Detection of Plural Cracks in Steel using Horizontal Coils
— 3D FEM Analysis Considering Hysteresis and Non-Uniformity of Steel —

Yuji Gotoh* Member
Norio Takahashi** Member

The alternating magnetic flux leakage testing is used for the detection of cracks on the surface in steel plate. Generally, the permeability and conductivity in steel are not uniform. As a result, the detection signal of the electromagnetic inspection method may not be uniform. In this paper, an efficient detecting technique of plural cracks using the amplitude and phase angle of flux density of horizontal (x) coil is proposed. The detecting property of plural cracks in non-uniform steel is investigated using 3-D edge-based hexahedral FEM considering the hysteresis and the non-uniformity of permeability and conductivity in steel. The possibility of the detection of plural cracks using the amplitude and phase angle of the horizontal (x) component is illustrated.

**Keywords**: alternating magnetic flux leakage testing, 3D-FEM, hysteresis, non-destructive inspection.

1. Introduction

The alternating magnetic flux leakage testing has been applied in the inspection for detecting cracks on the surface of steel. This testing detects the leakage flux from the cracks in ferromagnetic material magnetized by an ac electromagnet (1×5). The permeability and conductivity in the steel are usually not uniform. Therefore, the leakage flux may not be uniform. The non-uniformity of conductivity in the steel is not controlled by the exciting magnetic field. However, the non-uniformity of permeability can be reduced when the exciting magnetic field is increased. The effect of such non-uniformity on the error of the detection of crack was not investigated systematically.

By the way, it is proved in some papers (4-5) that the result of FEM (finite element method) analysis is in agreement with the actual measurements of electromagnetic non-destructive testing. Therefore, FEM can be used to examine the electromagnetic phenomenon in the alternating magnetic flux leakage testing, and to investigate the detection method of crack.

In this paper, the leakage flux that is influenced by the non-uniformity of permeability and conductivity in the steel (SS400) is examined by 3-D edge-based hexahedral FEM (5). The method for detecting plural cracks using the parallel (Bx) component (7) of leakage flux density is investigated by considering the hysteresis and nonuniform conductivity. The behavior of the amplitude Bx and phase angle $\theta$ of leakage flux and the effect of hysteresis on the phase angle $\theta$ are also investigated. The possibility of detecting plural cracks using Bx and $\theta$ is examined.

2. Measurement of Non-uniformity of Permeability and Conductivity in Steel

2.1 Permeability

Fig.1 shows the electromagnet for measuring the non-uniformity of permeability in steel (SS400).

A column specimen (diameter: 9mm, length: 160mm) is put between pole pieces. The flux density in the steel is measured using a search coil, and the magnetic field strength is measured using a transverse type Hall probe. The search coil and the Hall probe are moved in the x-direction, and the non-uniformity of permeability in steel is measured. Fig.2 shows the maximum dispersion rate $\varepsilon_p$ of permeability of the steel. The dispersion rate $\varepsilon_p$ of relative permeability $\mu_r$ is defined by:

$$\varepsilon_p = \frac{\mu_r(\text{each position}) - \mu_r(\text{average})}{\mu_r(\text{average})} \times 100 \quad (1)$$

![Fig. 1. Electromagnet for measuring non-uniformity of permeability and conductivity in steel](image)

![Fig. 2. Dispersion rate $\varepsilon_p$ of permeability of steel SS400 (measured)](image)

* Department of Electrical and Electronic Engineering, Kurume National College of Technology, Kurume, Fukuoka 830-8555
**Department of Electrical and Electronic Engineering, Graduate School of Natural Science & Technology, Okayama University, 3-1-1, Tsushima, Okayama 700-8530
The figure denotes that the non-uniformity ($\varepsilon_i$) of permeability is reduced, when the magnetic field is increased. This is, because the direction of each magnetic moment becomes nearly the same at high flux density.

2.2 Conductivity The non-uniformity of conductivity in the steel (SS400) is measured using a column specimen (96 x 600mm) shown in Fig.3 (a). A coil is wound around the center of the steel. This is moved by 5mm pitch, and ac field (40Hz, $2.5 \times 10^5$ T) is applied. The impedance of the coil is measured using an LCR meter. On the other hand, the impedance is calculated for each conductivity and permeability using the axis-symmetric FEM. The uniform conductivity and permeability are given in the FEM model of Fig.3 (a), and the impedance $Z$ and phase angle $\phi$ are obtained for each conductivity and permeability as shown in Fig.3 (b). Then, the non-uniformity of the conductivity and permeability in the steel is evaluated using the calculated and measured results of impedance of the coil. Fig.4 shows the non-uniformity of conductivity $\sigma$ which is obtained from the results of Fig.3 at each position. The dispersion rate $\varepsilon_i$ of conductivity is defined similar to Eq. (1). The figure denotes that the non-uniformity of conductivity is about 30% at the maximum.

