Three-Dimensional Analysis of Light Scattering Phenomena in Index-ellipsoid

Taro Yoshida^{*} Member
 Kazuhiro Makishima^{**} Non-member
 Takeshi Yamada^{**} Non-member
 Mototaka Sone^{**} Member
 Yoshihiro Murooka^{***} Member

The three dimensional optical technique has been applied to investigate the electromagnetic wave scattering phenomena. Recently, we have developed an ideal equation for analyzing the three dimensional transport character of light scattered due to many index-ellipsoids arranged directly along the light axis in series. The size of index-ellipsoid, the distance between two adjacent index-ellipsoids, the ellipsoid rate and others are considered. By using an analyzing equation devised by us, we succeeded to derive the condition the light can pass through many spherical and transparent mediums connected in series with high efficiency.

Keywords : index-ellipsoid, light line equation, light intensity distribution

1. Introduction

The mechanism of the pre-breakdown phenomena in liquids has been optically studied⁽¹⁾⁻⁽³⁾. In this case, the laser-schlieren technique was used for observing the disturbance produced due to the movement of electrons together with the ultra-high speed camera (10⁷pps) in the non-uniform field⁽⁴⁾⁻⁽⁷⁾. Here, the disturbance is consisted of a lot of tiny bubbles whose size is smaller than the wave length of light. The disturbance is recorded as a refractive index change. From the consideration the light is a kind of electromagnetic wave⁽⁸⁾⁻⁽¹⁰⁾, we have measured the electromagnetic wave scattered due to one tiny spherical dielectric material⁽¹¹⁾. 86% of the scattered electromagnetic wave characters observed experimentally agrees with those of wave optics obtained theoretically⁽¹²⁾. The dielectric constant of materials used is a constant in the frequency range of 5~50GHz. For analyzing the optical phenomena, the following four methods are considered such as the geometric optics, the wave optics, the electromagnetic wave optics and the quantum optics. In this experiment, no quantum effect exists. And then it does not need to take account of the quantum optics. Furthermore, the medium used in the experiment is uniformly distributed and no polarized light effect exists. Therefore, it is possible to optically analyze the experimental results obtained using the electromagnetic wave optics replacing with the wave optics.

Under such a background, we decide to theoretically study the transport mechanism of light in the materials where many index ellipsoids are arranged directly in series. As well known, all of the materials existing on the earth are the anisotropic materials. And

then it is better to use the index-ellipsoid for studying the transmission character of light scattered due to the anisotropic materials. However, it is very hard to theoretically analyze the abovementioned character using the electromagnetic wave optics and also the wave optics at the present. And then, the geometric optics is applied to analyze the transition mechanism of light in advance, because the geometric optics equal to wave optics when the wavelength become zero. Based on such a consideration⁽¹³⁾⁻⁽¹⁹⁾, we have derived the three dimensional transmission equation of light lines along the light axis where the many index-ellipsoids are arranged in series⁽²⁰⁾⁽²¹⁾.

The present paper describes mainly the conditions the maximum transmission rate of light lines exists at the limited distance of two adjacent materials and that is strongly affected by the ellipsoid rate.

2. Equation of Light Lines in the Three Dimensional Frame of Coordinate

For analyzing the three dimensional refractive phenomena to the plural index-ellipsoid, it is need to rotate an index-ellipsoid for the an arbitrary angle in the three dimensional space.

2.1 Rotation Around y and z Axes Firstly, by rotating θ degree around the z axis, it has to create the first conversion matrix for transferring the point P(x,y,z) on the Y-Z plane to the point P'(x',y',z') on the surface of index ellipsoid, and also by rotating ϕ degree around the y axis, the point P'(x',y',z') to the other point P''(x'',y'',z'') on the same index ellipsoid. If the rotation matrix for rotating θ degree around the z axis assumes A and that for rotating ϕ degree around the y axis assumes B, The matrix for transferring the point P(x,y,z) to the point P''(x'',y'',z'') is given by the following equation.

