is considered that the temperature is uniform within the PTC and the PTC itself is in the thermal equilibrium. So, under the atmospheric constant pressure (\( P = 1 \text{ atm} \)), PTC resistance can be expressed by its internal energy \( U \) which is also a function of temperature.

From the energy conservation’s law, the time derivative of \( U \) is expressed as follows,

\[
dU/dt = R_{PTC}I^2 - P_{out}
\]

where \( R_{PTC}I^2 \) is an input joule heating power \( (P_{in}) \), and \( P_{out} \) is radiation power loss to an ambient space. Because of \( R_{PTC}I^2 \gg P_{out} \), in the fault current condition, \( U \) is simply calculated from the input joule heating energy \( E_{in} = \int_0^t R_{PTC}I^2dt \),

\[
U = U_0 + \int_0^t R_{PTC}I^2dt = U_0 + E_{in}
\]

where \( U_0 \) is a \( U \) at room temperature. And in the fault current condition, the function of \( R_{PTC} = f(U) \) is obtained from an experimental result of the relationship between \( R_{PTC} \) with \( E_{in} \). Furthermore, we carried out many short circuit tests for several PTC samples and determined the function of \( R_{PTC} = f(U) \) with specific parameters for characterizing individual PTC elements.

### 3. Experimental Results

In Fig. 3, we show typical waveforms of (a) the PTC voltage \( V_{PTC} \) and (b) the current \( I_{PTC} \) of four short circuit tests in case of \( V_{c0} = 60 \text{ V} \). After Thy1 is turned on, the current increased to the peak value of 1 kA at 0.3 ms and suddenly decreased less than 93 A within short period of 0.1 ms. The voltage spike at \( t = 0.4 \sim 0.5 \text{ ms} \) is due to the large increase of \( R_{PTC} \) and \(-dV/dt\) during the transient. The doted curve in Fig. 4 is \( R_{PTC} \) as the function of \( E_{in} \) where \( R_{PTC} = V_{PTC}/I_{PTC} \) and

\[
E_{in} = \int_0^f V_{PTC}I_{PTC}dt.
\]

This curve shows that \( R_{PTC} \) becomes large.

With 4 PTC samples, we measured the effect of interval time between short circuit tests in case of \( V_{c0} = 60 \text{ V} \). This result is shown in Table 2, where the interval time is 2, 5, 10 and 20 min. If an interval time is not long enough, the value of \( E_{th} \) may become small in comparison with its previous measured one because PTC can not be restored to the same initial condition. And from Table 2 there is no significant difference between \( E_{th} \) of interval time 2 ms and 20 ms. Thus, the interval time of 2 ms is sufficient for these tests. Furthermore, the scattering of \( E_{th} \) is less than 3 J for the same sample. Without the auto-discharge circuit of Thy2 and 1.6Ω, because of an over-heating of PTC for a long period, the interval time and the scattering of \( E_{th} \) become large.

Also further short circuit tests were done on eight PTC identical samples with \( L = 3.73 \mu \text{H} \) and \( L = 11.7 \mu \text{H} \) where the applied voltages \( V_{c0} \) were 50, 60, and 70 V. In Table 3, we show the average values of \( E_{th} \) for two PTC samples, used as typical case. In all the tests, the current and voltage waveforms were similar to the results discussed before, where the peak current and tripped

<table>
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<tr>
<th>Interval time (min)</th>
<th>2</th>
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<th>10</th>
<th>20</th>
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<tr>
<td>Thy1</td>
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<tr>
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<td>20.72</td>
<td>20.49</td>
<td>20.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( L ) (( \mu \text{H} ))</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{c0} ) (V)</td>
<td>8.37</td>
<td>19.85</td>
<td>19.18</td>
</tr>
<tr>
<td>11.7</td>
<td>18.10</td>
<td>19.06</td>
<td>20.74</td>
</tr>
</tbody>
</table>
Fig. 3. (a) The PTC voltage waveform, and (b) the current waveform during short circuit tests from the experiments where PTC sample No.4 used as a typical case, and four tests are plotted.

Fig. 4. PTC resistance as a function of input heating energy (joule heating) from the experiments and the PTC-TACS model.

Fig. 5. A PTC-TACS model image created by ATPDraw.

4. PTC-TACS Model Description and Numerical Comparison

Using TACS elements in EMTP, special elements of nonlinear resistances as a function of circuit variables can be treated in circuit transient analysis.

From the discussion results about the function of \( R_{PTC} = f(U) \) in Sec. 2, and the experimental results of the \( R_{PTC} = f(E_{in}) \) in Sec. 3, by expressing \( R_{PTC} \) as a function of \( E_{in} \), it is possible to analyze circuits including PTC elements.

From the dotted curve in Fig. 4, although \( R_{PTC} \) may be expressed by three partitioned functions in the range of (i) \( 0 < E_{in} < 15 \), (ii) \( 15 < E_{in} < 20 \) and (iii) \( 20 < E_{in} \), we simply express it by a single function as follows,

\[
R_{PTC} = R_{min} \times \left(1 + \exp\left(\frac{E_{in} - E_{th}}{E_N}\right)\right) \quad \cdots \cdots \quad (2)
\]

where \( R_{min} \) is the PTC resistance at normal temperature, \( E_{in} \) is the input energy to the PTC, \( E_{th} \) is the threshold energy value, and \( E_N \) is the forming factor related to the gradient of \( dR_{PTC}/dE_{in} \).