3. Inspection Method and Modeling of Non-uniform Material Characteristics and Hysteresis

3.1 Inspection Method Fig.5 (a) shows a model of alternating magnetic flux leakage testing that detects plural cracks. The amplitude of current is 1A (rms). The gap between the leg of yoke and the surface of steel is 0.2mm.

Fig.5 (b) shows the proposed differential type search coils for detecting the parallel component $B_x$ of leakage flux from plural cracks. The leakage flux $B_x$ which is uniformly distributed over the steel surface is measured using the search coil $\beta$. The local leakage flux $B_x$ from the cracks is measured using the search coil $\alpha$. The difference between $B_x$ obtained by the search coil $\alpha$ and that by the search coil $\beta$ is used for detecting the number and size of plural cracks. The x-length of the search coil $\beta$ is set to 0.1mm. The y-length (Sw-\alpha and Sw-\beta) of the $B_x$ search coils $\alpha$ and $\beta$ is 5.5mm. The x-length (Sl-\beta) of the $B_x$ search coil $\beta$ is 59mm. The y-length of the search coils $\alpha$ and $\beta$ is 0.06mm.

The sizes of these coils were optimized using the evolution strategy. The distance (lift-off (Lo)) between the search coils and the surface of steel is 0.1mm. Therefore, the search coils (thickness < 0.1mm) can be located under the legs of yoke. The frequency is chosen as 1kHz, of which the skin depth is equal to 0.15mm (the relative permeability is assumed as 2500).

3.2 Method of Analysis The basic equation with eddy current in the case of the $A-\phi$ method is given by:

$$\text{rot}(\nu \text{rot} A) = J_s - \sigma \left( \frac{\partial A}{\partial t} + \nabla \phi \right)$$

$$\text{div}\left(-\sigma \left( \frac{\partial A}{\partial t} + \nabla \phi \right) \right) = 0$$

Fig. 3. Dispersion of conductivity and relative permeability of steel (40Hz)

Fig. 4. Dispersion of conductivity in steel (SS400, 40Hz, measured)

Fig. 5. Model for alternating flux leakage testing of plural cracks
Table 1. Conditions of analysis and experiment

<table>
<thead>
<tr>
<th>Exciting coil</th>
<th>1kHz, 1A rms, 30turns × 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes and elements</td>
<td>88095, 83232</td>
</tr>
<tr>
<td>Convergence criterion</td>
<td>N-R method: 0.01T ICCG method: 1.0 × 10⁻³</td>
</tr>
</tbody>
</table>

![B-H curves](image1)

(a) B-H curves

![μ-H curves](image2)

(b) μ-H curves

Fig. 6. Dispersion of Magnetization curves (SS400)

where $A$ is the magnetic vector potential, $\phi$ is the scalar potential, $v$ is the reluctivity, $J_e$ is the current density and $\sigma$ is the conductivity.

3-D FEM using the 1st order hexahedral edge element is applied. The flux and eddy current are analyzed by the step-by-step method taking account of the non-linearity of steel plate. In order to get the steady state result, the calculation is carried out during 2.5 periods (≈160 steps). The time interval $\Delta t$ of the step-by-step method is chosen as $1.5625 \times 10^6$ sec. The yoke is assumed to be linear (relative permeability: $\mu_r$ ~60,000) and the eddy current in it is neglected. The laminations of yoke is not taken into account. The conditions of analysis and experiment are shown in Table 1.

3.3 Modeling of Non-Uniform Initial Magnetization Curve and Conductivity in Steel

Fig. 6 shows the dispersion of magnetization curves. Forty-seven magnetization curves having 46% variation of maximum permeability are created. A curve is chosen at random among forty-seven magnetization curves following the normal random number (standard deviation: 5.0), and the selected curve is given in each finite element in the steel region. Similarly, the conductivity is generated according to a normal random number (standard deviation: 0.5) within the range between $3.0 \times 10^6$ and $6.0 \times 10^6$ S/m referring to Fig. 4. Fig. 7 shows the calculated results of the distribution of conductivity and relative permeability in the steel without crack. The figure denotes that the non-uniformity of relative permeability is reduced, if the magnetizing current, then the flux density is increased, but the non-uniformity of conductivity does not change.

