Natural ELF Pulses in the Time Domain : Series with Accelerated Convergence

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We present an algorithm for accelerating the convergence of the time domain formal solution constructed for the natural electromagnetic pulses propagating in the Earth – ionosphere cavity. Such pulses originate from vertical lightning discharges and travel around the globe at extremely low frequencies (ELF). A compact formula is found for horizontal magnetic field component. Results of computations are demonstrated for a set of the source – observer distances and properties of time domain solution are discussed.

Keywords : ELF waves, Earth-ionosphere waveguide, Time domain solution

1. Introduction

The solution of an electrodynamic problem is constructed usually in the frequency domain, especially when one treats the problem of monochromatic radio wave propagation or resonance oscillations. Experimental data are collected as a function of time, which are transformed into the functions of frequency by using the Fourier transform. Schumann resonance is the electromagnetic phenomenon that takes place in the spherical cavity formed by the surface of the Earth and lower edge of the ionosphere. The problem is usually solved in the frequency domain by applying the mode theory (Wait, 1962; Galejs, 1972; Bliokh et al., 1980; Nickolaenko and Hayakawa, 2002). Treatment of a problem in the time domain is rare, however, many practical applications require the constructing such a solution. These become necessary when analyzing transient excitations of the cavity (Q-bursts) by huge lightning strokes exceeding the median level by a factor of ten or higher. Such solutions are desirable in realistic models of natural noise being a combination of many individual pulses. Time domain waveforms are required when relating the results of classic mode theory with those obtained recently in the framework of FDTD method or with transmission line models (Hayakawa and Otsuyama, 2002; Simpson and Taflove, 2002; Morente et al., 2003a, b; Otsuyama et al., 2003).

The most straightforward way of obtaining the time domain waveforms is application of Fourier transform to the complex spectra of the signal (Jones, 1970b; Sukhorukov and Stubbe, 1997; Nickolaenko and Hayakawa, 1998):

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_r(\omega) e^{i\omega t} d\omega \quad (1)$$

$$H(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_\varphi(\omega) e^{i\omega t} d\omega \quad (2)$$

Here $E_r(t)$, $E_r(\omega)$, $H_\varphi(t)$, and $H_\varphi(\omega)$ are the waveforms

and complex spectral components of vertical electric and horizontal magnetic fields. The polar spherical coordinate system is used $\{r, \theta, \varphi\}$ with the observer placed on the ground surface at the pole $\theta = 0$.

Only the vertical electric and transverse horizontal magnetic fields propagate at extremely low frequencies (ELF), being the TEM mode. The frequency domain solutions $E_r(\omega)$ and $H_\varphi(\omega)$ are obtained by using the well-known zonal harmonic series representation for the zero order mode (Wait, 1962, Jones, 1970a, b, Galejs, 1972; Bliokh et al., 1980; Nickolaenko and Hayakawa, 2002):

$$E_r(\omega) = \frac{iv(\nu+1)}{\omega} \frac{M_c(\omega)}{4\pi \varepsilon h a^2} \sum_{n=0}^{\infty} \frac{(2n+1)P_n(\cos\theta)}{(n-\nu)(n+1+\nu)} \quad (3)$$

$$H_\varphi(\omega) = \frac{M_c(\omega)}{4\pi h a c^2} \sum_{n=1}^{\infty} \frac{(2n+1)P_n^1(\cos\theta)}{(n-\nu)(n+1+\nu)} \quad (4)$$

Here $E_r(\omega)$ and $H_\varphi(\omega)$ are measured in $V \cdot s \cdot m^{-1}$ and $A \cdot s \cdot m^{-1}$ correspondingly, ω is the angular frequency in s^{-1} , a is the Earth's radius in m, h is the ionosphere height above the ground (in m), ε and μ are the permittivity and the permeability of the vacuum, $M_c(\omega)$ is the current moment of the source in $A \cdot m \cdot s$, $\nu = \nu(\omega)$ is the dimensionless complex propagation constant of the radio wave with $\text{Re}\{\nu\} \geq 0$ and $\text{Im}\{\nu\} \leq 0$ defined for $\omega > 0$, $P_n(\cos\theta)$ and $P_n^1(\cos\theta)$ are the Legendre and associated Legendre polynomials, and θ is the angular distance between the source and observer.

