Electromagnetic Scattering by Multilayered Periodic Arrays of Circular Cylinders —Activity Report

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A very efficient and accurate method to characterize electromagnetic scattering by multilayered periodic arrays of circular cylinders is presented, using the lattice sums technique, the aggregate T-matrix algorithm, and the generalized reflection and transmission matrices for a layered system. The method is quite general and applies to various configurations of two-dimensional or three-dimensional periodic arrays. The unit cell of array can contain two or more cylinders, which may be dielectric, conductor, gyrotropic medium, or their mixture with different sizes. The periodic spacing of cylinders along each array plane should be same over all layers, otherwise the cylinders in different layers may be different in material properties and dimensions.

Keywords: scattering, electromagnetic crystals, lattice sums, T-matrix

1. Introduction

Electromagnetic crystals are periodic dielectric or metallic structures in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. This frequency range is called the electromagnetic bandgap which is analogous to the electron bandgap in natural crystals. Recently the electromagnetic crystals have received a growing attention⁽¹⁾⁽²⁾, because of their promising applications to the bandgap based devices in microwave to optical wave region, including narrow-band filters, laser mirrors, high-quality resonant cavities, optical waveguides, and substrates for microwave antennas. A periodic array of infinitely long parallel circular cylinders is typical of discrete periodic structures. The frequency response of the array is characterized by the scattering properties of individual cylinders and the multiple scattering due to the periodic arrangement of scatterers. When the array is layered, the multiple interaction of space-harmonics scattered from each of array layers suppresses the propagation of electromagnetic waves within a particular frequency range and an electromagnetic bandgap is formed. When the layered arrays are embedded in a dielectric slab, the frequency response is significantly modified due to the presence of a multiple scattering of fields between the array planes and the slab boundaries. Although electromagnetic crystals consisting of layered arrays of parallel cylinders exhibit various interesting frequency responses, it is essentially a two-dimensional structure. A fully three-dimensional structure is realized when the axes of parallel cylinders are rotated by a constant angle in each successive layer of the stacking sequence.

In this paper, we shall report our recent activity in the research on the multilayered periodic arrays of circular cylinders. An accurate and efficient approach developed by our group is outlined and discussed for modeling the two-dimensional or three-dimensional scattering by electromagnetic crystals consisting of layered parallel or crossed arrays of circular cylinders standing in free-space or embedded in a dielectric slab. Following the method, the reflection, transmission, and absorption characteristics of electromagnetic crystals with a lattice of circular cylinders can be easily calculated by a simpler matrix calculus based on the T-matrix of an isolated single cylinder and the one-dimensional lattice sums. Due to the limited space, we shall cite only our recent relevant investigations. An enormous amount of works on electromagnetic crystals reported during the past decade can be found in the referenced literatures.

2. Two-dimensional Scattering

2.1 Reflection and Transmission Matrices A periodic array of circular cylinders is free-standing in a background medium with a wave number k_0 as shown in Fig. 1. The cylinders are infinite long, parallel to each other, and spaced with a distance h along the x axis on the plane $y = -y_j$, which separates the whole space into two semi-infinite regions assigned j and $j + 1$. The local origin of the array is located at $(x_j, -y_j)$. We consider the scattering of an electromagnetic plane wave whose direction of incidence is normal to the cylinder axis. The problem is reduced to a two-dimensional one and may be treated separately for TM and TE waves relative to the z axis. Let $k_{x0} = k_0 \cos \phi_0$ be the x -component of the wave vector of the incident plane wave, where ϕ_0 denotes the incident angle. The periodic array scatters the incident wave into a series of space harmonics with the wavenumber $k_{xl} = k_0 \cos \phi_0 + 2l\pi/h$ where l is an integer. The