3.4 Modeling of Hysteresis

The hysteresis curves used for nonlinear analysis are shown in Fig. 8. These curves were measured under 1mHz alternating magnetic field by an automatic measuring system using a permeameter. The hysteresis curve having maximum flux density of arbitrary amplitude is

![Hysteresis curves](image3)

Fig. 8. Measured hysteresis curves (SS400)
interpolated using the adjacent hysteresis curves \(^{(9)}\). The reluctivity \(\nu\) tends to infinity at \(B=0\) T when the hysteresis is taken into account. The reluctivity at \(|B|=1.0 \times 10^{0.1}\) T is used as the value of \(\nu\) at \(B=0\) T.

4. Examination of Effect of Non-uniformity of Steel and Hysteresis

4.1 Effect of Non-Uniformity of Permeability and Conductivity

The non-uniform flux distribution in the steel due to the non-uniformity of \(\sigma\) and \(B-H\) curve is examined using the differential type \(Bx\) search coil shown in Fig.5. The movement of yoke is simulated by shifting the permeability and conductivity inside the steel in the x-direction by 0.05mm step.

Fig.9 (a) shows the calculated results of dispersion rate \(\epsilon_{Bx}\) at 0.5A when there is no crack. The dispersion rate \(\epsilon_{Bx}\) of \(|Bx|\) is defined by:

\[
\epsilon_{Bx} = \frac{|Bx| \text{ (each position)} - |Bx| \text{ (average)}}{|Bx| \text{ (average)}} \times 100
\]

(i) Non-uniform \(\sigma\)

(ii) Non-uniform \(B-H\) curve

The curves (i) and (ii) are obtained under the condition that only conductivity or only permeability is not uniform. The figure denotes that the dispersion \(\epsilon_{Bx}\) is mainly affected by the non-uniform conductivity. The non-uniformity of the leakage flux is within about \(\pm 1\%\). When the magnetizing current is equal to 2A, and the flux density is high (the maximum flux density in steel: 1.81T), \(Bx\) is mainly affected by the non-uniformity of conductivity as shown in Fig.9 (b).

Fig.10 shows the maximum non-uniformity \(\epsilon_{Bx}\) of \(|Bx|\) in the search coil which is estimated by the calculation when there is no crack. The maximum value of \(\epsilon_{Bx}\) is always about \(\pm 1\%\).

4.2 Effect of Hysteresis

The effect of hysteresis is examined using \(|Bx|\) search coil shown in Fig.11 (a). There are only one search coil and one crack, and the crack width (x-direction, \(Cw\)), crack depth (z-direction, \(Cd\)) and crack length (y-direction, \(Cl\)) are 0.5mm, 1mm and 100mm, respectively. The same magnetic yoke in Fig.5 (a) is used and the current in the exciting coil is equal to 1A (rms) (1kHz). The gap between the leg of yoke and the surface of steel is equal to 0.2mm. The gap between the leg of yoke and the surface of steel is 0.2mm. Fig.11 (b) shows the x-component of leakage flux detected by the \(|Bx|\) search coil. The hysteresis loop or initial magnetization curve is considered in the analysis. The measured result is also shown in the figure.

The figure shows that the results with hysteresis and without...
hysteresis are both in agreement with the measurement. However, the amplitude of \( |Bx| \) in a search coil, which is obtained considering hysteresis, is nearer to the measured one compared with the result without hysteresis.

5. Detection of Plural Cracks using Horizontal Coils

5.1 Amplitude of Leakage Flux The detection characteristics are analyzed by the nonlinear calculation considering the hysteresis curves and non-uniform conductivity. The evaluation of the number of plural cracks is performed using the differential type search coil shown in Fig.5 (b). Fig.12 shows the distribution of \( Bx \) of one, two and four cracks detected using the differential type coil. The crack width (\( C_w \)), depth (\( C_d \)) and distance (\( L \)) of plural cracks are 0.01mm, 1mm and 0.5mm, respectively. The figure illustrates that even if \( C_d \) is constant, the amplitude of \( Bx \) changes with the number of cracks.