Schumann resonance peaks are observed at 8, 14, 20, 26 Hz, etc. At such low frequencies only the vertical electric source current is important, and it is accepted usually that the current moment is independent of frequency: $M_c(\omega) = \text{const}$. The latter condition means that the current pulse is very short, and we are able to consider it as being a delta-function in the time domain $M_c(t) = \delta(t)$.

Direct application of Eqs.(1) and (2) to (3) and (4) is coupled with the problems. Say, the pulsed waveform could not be obtained at the source point. Difficulties are often met when applying FFT at the point of the source antipode. The analytic

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time domain solution provides us with the presentations free from the above problems (Nickolaenko *et al.*, 1999; Nickolaenko and Hayakawa, 2002).

2. Formal Solution in the Time Domain

We solve the problem for the postulated $\nu = \nu(\omega)$ dependence and explore the eigenfrequencies in the complex frequency plane. The $\nu = \nu(\omega)$ function is taken usually from the measurements (Ishaq and Jones, 1977; Bliokh *et al.*, 1980) or from the model considerations (Madden and Thompson, 1965; Jones, 1967; Hynninen and Galuk, 1972; Sentman, 1995; Fuellekrug, 2000). As a result, one can compute the complex spectra of fields Eqs.(3) and (4) defined for the positive real part of frequency $\text{Re}\{\omega\} > 0$. When obtaining the time domain solutions, we follow the standard procedure of the Fourier transform (Wait, 1962; Galejs, 1972; Jones, 1970b; Nickolaenko *et al.*, 1999; Nickolaenko and Hayakawa, 2002; Mushtak and Williams, 2002).

Integrals (1) and (2) are treated as contour integrals of functions of complex frequency. Since the functions of frequency are defined in the right half-space, we have to extend them into the whole complex frequency plane by using the Hermitian symmetry: the change of the sign in the argument is equivalent to the complex conjugate $F(-\omega) = F^*(\omega)$. This formal property follows from the demand that the time domain functions (being the Fourier conjugates of the spectral functions) are the real functions of time.

Thus we define the functions of frequency in the left half-plane and next we close the integration contour into the upper half-plane where the Jordan's Lemma is held. As a result, the infinite integrals Eqs.(1) and (2) are transformed into the sum of residuals at the poles positioned in the complex frequency plane (see the diagram in Fig. 1).

Spectra Eqs.(3) and (4) have the same system of the poles, which are found as the solutions of equation:

$$[n - \nu(\omega)][n + 1 + \nu(\omega)] = 0 \quad (5)$$

The static electric field in the Earth-ionosphere cavity corresponds to uniformly charged boundaries, so that the solution

is:

$$n = \nu = 0, \quad \omega = 0 \quad (6)$$

while the magnetic field does not contain the static $n=0$ term. Poles corresponding to the resonance modes are found in the right half-plane from the relation:

$$n - \nu(\omega) = 0 \quad (n = 1, 2, \dots) \quad (7)$$

It follows from Eq.(7) that resonance modes with different n correspond to the roots Ω_n positioned in the first quadrant:

$$\text{Re}\{\nu(\Omega_n)\} = n \quad (n = 1, 2, \dots) \quad (8)$$

Condition Eq.(8) clarifies the physical meaning of dimensionless propagation constant $\nu(f)$: the n -th Schumann resonance takes place at the frequency where the real part of propagation constant becomes an integer.

In accordance with the Hermitian symmetry, there exists the symmetric root $-\Omega_n^*$ for each index $n > 0$. Symmetric roots are located in the second quadrant of complex frequency plane. We show a few pole positions and the integration path in Fig. 1.

One obtains after substituting Eqs.(3), (4), and (8) into Eqs.(1) and (2), integration by computing the sums of residuals at simple poles Eq.(8), and re-arrangements:

$$E(t) = E_o + E_A \text{Re} \left\{ \sum_{n=1}^{\infty} \frac{n(n+1)e^{i\Omega_n t} P_n(\cos\theta)}{\Omega_n} \right\} \quad (9)$$

$$\text{Here } E_A = \frac{M_C}{2\pi h a^2 \varepsilon} \text{ is the electric amplitude.}$$

The static electric field E_o is maintained by the charge moment of the lightning stroke $M_Q = -\int M_C(t)dt$. It exists before the stroke initiation at the time moment t_s and it vanishes afterwards:

$$E_o = \frac{M_Q}{8\pi h a^2 \varepsilon} [1 - \text{sign}(t - t_s)] \quad (10)$$