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Fig. 1. Cross section of a periodic array of circular cylinders located at $y = -y_j$

single scattering is characterized by the reflection and transmission coefficients of the array which relate the amplitudes of the reflected and transmitted l -th space harmonics to that of the incident plane wave. When the arrays are multilayered as shown in Fig. 2, the scattered space harmonics impinge on the neighbor arrays as new incident waves and are scattered again into another series of space harmonics. This multiple scattering process is described in terms of the reflection and transmission matrices for each of layered arrays, which relate the amplitudes of the reflected and transmitted l -th space harmonics to that of the incident n -th space harmonic. Taking into account the orthogonality of space harmonic fields, after several manipulations, the reflection matrix $\mathbf{R}_j^{(+)}$ ($\mathbf{R}_j^{(-)}$) and the transmission matrix $\mathbf{F}_j^{(-)}$ ($\mathbf{F}_j^{(+)}$) for the incidence of space harmonics downgoing (upgoing) toward the j -th array are obtained as follows ^{(3) (4)}:

$$\mathbf{R}_j^{(\pm)} = \mathbf{X}_j^{(\pm)} \mathbf{U}^{(\pm)} \bar{\mathbf{T}}_j \mathbf{P}^{(\pm)} \mathbf{X}_j^{(-)} \dots \dots \dots (1)$$

$$\mathbf{F}_j^{(\mp)} = \mathbf{X}_j^{(+)} (\mathbf{I} + \mathbf{U}^{(\mp)} \bar{\mathbf{T}}_j \mathbf{P}^{(\pm)}) \mathbf{X}_j^{(-)} \dots \dots \dots (2)$$

with

$$\mathbf{U}^{(\pm)} = [u_{lm}^{(\pm)}] = \left[\frac{2(-i)^m}{k_0 h \sin \phi_l} e^{\pm i m \phi_l} \right] \dots \dots \dots (3)$$

$$\mathbf{X}_j^{(\pm)} = [x_{j,l}^{(\pm)} \delta_{ll'}] = [e^{\pm i k_{x,l} x_j} \delta_{ll'}] \dots \dots \dots (4)$$

$$\cos \phi_l = \cos \phi_0 + \frac{2l\pi}{k_0 h} \dots \dots \dots (5)$$

$$\sin \phi_l = \sqrt{1 - \cos^2 \phi_l}, \quad \text{Im}\{\sin \phi_l \geq 0\} \dots \dots \dots (6)$$

$$\bar{\mathbf{T}}_j = (\mathbf{I} - \mathbf{T}_j \mathbf{L})^{-1} \mathbf{T}_j \dots \dots \dots (7)$$

$$\mathbf{P}^{(\pm)} = [p_{mn}^{(\pm)}] = [i^m e^{\pm i m \phi_n}] \dots \dots \dots (8)$$

where $l, m, n = 0, \pm 1, \pm 2, \dots$, $\delta_{ll'}$ denotes the Kronecker's delta, and $\mathbf{I}$ is the unit matrix. In Eq. (7), $\mathbf{T}_j$ is the T-matrix of the isolated single cylinder located at the local origin $(x_j, -y_j)$, $\bar{\mathbf{T}}_j$ is the T-matrix of the periodic array, and $\mathbf{L}$ is the lattice sums matrix ⁽⁵⁾ whose element L_{mn} is given by

$$L_{mn} = S_{m-n}(k_0 h, \cos \phi_0) \dots \dots \dots (9)$$

$$S_{m-n}(k_0 h, \cos \phi_0) = \sum_{l=1}^{\infty} H_{m-n}^{(1)}(lk_0 h) [e^{-il k_0 h \cos \phi_0} + (-1)^{m-n} e^{il k_0 h \cos \phi_0}] \dots \dots \dots (10)$$

Fig. 2. Cross section of N -layered periodic array of circular cylinders

where $H_n^{(1)}$ is the n -th order Hankel function of the first kind. If both the cylinders and background medium consist of isotropic materials and the cross section of cylinders is up-down symmetric relative to the array plane $y = -y_j$, we have the following relations

$$\mathbf{R}_j^{(+)} = \mathbf{R}_j^{(-)}, \quad \mathbf{F}_j^{(-)} = \mathbf{F}_j^{(+)} \dots \dots \dots (11)$$

These relations can be effectively used when the arrays of up-down symmetric cylinders are multilayered.

2.2 Lattice Sums and T-matrix The lattice sums matrix $\mathbf{L}$ defined by Eqs. (9) and (10) characterizes uniquely the periodic arrangement of scatterers and are independent of the polarization of the incident field and the individual configuration of scatterers. The matrix $\mathbf{L}$ calculated once can be commonly used for both TM and TE waves and for any arrays of cylindrical objects whenever the periods are same. The details of scattering from each array element within a unit cell are described by the T-matrix $\mathbf{T}_j$ in Eq. (7). This is a main advantage of using the lattice sums in the scattering problems for discrete periodic structures. It is well known that the direct sum of Hankel functions in Eq. (10) converges very slowly. To overcome this difficulty, we have derived an integral form ⁽⁵⁾ for the lattice sums which is efficiently calculated using a simple formula of numerical integration for elementary functions.