Fig.13 shows the effect of depth of cracks when two cracks exist. The figure illustrates that the amplitude of \( Bx \) is increased as the crack depth becomes deep.

These figures denote that the non-uniformity of conductivity hardly influences the inspection of cracks.

5.2 Phase Angle of Leakage Flux The phase angle of leakage flux under AC excitation is influenced by both the eddy current and the hysteresis characteristics in steel. Therefore, the hysteresis should be considered in the analysis of phase angle. Fig.14 shows the Lissajous of flux density in the search coil \( \alpha \) at each position \( D \) near the crack when four cracks exist. The figure illustrates that the direction of leakage flux changes with time and position. Figs.15 and 16 show the \( Bx \) waveforms detected by the search coil \( \alpha \) at \( D=0.755 \)mm (above crack) and \( D=0.85 \)mm (right side of crack). The results of which the hysteresis is considered are neglected are shown in the figure. The waveforms of the magnetizing current \( i_0 \) and the \( z \)-component of flux density \( Bz \) are also shown. The figure illustrates that the phase angle between \( Bx \) and \( i_0 \) becomes large when the hysteresis is considered. Moreover, the phase angle between \( i_0 \) and \( Bx \) becomes small when the search coil \( \alpha \) approaches the crack (\( D=0.755 \)mm). Therefore, the position crack can be presumed from the phase angle of \( Bx \).

Fig.17 shows the change of the phase angle \( \theta \) of the parallel component \( Bx \) of leakage flux detected by the differential search coil when hysteresis is taken into account. \( \theta \) is the angle of the waveform, which is the difference between \( Bx \) in the coil \( \alpha \) near crack and \( Bx \) in the coil \( \beta \) in the adjacent region, and this denotes

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**Fig. 12.** Detection of \( Bx \) using differential search coil (\( C_w=0.01 \)mm, \( C_d=1 \)mm, \( L=0.5 \)mm, 1kHz, 1A, calculated, with hysteresis)

**Fig. 13.** Effect of depth of cracks using on \( Bx \) (1kHz, 1A, calculated, with hysteresis)

**Fig. 14.** Lissajous waveforms of flux density of search coil \( \alpha \) at each position \( D \) near cracks (four cracks, 1kHz, 1A, calculated)

**Fig. 15.** Waveform of \( Bx \) detected by search coil \( \alpha \) (\( D=0.755 \)mm (above crack), four cracks, 1kHz, 1A, calculated)
the effect of crack. The result of four cracks when hysteresis is not taken into account is also shown. The figure denotes that the phase angle $\theta$ becomes small at the crack position. This is, because the opposing flux due to the eddy current is large near the crack. It is also shown that the change of $\theta$ becomes remarkable when the hysteresis is taken into account. Moreover, the phase angle $\theta$ is hardly influenced by the non-uniformity of conductivity in steel.

6. Conclusions

The results obtained by this research are summarized as follows:

(1) It is shown that the non-uniformity of permeability becomes small when the steel is saturated.

(2) The dispersion of flux density is mainly affected by the non-uniformity of conductivity in steel. But it is within about 1%. The non-uniformity of permeability and conductivity does not influence the inspection result.

(3) It is possible to evaluate plural cracks by using the amplitude and phase angle of the parallel component $B_x$ of leakage flux detected by the differential search coil.

(4) The change of phase angle due to cracks becomes remarkable when the hysteresis is taken into account.

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References


Yuji Gotoh (Member) was born in Kanagawa, on May 28, 1972. He received B.E. and M.E. degrees from Polytechnic University in 1996 and 1998, respectively, and Ph.D degree from Okayama University in 2002. He is presently a research associate of Department of Electrical and Electronic Engineering, Kurena National College of Technology. His major interest is the development of electromagnetic non-destructive inspection method for the steel material. He is a member of The Japanese Society for Non-destructive Inspection, The Japan Society of Applied Electromagnetics and Mechanics and International COMPUMAG Society.

Norio Takahashi (Member) was born in Hyogo, on April 28, 1951. He received a B.E. degree from Okayama University in 1974 and M.E. and Ph.D. degrees from Kyoto University in 1976 and 1982, respectively. He is presently a professor of Department of Electrical and Electronic Engineering, Graduate School of Natural Science & Technology, Okayama University. His major interests are the development of numerical methods for calculating magnetic fields and optimal design methods for magnetic devices. Vice-President of International COMPUMAG Society, IEEE Fellow.