Here $\text{sign}(x)$ is a unit step function. In what follows, we exclude the static field from consideration because such a field is not measured in the Schumann resonance studies. Relations for the magnetic field component are obtained in a similar way:

$$H(t) = \frac{M_C}{2\pi h a c^2} \text{Im} \left\{ \sum_{n=1}^{\infty} P_n^1(\cos\theta) e^{i\Omega_n t} \right\} \quad (11)$$

Expansions (9) and (11) were already obtained by Nickolaenko *et al.* (1999). Our goal is to accelerate the convergence of series Eqs.(9) and (11) and to obtain (if possible) the compact analytical time domain solutions.

3. Accelerating the Convergence

We try to accelerate the convergence and obtain the compact time domain series for the propagation constant that is a linear function of frequency:

$$\nu(\omega) = A\omega + B \quad (\omega > 0) \quad (12)$$

The following considerations validate the application of this linear frequency dependence. The propagation constant as a function of frequency is found from the Schumann resonance spectra, i.e., from the succession of peaks in the power spectra of the electromagnetic field. The lowest peak frequency is 8 Hz, and the highest peak is observed around 32 Hz, the fifth mode. Higher

Poles in the complex frequency plane and the FT integration path

Fig. 1. Location of four initial poles in the complex frequency plane. The Fourier transform integration path goes from left to right below the real axis and is closed into the upper half-plane (not shown). A pole of the static field in the Earth-ionosphere cavity Ω_0 ($n=0$) occupies the origin, so the integration path is a half-circle below this pole

modes are not used owing to the low quality factor of the cavity, which does not exceed 5 – 6. As a result, the 6 Hz separation between the peaks becomes smaller than the widths of 5th – 6th peaks. Therefore the maxima higher than the fifth substantially overlap and are not resolved. Since the frequency range is relatively narrow where the spectral parameters and propagation constant might be deduced, only a linear $\nu(f)$ function of frequency is found with sufficient accuracy from the experimental data. One has to extrapolate the $\nu(f)$ experimental dependence to higher frequencies and thus to find the poles above the fifth. It follows from the condition Eq.(8) for the linear function Eq.(12) that $\Omega_n = (n-B)/A$, and the relation Eq.(9) turns into:

$$E(t) = E_A \cdot \text{Re} \left\{ \sum_{n=1}^{\infty} A \frac{n(n+1)}{n-B} P_n(\cos \theta) e^{it((n-B)/A)} \right\} \quad (13)$$

An arbitrary frequency dependence of the propagation constant is treated similarly, but the only distinction is in the pole coordinates and in relevant exponential factors included into Eq.(13). In a particular case of the heuristic frequency dependence $\nu(f) = (f-2)/6 - i \cdot (f)/100$ (Bliokh et al., 1980; Nickolaenko, 1997; Nickolaenko and Hayakawa, 2002), we obtain the following values for the model parameters:

$$A = (1/6 - i/100)/2\pi, \quad B = -1/3 \quad (14)$$

The heuristic dependence corresponds to the regular succession of Schumann resonance peaks occupying the frequencies 8, 14, 20, 26, 32, etc. Hz. The wave attenuation in this model corresponds to that pertinent to the cross-spectra of vertical electric field component (Bliokh et al., 1980).

Separate terms or “harmonics” in Eq.(13) depend on time as $\exp((n-B)it/A)$, so the solution is a finite function of time: it turns into zero as these terms decrease to zero when $t \rightarrow \infty$. It is clear that convergence of series Eq.(13) is improved exponentially when we compute the field for progressively increasing time. This is why a greater number of terms must be used in the field computations at small times and at small distances. Indeed, when time is relatively small, the exponential factors in Eq.(13) are close to unity, and general term of the sum starts to decrease noticeably when the summation index is sufficiently large: $0 < n < N$ and $N \gg 1$. We estimate the normalized inaccuracy of reducing (13) as being equal to:

$$\frac{\Delta E}{E} \approx N \cdot |\delta|^{(N-1)} = N \cdot |1-\eta|^{(N-1)} \quad (15)$$

Here we denote $\delta = (1-\eta) = e^{i(n-B)/A \cdot it}$. The upper bound estimate Eq.(15) indicates that the 1% error is guaranteed when 88 terms are included into the series Eq.(13), provided that the temporal factor $|\delta| = 0.9$.