The T-matrix of cylindrical objects located in a unit cell plays another important role in the present formulation. Any analytical or numerical techniques may be employed to calculate the T-matrix. When the cylindrical objects are circular cylinders, in particular, the T-matrix for both two-dimensional TM wave and TE wave excitations is obtained in closed form under various configurations. ^{(4) (6) - (11)} If two or more circular cylinders are contained within a unit cell, which may be of dielectrics, conductors, gyrotropic materials, or their mixture with different dimensions, the recursive algorithm ⁽¹²⁾ is used to obtain the aggregate T-matrix for the composite cylindrical system in terms of the reduced T-matrix for each cylinder in isolation.

2.3 Layered Periodic Arrays The generalized reflection and transmission matrices of the N -layered arrays as shown in Fig. 2 can be obtained by successively

concatenating the reflection and transmission matrices given by Eqs. (1) and (2). Let $\bar{\bar{R}}_j^{(\pm)}$ and $\bar{\bar{F}}_j^{(\mp)}$ be the generalized reflection and transmission matrices for the entire system of j -layered arrays, where the superscripts $\pm$ are used in the same way as Eqs. (1) and (2). When the j -th array is stacked at $y = -y_{j+1}$ above the j -th array located at $y = -y_j$, the generalized reflection and transmission matrices $\bar{\bar{R}}_{j+1}^{(\pm)}$ and $\bar{\bar{F}}_{j+1}^{(\pm)}$ for this $(j+1)$ -layered system are calculated using the following recursive relations^{(6) (14)}:

$$\bar{\bar{R}}_{j+1}^{(+)} = \mathbf{R}_{j+1}^{(+)} + \mathbf{Q}_{j+1} \bar{\bar{R}}_j^{(+)} \mathbf{Y}_{j+1} \mathbf{F}_{j+1}^{(-)} \dots \dots \dots (12)$$

$$\bar{\bar{F}}_{j+1}^{(-)} = \mathbf{S}_j \mathbf{Y}_{j+1} \mathbf{F}_{j+1}^{(-)} \dots \dots \dots (13)$$

$$\bar{\bar{R}}_{j+1}^{(-)} = \bar{\bar{R}}_j^{(-)} + \mathbf{S}_j \mathbf{Y}_{j+1} \mathbf{R}_{j+1}^{(-)} \mathbf{Y}_{j+1} \bar{\bar{F}}_j^{(+)} \dots \dots \dots (14)$$

$$\bar{\bar{F}}_{j+1}^{(+)} = \mathbf{Q}_{j+1} \bar{\bar{F}}_j^{(+)} \dots \dots \dots (15)$$

with

$$\mathbf{Q}_{j+1} = \mathbf{F}_{j+1}^{(+)} \mathbf{Y}_{j+1} [\mathbf{I} - \bar{\bar{R}}_j^{(+)} \mathbf{Y}_{j+1} \mathbf{R}_{j+1}^{(-)} \mathbf{Y}_{j+1}]^{-1} \dots (16)$$

$$\mathbf{S}_j = \bar{\bar{F}}_j^{(-)} [\mathbf{I} - \mathbf{Y}_{j+1} \mathbf{R}_{j+1}^{(-)} \mathbf{Y}_{j+1} \bar{\bar{R}}_j^{(+)}]^{-1} \dots \dots \dots (17)$$

$$\mathbf{Y}_{j+1} = [e^{ik_0 d_{j+1} \sin \phi_m \delta_{mm'}}] \dots \dots \dots (18)$$