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = A \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = AB \begin{pmatrix} x \\ y \\ z \end{pmatrix} \dots\dots\dots (1)$$

* Technical Research and Development Institute 2nd Research Center, Japan Defense Agency

1-2-24, Ikejiri, Setagaya-ku, Tokyo 154-8511

** Musashi Institute of Technology

1-28-1, Tamazutsumi, Setagaya-ku, Tokyo 158-8557

*** Shibaura Institute of Technology

3-9-14, Shibaura, Minato-ku, Tokyo 108-8548

where the two matrixes are given as follows:

$$B = \begin{pmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{pmatrix}, \quad A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If the product of matrixes A and B is equal to C, the matrix C becomes as follows;

$$C = AB = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \dots \dots \dots (2)$$

2.2 Fundamental Equation of the Rotated Index-ellipsoid

The equation of light lines for one index-ellipsoid arranged in the three dimensional frame coordinate is given as follows;

$$F(x, y, z) = \frac{(x-x_c)^2}{n_x^2} + \frac{(y-y_c)^2}{n_y^2} + \frac{(z-z_c)^2}{n_z^2} - 1 \dots \dots \dots (3)$$

where, n_x, n_y and n_z are the main refractive index of x, y and z axes, and $n_x = \alpha n_n, n_y = \beta n_n, n_z = \gamma n_n$. Here, n_n is the proper refractive index of medium and α, β, γ are the ellipsoid rate of the each axis. From Eq. (2), Eq. (1) can be rewritten as follows;

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C^{-1} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \dots \dots \dots (4)$$

where, $|C| \neq 0$ and then C^{-1} is given by

$$C^{-1} = \frac{\Delta}{|C|}$$

Here, Δ is a cofactor.

From these assumptions, Eq. (3) can be rewritten as follows:

$$F(x, y, z) = \frac{(a_1'x'' + a_2'y'' + a_3'z'' - x_c)^2}{(n_x)^2} + \frac{(b_1'x'' + b_2'y'' + b_3'z'' - y_c)^2}{(n_y)^2} + \frac{(c_1'x'' + c_2'y'' + c_3'z'' - z_c)^2}{(n_z)^2} - 1 \dots \dots \dots (5)$$

2.3 Equation of Light Lines for the Plural Index Ellipsoid

2.3.1 In the case of one index ellipsoid

Figure 1 is a typical model of three dimensional trace of light lines scattered due to one index ellipsoid arranged in the three dimensional frame of coordinate. In the figure, $F_i(x, y, z)$ indicates the index-ellipsoid given by Eq. (5). Here, the attached i is a parameter indicating the number of many index-ellipsoids arranged. The light line emitted from an arbitrary point $P_{0i}(x_{0i}, y_{0i}, z_{0i})$ on the incident light source plane is supplied to the point $P_{1i}(x_{1i}, y_{1i}, z_{1i})$ on the surface of index-ellipsoid. Here, the direction vector of incident light is given by $\vec{d}_{0i} = (a_{0i}, b_{0i}, c_{0i})$. The light line refracted at the point $P_{1i}(x_{1i}, y_{1i}, z_{1i})$ reaches the point $P_{2i}(x_{2i}, y_{2i}, z_{2i})$ on the surface of the index ellipsoid. Here, the direction vector of light line passing through in the index ellipsoid is given by $\vec{d}_{1i} = (a_{1i}, b_{1i}, c_{1i})$. The light line passing through the index ellipsoid is again refracted at the point $P_{2i}(x_{2i}, y_{2i}, z_{2i})$. Finally, this light line is supplied to the observation plane M. Here, the direction vector of light lines passing through the index ellipsoid is given by

Fig. 1. Trace of light lines scattered due to the index-ellipsoid

$\vec{d}_{2i} = (a_{2i}, b_{2i}, c_{2i})$. Similarly, by changing the point $P_{0i}(x_{0i}, y_{0i}, z_{0i})$ on the light source, the optical intensity is obtained from the number of light lines reached to the observation plane. In this case, the equation of light lines $\overline{P_{0i}P_{1i}}$ passing through the index ellipsoid can be explained as follows.

$$\overline{P_{0i}P_{1i}}: \begin{pmatrix} x_{1i} \\ y_{1i} \\ z_{1i} \end{pmatrix} = \begin{pmatrix} x_{0i} \\ y_{0i} \\ z_{0i} \end{pmatrix} + t_1 \begin{pmatrix} a_{0i} \\ b_{0i} \\ c_{0i} \end{pmatrix} \dots \dots \dots (6)$$