It is desirable to accelerate the convergence of time series, or even to convolve the sums into analytical functions. We implement the accelerating algorithm used for the zonal harmonic series representation in the frequency domain (Johler and Lewis, 1969, Nickolaenko and Rabinowicz, 1974, Bliokh et al., 1980, Jones and Burke, 1990, Nickolaenko and Hayakawa, 2002). Consider the auxiliary series being the generating function of the Legendre polynomials (Bateman and Erdelji, 1973):

$$R_0(g, x) = \sum_{n=0}^{\infty} g^n P_n(x) = \frac{1}{(1-2xg+g^2)^{1/2}} \quad (16)$$

where $g = e^{i(t/A)}$ and $x = \cos \theta$.

We obtain the following relation after differentiating Eq.(16) with respect to g inside the convergence circle and by combining the result with the generating function itself:

$$\begin{aligned} R_{-1}(g, x) &= \sum_{n=0}^{\infty} (n+1) g^n P_n(x) \\ &= g \frac{\partial R_0(g, x)}{\partial g} + R_0(g, x) - 1 \\ &= \frac{1-xg}{(1-2xg+g^2)^{3/2}} - 1 \end{aligned} \quad (17)$$

By adding and extracting Eq.(17) into Eq.(13), one obtains:

$$E(t) = E_A \cdot \text{Re} \left\{ A \cdot g^{-B} \left[R_{-1} + B \sum_{n=1}^{\infty} \frac{(n+1)g^n}{n-B} P_n(x) \right] \right\} \quad (18)$$

The general term in sum Eq.(18) acquired the $1/n$ factor; hence, the series converges faster than the original sum Eq.(13). The sum Eq.(18) already converges uniformly and absolutely in all the points including the source point $\theta = 0$, provided that $t > 0$. We proceed with the accelerating algorithm by applying it to Eq.(17) and substituting the result into Eq.(18), etc. The following formula is obtained after the third step of accelerating the convergence:

$$\begin{aligned} E(t) &= E_A \cdot \text{Re} \left\{ A g^{-B} \left[R_{-1} + (R_0 - 1)B \right. \right. \\ &\quad \left. \left. + \left(\frac{R_1}{g} - 1 \right) B(B+1) + B(B+1)^2 \sum_{n=1}^{\infty} \frac{g^n P_n(x)}{(n-B)(n+1)} \right] \right\} \end{aligned} \quad (19)$$

where

$$\begin{aligned} R_1(g, x) &= \int_0^g R_0(s, x) ds = \sum_{n=1}^{\infty} \frac{g^{n+1}}{(n+1)^2} P_n(x) \\ &= \ln \frac{g-x + (1-2xg+g^2)^{1/2}}{1-x} \end{aligned} \quad (20)$$

Now, the normalized inaccuracy of computations with reduced Eq.(19) can be evaluated by,

$$\frac{\Delta E}{E} \leq \frac{|\delta|^{(N-1)}}{N} \quad (21)$$

A substantially smaller number of terms is necessary to obtain the fixed accuracy of computations. The 1% error is reached now when only 18 terms are included into the summation of Eq.(19) for the same temporal factor $|\delta| = 0.9$. So, we have decreased the necessary number of terms by a factor exceeding 3.

The formulas for the magnetic field component are obtained similarly:

$$H(t) = H_A \text{Im} \left\{ \sum_{n=1}^{\infty} e^{it((n-B)/A)} P_n^1(\cos \theta) \right\} \quad (22)$$

with the amplitude $H_A = M_0 / 2\pi a h c^2$.

Magnetic solution Eq.(22) is transformed into the closed form after we differentiate Eq.(16) with respect to θ and substitute

the result into Eq.(22):

$$H(t) = H_A \cdot \operatorname{Im} \left\{ \frac{g^{(1-B)} (1-x^2)^{1/2}}{(1-2xg+g^2)^{3/2}} \right\} \dots\dots\dots (23)$$

The electric and magnetic fields in the time domain are expressed in a closed analytical form when the linear frequency dependence of the propagation constant is a uniform one, i.e., when parameter $B = 0$:

$$E(t) = E_A \cdot \operatorname{Re} \left\{ A_U \left[\frac{1-xg}{(1-2xg+g^2)^{3/2}} - 1 \right] \right\} \dots\dots\dots (24)$$

and

$$H(t) = H_A \cdot \operatorname{Im} \left\{ \frac{g (1-x^2)^{1/2}}{(1-2xg+g^2)^{3/2}} \right\} \dots\dots\dots (25)$$

We here comment on the limitations of Eq.(18) as follows: (1) the frequency should be below 100Hz and (2) the distance is greater than 500km.