where $d_{j+1} = y_j - y_{j+1}$. The generalized reflection and transmission matrices $\bar{\bar{R}}_N^{(\pm)}$ and $\bar{\bar{F}}_N^{(\mp)}$ for the N -layered arrays are calculated from Eqs. (12) ~ (15) through the $(N-1)$ times recursion process starting with $\bar{\bar{R}}_1^{(\pm)} = \mathbf{R}_1^{(\pm)}$ and $\bar{\bar{F}}_1^{(\mp)} = \mathbf{F}_1^{(\mp)}$. If the arrays consist of periodically layered identical arrays with an equal separation, we may calculate $\bar{\bar{R}}_N^{(\pm)}$ and $\bar{\bar{F}}_N^{(\mp)}$ by using the concept of Floquet modes⁽¹⁴⁾ in the y direction. This approach is efficient when the number of arrays is increased. The reflection and transmission efficiencies of the N -layered arrays into the l -th space harmonic can be easily calculated from $\bar{\bar{R}}_{N,l}^{(\pm)}$ and $\bar{\bar{F}}_{N,l}^{(\mp)}$ for the incidence of downgoing and upgoing plane waves, respectively.

2.4 Layered Arrays Embedded in Slab When the array are embedded in a slab of transparent medium as shown in Fig. 3, additional effects are incorporated into the multiple scattering process of the space harmonic fields. These are the change in an effective propagation length of fields inside the slab medium and the multiple scattering of fields between the array planes and the slab boundaries. The combined effects in scattering lend variety to the frequency response of the arrays which may be controllable by varying the slab thickness and material combination. Let the permittivity and permeability of the slab medium be ϵ_s and μ_s . For the array located in the slab, the reflection and transmission matrices are still given in the form of Eqs. (1) and (2), but the expressions (3) ~ (8) must be slightly modified due to the Fresnel reflection of space harmonic fields at the plane boundary between the slab medium and the exterior free space as follows^{(6) (10) (15) ~ (17)}:

Fig. 3. Cross section of N -layered periodic array of circular cylinders embedded in a dielectric slab

$$\mathbf{U}^{(\pm)} = [u_{lm}^{(\pm)}] = [\frac{2(-i)^m}{k_s h \sin \alpha_l} e^{\pm i m \alpha_l}] \dots \dots \dots (19)$$

$$\mathbf{X}_j^{(\pm)} = [x_{j,l}^{(\pm)} \delta_{ll'}] = [e^{\mp i x_j k_0 \cos \phi_l} \delta_{ll'}] \dots \dots \dots (20)$$

$$\cos \alpha_l = \frac{k_0}{k_s} \cos \phi_0 + \frac{2l\pi}{k_s h} \dots \dots \dots (21)$$

$$\sin \alpha_l = \sqrt{1 - \cos^2 \alpha_l}, \quad \text{Im}\{\sin \alpha_l\} \geq 0 \dots \dots \dots (22)$$

$$\bar{\bar{P}}^{(\pm)} = [p_{mn}^{(\pm)}] = [i^m e^{\pm i m \alpha_n}] \dots \dots \dots (23)$$

Since the x component of the wavevector is conserved through the Fresnel reflection, the lattice sums matrix $\mathbf{L}$ is same as Eqs. (9) and (10) and hence the array T-matrix $\bar{\bar{T}}_j$ for the embedded array is still given by Eq. (7).

For the N -layered array embedded in the slab, the lower boundary of the slab is treated as the zero-th layer and the upper boundary is as the $(N+1)$ -th layer. Then $\mathbf{R}_{N+1}^{(\mp)}$, $\mathbf{F}_{N+1}^{(\pm)}$, $\mathbf{R}_0^{(\pm)}$, and $\mathbf{F}_0^{(\mp)}$ are replaced by the conventional Fresnel reflection and transmission matrices which are diagonal matrices. The generalized reflection and transmission matrices $\bar{\bar{R}}_N^{(\pm)}$ and $\bar{\bar{F}}_N^{(\mp)}$ for the embedded N -layered arrays are obtained from Eqs. (12) ~ (15) through the $(N+1)$ times recursion process. If the arrays consist of periodically layered identical arrays with an equal separation, we may calculate first the generalized reflection and transmission matrices for the N -layered arrays free-standing in the slab medium of infinite extent by using the concept of Floquet modes⁽¹⁴⁾ in the y direction and then concatenate them to the Fresnel reflection and transmission matrices at the upper and lower slab boundaries. The reflection and transmission efficiencies can be calculated using Eqs. (12) ~ (15).