Comparing the experimental data of \( R_{PTC}(E_{in}) \), we adopt \( R_{min}=0.05 \Omega, E_{th}=20 \text{ J}, \) and \( E_N=1 \text{ J} \) and show its function curve in Fig. 4 as a solid line.

To confirm this model availability (for PTC-TACS model validation), transient circuit of \( I-V \) characteristics of the PTC is calculated using Eq.(2). In Fig. 5, we show a PTC-TACS model image created by ATPDraw where: (1) The voltage \( V_{PTC} \) and the current \( I_{PTC} \) of the PTC are probed by the TACS probes (passing EMTP information to TACS) T1 and T2 respectively, P1 is the probe branch voltage request across the PTC, P2 and P3 are the branches current output request. (2) The input heating power \( P_{in} = I_{PTC} \times V_{PTC} \) is calculated by TACS function FORTRAN block 1, (3)
the input heating energy $E_{in} = \int_0^t V_{PTC} I_{PTC} dt$ is calculated by TACS controlled integrator block 2 and (4) the value of the $R_{PTC}$ is adjusted according to Eq.(2) by using TACS FORTRAN block 3. In Fig.6, we show the analyzed results of (a) the voltage $V_{PTC}$ and (b) the current $I_{PTC}$ waveforms of the PTC under the condition of $V_{c0} = 60$V. From these results, the maximum current value of 1kA and the time interval of 0.45 ms for the PTC to trip coincide with the experimental values.

The small difference shown in the two curves in Fig.4 of $R_{PTC}$ (Experiment) and $R_{PTC}$ (PTC-TACS) does not have large effect on the voltage and current waveforms in Fig.6, but may have an effect on the peak spike voltage or tripped time. These results show that our PTC model as a function of input joule heating energy is valid for the analysis of PTC during the fault current condition.

5. Effects of Radiation Power Loss

In case of small current or small joule heating power, the radiation power loss $P_{out}$ can not be neglected. In this section, we discuss $P_{out}$ effects on $E_{th}$ and trip time.

Using Fourier’s law for $P_{out}$, Eq.(1) is modified as follows,

$$\frac{d\Delta U}{dt} = \frac{P_{in} - \Delta U}{\tau} \quad \text{(3)}$$

$$\Delta U = U - U_0 \quad \text{(4)}$$

where $\tau$ is the time constant relating to geometry and the ambient condition of the PTC. Moreover, $\Delta U$ differs from $E_{th}$ by an amount of $\int_0^t P_{loss} dt$.

In Fig.7, we show $E_{th}$ and $\Delta U_{th}$ as a function of the capacitor charging voltage $V_{c0}$. We define $\Delta U_{th}$ ($\tau = 4$ms) as the threshold value of $\Delta U$ where $R_{PTC}$ crosses 0.5 $\Omega$. $E_{th}$ remains nearly 20J in the range of $V_{c0} \geq 50$V, whereas it becomes large as $V_{c0}$ decreases due to the radiation power loss. But estimating the radiation power loss using $\tau = 4$ms, $\Delta U_{th}$ remains constant value of 20J over a wide range of $V_{c0}$.
Furthermore, to indicate the adequacy of $\tau = 4\text{ms}$, in Fig.8, we compared the trip time measured by experiments when $I_{PTC}$ decreases to 90% of its peak value with calculated one at which $\Delta U$ becomes 20J for the conditions of $\tau = 2, 4, 6, 8\text{ms}$ and $R_{PTC} = 0.05\Omega$. From these results, calculated values of $\tau = 4\text{ms}$ are closest to the experimental ones.

From these analysis, it is shown that the radiation power loss can be treated as $\Delta U/\tau$ where in this case $\tau$ is 4\text{ms}. Especially, this radiation effect can be neglected in case that the trip time $\ll 4\text{ms}$. And $R_{PTC}$ is expressed by $\Delta U$ with threshold value of 20J.

6. Conclusions

In this paper, more than 80 short circuit tests were carried out in experiments on four samples of PTC resistors and the results show that all PTC samples tripped where the input energy nearly equals 20 J. Therefore, this demonstrates that the PTC resistance is a function of joule heating. A new PTC-TACS model equation is proposed to determine useful electrical information of the PTC under the fault current condition. The results obtained from the experimental technique used in this paper are compared with the results obtained by using EMTP simulation, which validated the model. To our knowledge, it has been shown here for the first time that the EMTP with the PTC-TACS model is used as a function of input joule heating energy and tripped PTC resistance in a simple model. Furthermore, we discussed the effect of the radiation power loss on the trip time of the PTC and showed that the time constant related to the radiation power loss is nearly 4\text{ms} and the radiation power loss can be neglected in the case of $V_{c0} > 50\text{V}$.

Detailed research on a new over-current limiter based on PTC materials in our laboratory is under experiment. (Manuscript received March 26, 2004, revised June 24, 2004)

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