A satisfactory correspondence to the experimental Schumann resonance spectra is obtained for the uniform dependence when coefficient A_U is equal to:

$$A_U = \left(\frac{1}{7} - \frac{i}{100} \right) \frac{1}{2\pi}, \quad B = 0 \dots\dots\dots (26)$$

We managed to construct an effective analytical solution for the electromagnetic problem in the time domain. Attempts to obtain compact presentations were undertaken for a long time, and the last one was obtained by *Wait* (1962). *Wait* treated the problem of a radio pulse propagating from a lightning stroke in the flat Earth – ionosphere waveguide. He used the Laplace transform rather than the Fourier transform used here and obtained the result in the time domain. The source was assumed to be a vertical electric dipole on the ground surface with the current moment $p(t)$ being zero for $t < 0$. At a distance ρ along the Earth's surface, the resulting vertical electric field was (*Wait*, 1962):

$$e_z(t) \cong \frac{\mu}{4\pi h} \cdot \left(\frac{c}{2\rho} \right)^{1/2} p_0 \cdot \gamma^{-3/2} \exp \left[\frac{\rho c}{8h^2} \left(\frac{\varepsilon}{\sigma} \right) \right] P \left(\frac{t'}{\gamma} \right) \dots\dots\dots (27)$$

where μ and ε are the permeability and permittivity of free space, c is the light velocity, h and σ are the height and conductivity of the ionosphere correspondingly, $t' = t - (\rho/c)$ is the time with propagation delay,

$$(\gamma)^{1/2} = \frac{\rho}{2h} \left(\frac{\varepsilon}{\sigma} \right)^{1/2} + (6t_m)^{1/2},$$

$$P(T) = \left(\frac{1}{2T} - 1 \right) \sqrt{\frac{1}{T^3}} \exp \left(-\frac{1}{4T} \right)$$

is the transient response of the zero – order mode for an impulsive vertical dipole source. The special analytical form for the current moment $p(t)$ was chosen such as

$$p(t) = \frac{p_0}{2} \left(\frac{6t_m}{\pi} \right)^{1/2} \frac{1}{t^{3/2}} \cdot \exp \left(-\frac{3t_m}{2t} \right) \dots\dots\dots (28)$$

Wait had also expanded the treatment to the case of a tilted

electric dipole. The principal distinction of solutions Eqs.(19) and (23) from (27) is related to the spherical geometry of our problem. The focusing of radio signal takes place at the source antipode $\theta = \pi$ (or $\rho = 20$ Mm). Such an enhancement of the signal occurs only in the closed spherical cavity and it is absent in the flat geometry.

4. Results of Computations and Discussion

Figure 2 presents the waveforms of the vertical electric and horizontal magnetic fields computed by using Eqs.(19) and (23). Time is shown from the stroke initiation in milliseconds along the abscissa in linear scale. The field amplitudes are given in arbitrary units on the ordinates. Waveforms were computed for source – observer distances ranging from 3 to 20 Mm with the step of 1 Mm (1 Mm = 1000 km). The waveforms coincide with the results published (*Nickolaenko and Hayakawa*, 1998; *Nickolaenko et al.*, 1999; *Nickolaenko and Hayakawa*, 2002) with the graphic accuracy, provided that the same propagation model $\nu(f)$ is used.

Lower plots in Fig.2 depict the pulse variations. They demonstrate in particular how the arrival time of the direct pulse increases with the source distance. Amplitude of the direct pulse is reduced and its width increases noticeably with the source distance. Such a behavior is conditioned by the growth of the wave attenuation with frequency: the waveguide acts as a low – pass filter. Antipodal and round – the – world waves are seen as a weak secondary pulse that arrives at appropriate delay. Interference between the direct and antipodal waves is responsible for the Schumann resonance pattern observed in the spectra.