3. Three-dimensional Scattering

3.1 Reflection and Transmission Matrices If the wavevector of an incident plane wave is not perpendicular to the cylinder axis, the scattering by a cylindrical object turn to a three-dimensional problem. The incident TM or TE wave is scattered into both TM and

Fig. 4. Schematic view of a plane wave obliquely incident on a periodic array of circular cylinders

TE waves. In a periodic array, the scattering process between the TM and TE waves takes place sequentially along the array elements. Although such a multiple coupling of scattered fields makes the treatment intricate, being compared with the two-dimensional problem, the approach based on the lattice sums and T-matrix can be also applied to the three-dimensional problem.

Figure 4 shows the schematic view of a periodic array of circular cylinders and the coordinate for the incident wave. The z components of the incident fields parallel to the cylinder axis are expressed as follows:

$$E_z^i = E_0 e^{i(\xi_0 x - \kappa_0 y + \beta_0 z)} \quad (24)$$

$$\tilde{H}_z^i = \tilde{H}_0 e^{i(\xi_0 x - \kappa_0 y + \beta_0 z)} \quad (25)$$

with

$$E_0 = \sin \theta_0 \cos \psi^i, \quad \tilde{H}_0 = \sin \theta_0 \sin \psi^i \quad (26)$$

$$\xi_0 = k_0 \sin \theta_0 \cos \phi_0, \quad \beta_0 = k_0 \sin \theta_0 \sin \phi_0 \quad (27)$$

$$\kappa_0 = k_0 \cos \theta_0 \quad (28)$$

where $\tilde{H}_z^i = \sqrt{\mu_0/\varepsilon_0} H_z^i$ denotes the normalized magnetic field, (θ_0, ϕ_0) denotes the angle of incidence as depicted in Fig. 4, and ψ^i is the polarization angle of the incident electric field.

Following a similar analytical treatment discussed in 2.1, the reflection matrix $\mathbf{R}_j^{(\pm)}$ and the transmission matrix $\mathbf{F}_j^{(\mp)}$ for the incident space-harmonics downgoing and upgoing towards the j -th array are deduced as follows⁽¹⁸⁾:

$$\mathbf{R}_j^{(\pm)} = \begin{bmatrix} \mathbf{R}_j^{ee(\pm)} & \mathbf{R}_j^{eh(\pm)} \\ \mathbf{R}_j^{he(\pm)} & \mathbf{R}_j^{hh(\pm)} \end{bmatrix} \quad (29)$$

$$\mathbf{F}_j^{(\mp)} = \begin{bmatrix} \mathbf{F}_j^{ee(\mp)} & \mathbf{F}_j^{eh(\mp)} \\ \mathbf{F}_j^{he(\mp)} & \mathbf{F}_j^{hh(\mp)} \end{bmatrix} \quad (30)$$

$$\mathbf{R}_j^{\mu\nu(\pm)} = \mathbf{X}_j^{(+)} \mathbf{U}^{(\pm)} \bar{\mathbf{T}}_j^{\mu\nu} \mathbf{P}^{(\pm)} \mathbf{X}_j^{(-)} \quad (31)$$

$(\mu, \nu = e, h)$

$$\mathbf{F}_j^{\mu\nu(\mp)} = \mathbf{X}_j^{(+)} [\mathbf{I} + \mathbf{U}^{(\mp)} \bar{\mathbf{T}}_j^{\mu\nu} \mathbf{P}^{(\pm)}] \mathbf{X}_j^{(-)} \quad (32)$$

$(\mu = \nu)$

$$\mathbf{F}_j^{\mu\nu(\mp)} = \mathbf{X}_j^{(+)} \mathbf{U}^{(\mp)} \bar{\mathbf{T}}_j^{\mu\nu} \mathbf{P}^{(\pm)} \mathbf{X}_j^{(-)} \quad (33)$$