Here, t_1 is an arbitrary constant. The tangential plane equation at the intersection point of the light lines $\overline{P_{0i}P_{1i}}$ and the index-ellipsoid is given by Eq. (7).

$$\frac{\partial F_i}{\partial x}(x - x_{1i}) + \frac{\partial F_i}{\partial y}(y - y_{1i}) + \frac{\partial F_i}{\partial z}(z - z_{1i}) = 0 \dots \dots \dots (7)$$

And the normal direction vector at the intersection point $\vec{n}_{orm}$ is given by Eq. (8).

$$\vec{n}_{orm} = \left(\frac{\partial F_i}{\partial x}, \frac{\partial F_i}{\partial y}, \frac{\partial F_i}{\partial z} \right) \dots \dots \dots (8)$$

Figure 2 shows a typical ideal model of light refracted at the contour of index-ellipsoid. Here, it is assumed that the index-ellipsoid $F_i(x, y, z)$ existing in the medium with the refractive index n_2 exists in the medium with the refractive index n_1 . In this case, the incident light angle is θ_{1i} and the refracted light angle is θ_{2i} . Here, the suffix i indicates the number of the index-ellipsoid as mentioned above. The normal line vector $\vec{n}_{orm}$ is orthogonal to the tangential plane at the point $P_{1i}(x_{1i}, y_{1i}, z_{1i})$ and then the normal line vector and the direction vector of the incident light line are existing in the same plane i . According to Fig. 2(a), the direction vector $\vec{d}_{1i}$ exists on the plane i and the direction vector of refracted light line also exists on the same plane. The details of the direction vector of refracted light line are shown in Fig.2 (b). From the figure, it is possible to find out the direction vector of refracted light line $\vec{d}_{1i}$. The tangent bowstring of the refractive angle θ_{2i} and the incidental angle θ_{1i} existing in the plane i can be explained by Eq. (9).

$$\cos \theta_{1i} = \frac{|\vec{d}_{0i} \cdot \vec{d}_{1i}|}{|\vec{d}_{0i}| |\vec{d}_{1i}|}, \quad \cos \theta_{2i} = \sqrt{1 - \frac{n_1^2}{n_2^2} (1 - \cos^2 \theta_{1i})} \dots \dots \dots (9)$$

In this case, it is possible to derive the direction vector of refracted

Fig. 2. Cross section of refractive phenomena at boundary surface

light line $\vec{d}_{1i}$ as Eq. (10).

$$\vec{d}_{1i} = \frac{n_1}{n_2} \cdot \frac{\cos \theta_{1i}}{\cos \theta_{2i}} (\vec{d}_{0i} - \vec{n}_{orm}) + \vec{n}_{orm} \quad (10)$$

Here, $\vec{n}_{orm}$ is the normalized vector of the vector $\vec{d}_{0i}$ against the vector $\vec{n}_{orm}$ given in Eq. (11).

$$\vec{n}_{orm} = \frac{\vec{n}_{orm}}{|\vec{n}_{orm}|} |\vec{d}_{0i}| \cos \theta_{1i} \quad (11)$$

And then, the equation of light lines $\overline{P_{1i}P_{2i}}$ can be explained by Eq. (12).

$$\overline{P_{1i}P_{2i}}: \begin{pmatrix} x_{2i} \\ y_{2i} \\ z_{2i} \end{pmatrix} = \begin{pmatrix} x_{1i} \\ y_{1i} \\ z_{1i} \end{pmatrix} + t_2 \begin{pmatrix} a_{1i} \\ b_{1i} \\ c_{1i} \end{pmatrix} \quad (12)$$

Here, t_2 is an arbitrary constant. By using the same method in the case of Eq. (7), the equation of light lines $\overline{P_{2i}P_{3i}}$ is as Eq. (13).

$$\overline{P_{2i}P_{3i}}: \begin{pmatrix} x_{3i} \\ y_{3i} \\ z_{3i} \end{pmatrix} = \begin{pmatrix} x_{2i} \\ y_{2i} \\ z_{2i} \end{pmatrix} + t_3 \begin{pmatrix} a_{2i} \\ b_{2i} \\ c_{2i} \end{pmatrix} \quad (13)$$

Here, t_3 is an arbitrary constant. Eq.(13) can not use for estimating the intensity of light lines.