Focusing of the signal is apparent in the vertical electric field: the pulse amplitude initially decreases and it must grow after the signal has passed the equatorial 10 Mm distance when the wave front starts to converge toward the antipode. Focusing is obscured by the attenuation, so the electric pulse amplitude begins to increase obviously after 15 Mm distance. Here the amplitude of direct wave becomes greater than that observed at 10 Mm. The upper left plot separately shows the pulses at 10 and 20 Mm

Fig. 2. Vertical electric (left) and horizontal magnetic (right) fields computed for the source – observer distances ranging from 3 to 20 Mm ; Upper plots separately show electric and magnetic waveforms at 10 and 20 Mm distance, Lower plots survey the evolution of pulses

distances. Magnetic field is equal to zero at the antipodal distance, and no pulse is observed here (see right upper plot in Fig. 2).

Upper plots of Fig. 2 also demonstrate a characteristic difference between electric and magnetic waveforms. The reason of such a distinction is that the vertical electric pulse has the same polarity when it arrives via the direct or antipodal path. Therefore the positive onset of antipodal wave is superimposed on the negative half-wave of the direct pulse in the electric field (10 Mm distance). The sign (orientation) of the horizontal magnetic field component depends on the arrival direction, and direct and antipodal pulses acquire the opposite signs. Thus a 'square waveform' appears in the magnetic field seen in the right upper plot of Fig. 2 for 10 Mm distance.

Time-distance dependence shown in the lower part of Fig. 2 can be transformed into contour maps, which distinctly show the pulse evolution in the spherical Earth-ionosphere cavity (see Fig. 3). The delay from the stroke initiation is plotted on the abscissa of Fig. 3 in milliseconds, and the source-observer distance is depicted on the ordinate in Mm. The lower boundary of the maps corresponds to the 0.5 Mm neighborhood of the source, and the upper boundary is the antipodal 20 Mm distance. Focusing of electric pulse is observed around the antipode and the source at about 80 and 160 ms after the stroke initiation. Focusing is less pronounced in the magnetic component owing to its nodes at 20 and 40 Mm distances.

The structure of Fig. 3 plots is connected with two straight lines over the delay-distance plane. Such a 'flat' geometry is unexpected for the pulse propagation in a spherical cavity, which offers many 'side wave' paths.

The property becomes evident after we consider a pulse 'launched' from an arbitrary point. The wave initially expands from the source forming concentric circles of growing radius. After the front passes the 10 Mm distance (90°) from the source, it starts to converge towards the source antipode. The circular waves collapse into a point producing the antipodal increase of finite amplitude though.

In the magnetic field, the individual pulses arrive from opposite directions and compensate each other, and the resulting field turns into zero. The front proceeds from the antipode towards the source point. A new circular wave appears of smaller amplitude expanding from the antipode. It crosses the observer at appropriate time on its way to the source. A new, finite, focusing of the field takes place at the source, etc. These considerations explain the straight line structure of the maps in Fig. 3.

An observer at distance D from the source detects the direct and the antipodal waves arriving in succession. Waves have the delays: $\tau_d = D/V_g$ and $\tau_a = (40 - D)/V_g$ where V_g is the

Fig. 3. Pulse propagation in the spherical Earth-ionosphere cavity; 'Plane' geometry is pertinent to the plots over the distance-delay plane

signal velocity, and we can estimate the latter from the plots as being equal to $V_g \approx 2.66 \times 10^5$ km/s.

5. Conclusion

To conclude the paper we list the main results.

(1) The time domain solution was obtained in the form of infinite series and its convergence was accelerated.

(2) The compact analytical form was found for the horizontal magnetic field, which is very convenient in the numerical modeling of Schumann resonance for practical applications.

(3) There exist the compact analytical solutions for vertical electric and horizontal magnetic fields in the time domain connected with the uniform linear frequency dependence of ELF propagation constant.

It is possible now to construct a real-time model of natural ELF random signal similar to that described by Nickolaenko and Hayakawa (2002). A user specifies the statistical and spectral characteristics (pulse arrival times, source distribution, amplitude probability distribution, etc.). Such artificial signals will enable us to investigate the role of separate factors involved in the signal. Synthetic signals might be especially useful when performing 'standard calibrations' of the equipment at different field sites by applying the same input signal of given properties.

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