$(\mu \neq \nu)$

with

$$\mathbf{U}^{(\pm)} = [u_{lm}^{(\pm)}] = \left[\frac{2(-i)^m}{\gamma_0 h \sin \gamma_l} e^{\pm i m \gamma_l} \right] \quad (34)$$

$$\mathbf{P}^{(\pm)} = [p_{mn}^{(\pm)}] = [i^m e^{\pm i m \gamma_n}] \quad (35)$$

$$\mathbf{X}_j^{(\pm)} = [x_{j,l}^{(\pm)} \delta_{ll'}] = [e^{\pm i x_j k_0 \cos \gamma_l} \delta_{ll'}] \quad (36)$$

$$\cos \gamma_l = \frac{k_0}{\eta_0} \sin \theta_0 \cos \phi_0 + \frac{2l\pi}{\eta_0 h} \quad (37)$$

$$\eta_0 = \sqrt{k_0^2 - \beta_0^2} \quad (38)$$

where η_0 is the projection of wavevector of the incident wave onto the x - y plane, $\bar{\mathbf{T}}_j^{\mu\nu}$ ($\mu, \nu = e, h$) represent the array T-matrices which relate the scattered TM or TE wave to the incident TM and TE waves. The superscripts e and h are used for the TM wave and TE wave components, respectively. Note that $\bar{\mathbf{T}}_j^{eh}$ and $\bar{\mathbf{T}}_j^{he}$ represent the cross scattering effects inherent in the three-dimensional scattering. If both the cylinders and background medium consist of isotropic materials and the cross section of cylinders is up-down symmetric relative to the array plane, we have the following relations

$$\mathbf{R}_j^{(+)} = \mathbf{R}_j^{(-)}, \quad \mathbf{F}_j^{(-)} = \mathbf{F}_j^{(+)} \quad (39)$$

3.2 T-matrix and Lattice Sums The array T-matrices appeared in Eqs. (31) ~ (33) are derived as follows⁽¹⁸⁾:

$$\bar{\mathbf{T}}_j^{ee} = (\mathbf{I} + \boldsymbol{\sigma}_1 \mathbf{L} \boldsymbol{\sigma}_2 \mathbf{L})^{-1} (\boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_1 \mathbf{L} \boldsymbol{\sigma}_2) \quad (40)$$

$$\bar{\mathbf{T}}_j^{eh} = (\mathbf{I} + \boldsymbol{\sigma}_1 \mathbf{L} \boldsymbol{\sigma}_2 \mathbf{L})^{-1} \boldsymbol{\sigma}_1 (\mathbf{I} + \mathbf{L} \boldsymbol{\sigma}_4) \quad (41)$$

$$\bar{\mathbf{T}}_j^{he} = -(\mathbf{I} + \boldsymbol{\sigma}_2 \mathbf{L} \boldsymbol{\sigma}_1 \mathbf{L})^{-1} \boldsymbol{\sigma}_2 (\mathbf{I} + \mathbf{L} \boldsymbol{\sigma}_3) \quad (42)$$

$$\bar{\mathbf{T}}_j^{hh} = (\mathbf{I} + \boldsymbol{\sigma}_2 \mathbf{L} \boldsymbol{\sigma}_1 \mathbf{L})^{-1} (\boldsymbol{\sigma}_4 - \boldsymbol{\sigma}_2 \mathbf{L} \boldsymbol{\sigma}_1) \quad (43)$$

with

$$\boldsymbol{\sigma}_1 = -(\mathbf{I} - \mathbf{T}_j^{ee} \mathbf{L})^{-1} \mathbf{T}_j^{he} \quad (44)$$

$$\boldsymbol{\sigma}_2 = (\mathbf{I} - \mathbf{T}_j^{hh} \mathbf{L})^{-1} \mathbf{T}_j^{eh} \quad (45)$$

$$\boldsymbol{\sigma}_3 = (\mathbf{I} - \mathbf{T}_j^{ee} \mathbf{L})^{-1} \mathbf{T}_j^{ee} \quad (46)$$

$$\boldsymbol{\sigma}_4 = (\mathbf{I} - \mathbf{T}_j^{hh} \mathbf{L})^{-1} \mathbf{T}_j^{hh} \quad (47)$$

where $\mathbf{L}$ is the lattice sums matrix and $\mathbf{T}_j^{\mu\nu}$ ($\mu, \nu = e, h$) represent the three-dimensional T-matrices of the isolated single cylinder. Note that the lattice sums and T-matrices still play an important role in the three-dimensional scattering problem. The elements L_{mn} of the lattice sums matrix in this three-dimensional case are calculated in terms of η_0 and the direction cosine $\sin \theta_0 \cos \phi_0$ as follows:

$$L_{mn} = S_{m-n}(\eta_0 h, \sin \theta_0 \cos \phi_0) \quad (48)$$

For an isolated circular cylinder of radius d , permittivity ε , and permeability μ , on the other hand, the three-dimensional T-matrices are obtained as follows⁽¹⁸⁾:

$$\mathbf{T}_j^{ee} = - \left[\frac{J_m(w)}{H_m^{(1)}(w)} \frac{\Delta_{2,m}}{\Delta_{1,m}} \delta_{mn} \right] \dots \dots \dots (49)$$

$$\mathbf{T}_j^{eh} = -\mathbf{T}_j^{he} = - \frac{2k_0}{\pi w^2 k} \left[\frac{K_m \delta_{mn}}{[H_m^{(1)}(w)]^2 \Delta_{1,m}} \right] \dots \dots \dots (50)$$

$$\mathbf{T}_j^{hh} = - \left[\frac{J_m(w)}{H_m^{(1)}(w)} \frac{\Delta_{3,m}}{\Delta_{1,m}} \delta_{mn} \right] \dots \dots \dots (51)$$

$$\Delta_{1,m} = M_{1,m} M_{2,m} - K_m^2 \dots \dots \dots (52)$$

$$\Delta_{2,m} = M_{2,m} M_{3,m} - K_m^2 \dots \dots \dots (53)$$

$$\Delta_{3,m} = M_{4,m} M_{1,m} - K_m^2 \dots \dots \dots (54)$$

$$M_{1,m} = \frac{J'_m(u)}{u J_m(u)} - \frac{\varepsilon_0}{\varepsilon} \frac{H'_m{}^{(1)}(w)}{w H_m^{(1)}(w)} \dots \dots \dots (55)$$

$$M_{2,m} = \frac{J'_m(u)}{u J_m(u)} - \frac{\mu_0}{\mu} \frac{H'_m{}^{(1)}(w)}{w H_m^{(1)}(w)} \dots \dots \dots (56)$$

$$M_{3,m} = \frac{J'_m(u)}{u J_m(u)} - \frac{\varepsilon_0}{\varepsilon} \frac{J'_m(w)}{w J_m(w)} \dots \dots \dots (57)$$

$$M_{4,m} = \frac{J'_m(u)}{u J_m(u)} - \frac{\mu_0}{\mu} \frac{J'_m(w)}{w J_m(w)} \dots \dots \dots (58)$$

$$K_m = \frac{m\beta_0}{k} \left(\frac{1}{u^2} - \frac{1}{w^2} \right) \dots \dots \dots (59)$$

$$w = \eta_0 d, \quad u = d \sqrt{k^2 - \beta_0^2} \dots \dots \dots (60)$$

where $k = \omega \sqrt{\varepsilon \mu}$. When the direction of incidence of plane wave is normal ($\phi_0 = 0^\circ$) to the cylinder axis, we have $\beta_0 = 0$, $\eta_0 = k_0$, $w = k_0 d$, $u = kd$, $K_m = 0$, and $\mathbf{T}_j^{eh} = \mathbf{T}_j^{he} = \mathbf{0}$. In this case, $\bar{\mathbf{T}}_j^{ee}$ and $\bar{\mathbf{T}}_j^{hh}$ in Eqs. (49) and (51) are reduced to the two-dimensional T-matrices for the separated TM and TE waves.

3.3 Layered Parallel Arrays When the periodic arrays are layered with the cylinder axes parallel to the z axis as shown in Fig. 2, the incident angle of the plane wave viewed from the array is the same over all layers. Then the three-dimensional T-matrix of the isolated single cylinder and the lattice sums matrix are defined using the same η_0 and direction cosine $\sin \theta_0 \cos \phi_0$. In this case, the generalized reflection and transmission matrices for the N -layered parallel arrays are calculated by applying the same recurrence formula as Eqs. (12) ~ (15) to the three-dimensional reflection and transmission matrices $\mathbf{R}_j^{(\pm)}$ and $\mathbf{F}_j^{(\mp)}$ defined by Eqs. (29) and (30), respectively. This extension is straightforward.