Figure 3 shows the details of the observation plane M. That shows the typical image figure where the refracted light lines inserted into the observation plane M. On the plane, small circles are arranged for counting the number of light lines. In the case of the light lines coming into the small circle called a cell, the number of light lines reached the cell on the xy plane is counted. Here, the intensity of light lines inserted into the cell which is given by the plural cells equation $(y - y_{ck})^2 + (z - z_{ck})^2 = r^2$ can be obtained by counting the number of incident light lines. From these facts, it is possible to estimate the value of t_3 in Eq. (13) when the $P_{3i}(x_{3i}, y_{3i}, z_{3i})$ counted becomes equal to that of the plural cells.

2.3.2 In the case of more two index-ellipsoids In the case of the plural index-ellipsoids ($F_{i(=1)}$), the refracted light lines of the last index ellipsoid become the incident light lines of the next one. For applying this condition into the following

Fig. 3. Observation plane M of light

Fig. 4. Parameter used in this calculation and trace of light lines

computation, it is require the parameters which are the two vectors indicating both position and direction. The condition for indicating position parameter is that the $P_{3i}(x_{3i}, y_{3i}, z_{3i})$ obtained using Eq. (13) relating to $F_i(x, y, z)$ becomes equal to the $P_{0j}(x_{0j}, y_{0j}, z_{0j})$ obtained using Eq. (6) relating to $F_{i+1}(x, y, z)$. Here, $j=i+1$. And the similar condition is also applied to derive the other direction vector. In the case of $F_i(x, y, z)$, the direction vector $\vec{d}_{2i} = (a_{2i}, b_{2i}, c_{2i})$ obtained at Eq. (8) becomes equal to $\vec{d}_{0j} = (a_{0j}, b_{0j}, c_{0j})$ obtained at Eq. (10). From these facts, by expressing the equation of light lines as the asymptotic equation Eq. (14) can be derived as follows.

For the position coordinate

$$P_{0j}(x_{0j}, y_{0j}, z_{0j}) \mid_{j=i+1} - P_{3i}(x_{3i}, y_{3i}, z_{3i}) = 0$$

and for the direction vector

$$\vec{d}_{0j} = (a_{0j}, b_{0j}, c_{0j}) \mid_{j=i+1} - \vec{d}_{2i}(a_{2i}, b_{2i}, c_{2i}) = 0$$

$$\dots \dots \dots (14)$$

Subsequently, the calculation technique used in the case of the plural index-ellipsoids connected in series is given.

Figure 4 is the relationship between the trace of the light lines and the parameter used for computing Eq. (14). It shows a typical model indicating the circumstance where the incident light reaches the index ellipsoid ($\#i$) and is reflected to the positive x direction due to the index ellipsoid ($\#i$) after incident light inserted into the index ellipsoid ($\#i-1$). Fig. 4 shows the last two mediums when the i_{th} mediums are arranged along the x axis direction in series.

The distance between the surface of the index ellipsoid ($\#i-1$) and that of the index ellipsoid ($\#i$) is given by the following equation with the main refractive index.

$$(x_{c(\#i)} - n_x) - (x_{c(\#i-1)} + n_x) = x_{c(\#i)} - x_{c(\#i-1)} - 2n_x$$

The focus point is explained by the distance x from the surface of the index-ellipsoid ($\#i-1$) where the number of light lines inserted into the observation plane becomes the maximum. According to the asymptotic equation of light lines shown in Eq.(14), it is possible to obtain the intensity of refracted light lines at the observation plane M. And at the same time the transmission rate of refracted light lines against the number of index-ellipsoid can be obtained.