3.4 Layered Crossed Arrays Although the layered parallel arrays discussed above exhibit various interesting resonance phenomena, it is essentially a two-dimensional structure. When the array is illuminated by electromagnetic waves propagating obliquely relative to

Fig. 5. Schematic view of layered crossed-arrays of circular cylinders

the cylinder axis, it is rather difficult to fully control the frequency response by the structural parameters alone. Such a deficiency inherent in the array of infinite parallel cylinders may be resolved by introducing a three-dimensional nature in the layering process.

The layered crossed arrays considered here is schematically shown in Fig. 5. The cylinders in each layer are infinitely long and parallel to each other, while the cylinder axes are rotated by 90° in each successive layer. The stacking sequence repeats every two layers. The array of z -directed cylinders periodically spaced with a distance h_x along the x axis is referred to as the *Z-array* and the array of x -directed cylinders periodically spaced with a distance h_z along the z axis is as the *X-array*. Since the structure is periodic both in x and z directions, the scattered fields consist of double space-harmonics varying as $e^{i(\xi_l x + \beta_m z)}$ where $\xi_l = \xi_0 + 2l\pi/h_x$, $\beta_m = \beta_0 + 2m\pi/h_z$, and ξ_0 and β_0 are defined by Eq. (27).

For the *Z-array*, E_z and $\tilde{H}_z$ fields are employed as the leading fields. Following the same procedure as discussed in Sec. 3.1, the reflection matrix $\mathbf{R}_z^{\mu\nu(\pm)}$ and transmission matrix $\mathbf{F}_z^{\mu\nu(\mp)}$ are obtained on the basis of the reflection submatrix $\mathbf{R}_z^{\mu\nu(\pm)}(m)$ and transmission submatrix $\mathbf{F}_z^{\mu\nu(\pm)}(m)$ calculated for each of the space harmonic components varying as $e^{i\beta_m z}$. For the *X-array*, on the other hand, E_x and $\tilde{H}_x$ fields are employed as the leading fields. The reflection matrix $\mathbf{R}_x^{\mu\nu(\pm)}$ and transmission matrix $\mathbf{F}_x^{\mu\nu(\mp)}$ are obtained on the basis of the reflection submatrix $\mathbf{R}_x^{\mu\nu(\pm)}(l)$ and transmission submatrix $\mathbf{F}_x^{\mu\nu(\mp)}(l)$ calculated for each of the space harmonic components varying as $e^{i\xi_l x}$. When the *Z-array* and the *X-array* are stacked in the y direction as shown in Fig. 6, a coordinate transformation for the scattered fields is introduced. The space harmonic fields expressed in terms of E_x and $\tilde{H}_x$ as the leading fields can be easily transformed into the expressions in terms of the E_z and $\tilde{H}_z$ fields. This yields the reflection and transmission matrices of the *X-array* based on E_z and $\tilde{H}_z$ as the

Fig. 6. Schematic view of a unit cell of crossed-arrays

leading fields. Then the reflection and transmission matrices of the *X-array* and *Z-array* are linked to obtain those of the *crossed-array* which forms a unit cell in the stacking sequence. Finally, the reflection and transmission matrices of the *crossed-array* are concatenated successively over the repeated number of cells, using the recursive formula given by Eqs. (12) ~ (15), to obtain the generalized reflection and transmission matrices⁽¹⁹⁾ for the layered crossed-arrays as shown in Fig.5. Thus the three dimensional electromagnetic scattering from the multilayered crossed-arrays can be calculated by a simpler matrix operation on the basis of the one-dimensional lattice sums and the three-dimensional T-matrix of the isolated single cylinder.

4. Conclusion

An accurate approach for analyzing two-dimensional or three-dimensional electromagnetic scattering by multilayered parallel or crossed arrays of circular cylinders have been discussed, using the lattice-sums technique, and the aggregate T-matrix of cylindrical objects, and the generalized reflection and transmission matrices in a layered system. The formulation is quite general and can be applied to various configurations of cylindrical arrays. The unit cell may include two or more cylinders which may be of dielectrics, conductors, gyrotropic materials, or their mixture with different dimensions. The array elements in different layers may be different. The approach can be also used to analyze photonic crystal waveguides consisting of the lattice of circular rods^{(20)~(22)}. The accuracy and efficiency of the approach has been tested through a substantial number of numerical examples which are presented in the referenced literatures.

Acknowledgment

This work was supported by the 2003 Research Grant of Hoso-Bunka Foundation.

(Manuscript received Feb. 26, 2004)

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