3. Calculation Results and Discussion

3.1 In the Case of One Index-ellipsoid

3.1.1 Refractive index change It is assumed that All main refractive index (n_x, n_y, n_z) is equal. In this case, the ellipsoid rate becomes $\alpha = \beta = \gamma = 1$ and the index-ellipsoid becomes a sphere. And then the refractive character to the spherical index ellipsoid can be easily calculated, when the center is located at ($x_c = -1, y_c = 0, z_c = 0$). By substituting the main refractive index ($n_x = n_y = n_z = 1.5, n_n = 1.5$) or ($n_x = n_y = n_z = 1.97, n_n = 1.97$) into Eq. (10), the trace of light lines and the light intensity distribution are calculated. Here, $\theta = \varphi = 0$. The size of the spherical medium is normalized to 1.0. And then, the light direction becomes no unit.

Figure 5 shows a typical computed result. In the trace of light lines in Fig.5(a), the focus point appears at the out side of the

Fig. 5. Intensity distribution scattered due to one isotropic medium

Fig. 6. Intensity distribution scattered due to one anisotropic medium

spherical medium, while in the case of Fig.5(b) the focus point moves into the inside of sphere and the observable peak of the refracted light intensity appear at the surface of the medium.

Fig. 7. Light intensity distribution obtained by changing the gap length between two adjacent index-ellipsoid surfaces :

- (a) Ellipsoid rate= 0.94, (b) Ellipsoid rate = 0.97,
- (c) Ellipsoid rate = 1.0, (d) Ellipsoid rate = 1.03,
- (e) Ellipsoid rate = 1.07

3.1.2 Ellipsoid rate change By using the similar method mentioned in Fig.5, the trace of light lines and the light line intensity distribution are calculated.

Here, the ellipsoid rate along the x axis is changed such as $\alpha=0.8, 0.85, 0.9$ and 0.95 . But $\theta=\varphi=0$ and the center coordinate is ($x_c = -1, y_c = 0, z_c = 0$). Fig.6 shows the typical computed results. From the figure, it is found that the peak of the refracted light intensity appears at the out side of the sphere. In the case of the ellipsoid rate being 1.0 in Fig.5(b), the peak value does not appear at the out side of the medium. Even if the proper refractive index used in Fig.6 is the same as that used in Fig.5(b) the peak of refracted light intensity moves to the out side of the material only by changing the ellipsoid rate from 1.0 to 0.95. From this, it is found that the ellipsoid rate plays the important role to the transport rate of light lines.

3.2 In the Case of Two Index Ellipsoids In this case, the ellipsoid rate of x axis coordinate α is changed 0.94, 0.97, 1.0, 1.03, 1.06. And $\beta=\gamma=1$. The resulting main refractive index n_x becomes 1.85, 1.91, 1.97, 2.03, 2.09, respectively, while $n_y=n_z=1.97$. The distance between the index ellipsoid (#1) and the index ellipsoid (#2) is changed from 0.0 to 5.0 and then the trace of light lines and the intensity distribution of light lines are obtained.

Figure 7 shows a typical computed result. In the figure, the horizontal axis indicates the distance along the x axis, while the vertical indicates the number of the intensity of refracted light.

In the Fig.7, the left side indicates the light intensity against the distance from the surface of index ellipsoid (#1) as a function of the gap length between the two adjacent mediums. Contrary to this, the right side of Fig.7 indicates the maximum light intensity against the distance from the index ellipsoid (#1) as a function of the two adjacent mediums. From the curves, it can be seen that when the ellipsoid rate increases a little bit the focus point does not depend on the gap length. For example; in the case of the ellipsoid rate being 0.85 in Fig.7(a), the refracted light intensity distribution and the maximum light intensity are much depending on the gap length in the range of 0 to 0.5, while in the case of the ellipsoid rate being 1.0 in Fig.7(d), the refracted light intensity distribution and the maximum light intensity does not depend on the gap length in the same range.

Table 1 shows the calculated results of the maximum rate of the light transportation at the ellipsoid rate of $\alpha=0.94, 0.97, 1.0, 1.03$ and 1.06. From the table, the following results are obtained: In the

case of Fig.7(a), the maximum rate of light transportation is 88.7% at the gap length is 0.5. In the case of Fig.7 (b), (c), (d) and (e), the maximum rate of light transportation is 93.0% at the gap length is 0.4, 95.5% at the gap length is 0.3, 97.5% at the gap length is 0.2 and 100.0% at the gap length is 0.0, respectively. For example, when the two spherical isotropic materials of 7.5 mm in radius are arranged in series the maximum intensity exists at around 30mm from the surface of the first material in the gap length from 3 to 12.

4. Conclusion

The conclusion mentioned in this paper is summarized as follows:

- (1) We succeeded to derive the three dimensional equation for analyzing the transition light scattered due to the index ellipsoid, many of which are arranged directly in series.
- (2) It is found that in the case of the two index ellipsoids arranged along the optical axis, the maximum light transmission rate is existing at the limited gap distance between two adjacent spherical mediums.
- (3) It is also found that in the case of the two index ellipsoids arranged along the optical axis, the maximum light transmission rate is strongly affected by the ellipsoid rate.

However, more research is required to develop the light transmission mechanism for other optical systems where the ellipsoid rate and the refractive index are changed and also many mediums are arranged in series and also rotated.

(Manuscript received Nov. 14, 2003, revised March 24, 2004)

References

- (1) E. O. Forster and P. Wong : "High Speed Laser Schlieren Studies of Electrical Breakdown in Liquid Hydrocarbons", *IEEE Trans. Electr. Insul.*, Vol.EI-12, No.6, pp.435-442 (1977-12)
- (2) E. Morikawa : "Optical Observation of Pre-breakdown Phenomena in Dielectric Oil", *T. IEE Japan*, Vol.92, No.1, p.27 (1972-1) (in Japanese)
- (3) H. Yamashita, H. Amano and T. Mori : "Optical observation of pre-breakdown and breakdown phenomena in transformer oil", *J. Phys. D: Appl. Phys.*, Vol.10, pp.1753-1760 (1977)
- (4) Y. Murooka : "Observation on the intermittent electron emission in liquefied nitrogen", *J. Appl. Phys.*, Vol.48, pp.136-142 (1977)
- (5) Y. Hirashima and Y. Murooka : "A study of breakdown phenomena in liquefied nitrogen using schlieren and mach zehnder methods", *T. IEE Japan*, Vol.103-A, No.7, pp.311-318 (1983-7) (in Japanese)
- (6) Y. Murooka, Y. Hirashima, and M. Kubo : "Optical studies on hopping-transport mechanism of excess electrons in liquefied nitrogen", *J. Appl. Phys.*, Vol.53, pp. 4987-4995 (1982)
- (7) Y. Hirashima, M. Kubo, and Y. Murooka : "Study of electron transport mechanism in liquefied nitrogen under the high electric field", *T. IEE Japan*, Vol.101-A, No.6, pp.311-318 (1981-6) (in Japanese)
- (8) J. A. Stratton: *Electromagnetic Theory*, pp.564-591, McGraw-Hill (1941)
- (9) G. Mie : "Beiträge zur optik trüber medien, speziell kolloidaler metallösungen", *Ann. Physik*, Vol. 25, pp. 377-445 (1908)
- (10) T. Yoshida, K. Hidaka, and Y. Murooka : "Optical and mm-electromagnetic wave scattering phenomena for micro dielectric materials", *Proc. of the Conference ICPADM*, Vol.1, pp.65-68, Xi'an, China (1985)
- (11) T. Yamada, K. Makishima, T. Yoshida, M. Sone, and Y. Murooka : "Study of light and elector-magnetic wave scattering phenomena due to a tiny material", 1998 Annual Conference of Fundamentals and Material Society, IEE Japan, No.174, p.1-28 (1998)
- (12) T. Yoshida, T. Yamada, K. Makishima, M. Sone, and Y. Murooka : "A study on optical observation of electronic conduction phenomena in liquids", *International Conference on Dielectric Liquids 13th*, pp.227-300 (1999)
- (13) E.Hecht : *OPTICS* (Fourth Edition), No.4, pp.148-158, Addison Wesley (2002)
- (14) D. Badouel, K. Bouatouch, and T. Priol : "Distributing data and control for ray tracing in parallel", *IEEE Computer Graphics & Applications*, Vol.14, No.4, pp.69-77 (1994)

Table 1. Maximum transmissibility of light lines between the index-ellipsoids

No.	Main refractive index	Gap length	Maximum transmissibility [%]
1	$n_x=1.85, n_y=n_z=1.97$ $\alpha=0.94$	0.5	88.7
2	$n_x=1.91, n_y=n_z=1.97$ $\alpha=0.97$	0.4	92.0
3	$n_x=n_y=n_z=1.97$ $\alpha=1.0$	0.3	95.5
4	$n_x=n_y=n_z=1.97$ $\alpha=1.03$	0.2	100
5	$n_x=2.09, n_y=n_z=1.97$ $\alpha=1.06$	0.0	100

Here, $\beta=\gamma=1$.

- (15) K. Yoshihara : Physical Optics, p.194, Kyouritsu-Shuppan Press (1966) (in Japanese)
- (16) E. Fushimi : Linear Algebra and Geometry, p.14, p.32; Sankaido Press (1971) (in Japanese)
- (17) S. Nemoto : Linear Algebra and Exercise, p.149, Morikita-Shuppan Press (1980) (in Japanese)
- (18) M. Ohtsu : Modern Optical Science, pp.149-152, Asakura Press (1994) (in Japanese)
- (19) Y. Murooka, T. Yoshida, M. Sone, K. Shirakawa, and T. Maeno : "Fundamental study on electromagnetic wave scattered by spherical dielectric material smaller than wave length (Experiment and Theory)", Proc. of 29th Meeting on Lightwave Sensing Technology, JSAP02316, LST29-9, pp.59-66 (2002)
- (20) K. Makishima, T. Yamada, T. Yoshida, M. Sone, and Y. Murooka : "3-Dimensional analysis of light scattering phenomena in indicatrix", The Papers of Technical Meeting, IEE Japan, ED-98-216, pp.43-48 (1998)
- (21) T. Yoshida, K. Makishima, M. Sone, and Y. Murooka : "Three-Dimensional Analysis of Light Scattering Phenomena in Index-ellipsoid", 2003 Japan-Korea Joint Symposium on Electrical Discharge and High Voltage Engineering, pp.229-232 (2003)

Taro Yoshida

(Member) He received his M.E. degree from the NDA of Japan in 1985. Appointed as a research staff at the Technical Research and Development Institute, Japan Defense Agency. He engages in developing the detection system of the moving target using the electromagnetic wave, the light wave and also the infrared light with two frequency waves.

Kazuhiro Makishima (Non-member)

He received his M.E. degree from the Musashi Institute of Technology in 1998. Appointed as a staff at the Hitachi-Kasei K.K. in 1998 ~. He is belongs to the Development of Indication Material and engaged mainly in the development of liquid crystal mediums.

Takeshi Yamada

(Non-member) He received his M.E. degree from the Musashi Institute of Technology in 1998. Appointed as a staff at the Fujita Kannko K.K. in 1998. He retired from the same company in 2003. Appointed as a staff at the Seiko K.K. in 2003.

Mototaka Sone

(Member) He received his M.E. degree from the Musashi Institute of Technology in 1965. His work is mainly the breakdown mechanism of organic compound, the discharge phenomena in gasses, the high-speed calculation on the power transmission system, the practical research work to the electric power system using the neural-network and fuzzy, the research on DSP. Appointed as a Professor at the Musashi Institute of Technology in 1990 ~. Appointed as the Director of Library at the Musashi Institute of Technology in 2003 ~. The members of Electronic and Information Society, the Institution of Electrical Engineering, Measurement and Automatic Control Society, the IEEE Society.

Yoshihiro Murooka (Member)

He received his D.E. degree from the University of Tohoku in 1963 and his Ph.D. degree from the University of Liverpool (UK) in 1968. Appointed as a Professor at the NDA of Japan in 1969. He retired from the NDA of Japan in 1997 and received an Emeritus Professor from the same Academy in 1997. Appointed as a Professor at the Musashi Institute of Technology in 1997. He retired from the Musashi Institute of Technology in 2001. Appointed as a Visiting Professor at Shibaura Institute of Technology in 2001 ~. His honors are; the Promotion Medals from the IEE of Japan in 1985 & 1998, the Duddell Premium from the IEE of England in 1986, the 100 years Anniversary Competition Medal from the IEE of Japan in 1988, the Scientific Achievement Medal from the Japanese Government in 1989.