Analysis of Lightning-induced Impulse Magnetic Fields in the Building with an Insulated Down Conductor

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Key words : lightning protection system, impulse magnetic field, coaxial cable, building

Recently insulated down conductors, such as high-voltage coaxial cables have been increasingly used for lightning protection in buildings. They may be routed in electrical ducts within or on façade of a building. During a direct strike the down conductors discharge the lightning current and generate substantial magnetic fields around the building. The potential interference to sensitive equipment in the building becomes of concern. This paper presents a modeling procedure and the analysis of impulse magnetic fields in the building with an insulated down conductor.

In this paper modeling of an insulated down conductor is described first. The insulated down conductor, as shown in Fig.1, is subdivided into a number of short sections according to the maximum frequency of the lightning transient current. Each section consists of two concentric conductive elements insulated by a dielectric layer, that is, core and screening. It is modeled as an equivalent π -type circuit as shown in Fig.2. The formulas for calculating the parameters of equivalent circuit are presented.

The models of the wire-grid structure, the earth terminal and the lightning stroke are also presented in the paper. The influence of the earth is included in these models using the image method. A system model is then formed by integrating the models of relevant components in a building. With the system model transient current

distribution during a lightning strike is obtained using EMTP. The impulse magnetic fields are calculated with the Biot-Savart law.

To validate the proposed system model, an experiment was conducted in a laboratory. Figure.3 shows the setup of this experiment. Both the branch currents and the magnetic fields inside the structure were measured and compared with the results numerically calculated. It was found that both measured and calculated values matched very well with an average error of less than 5%.

The modeling approach was applied to analyze impulse magnetic fields in a full-scale building equipped with an insulated down conductor. The simulation results indicate that the lightning-induced magnetic field around an insulated down conductor is significantly high, and can be as high as that generated from a standalone down conductor. It is highly recommended not placing sensitive equipment in the area near the insulated down conductor.

The metallic structure of a building can attenuate the impulse magnetic field from the insulated down conductor because of the induced currents if the structure is electrically interconnected. It is recommended moving the insulated down conductor either to a column or outside the building in order to achieve the significant field compensation in the area around the insulated down conductor.

Finally, a mitigation measure of the impulse magnetic fields is addressed. It was found that the impulse magnetic field could be reduced significantly if the down conductor was housed in an earthed metal pipe. The metal pipe and metal structure of the building are bonded together. During a direct lightning strike, a large induced current will be generated in the pipe to compensate the field from the insulated down conductor. The field can be reduced down to 10% of its value calculated without the pipe in some cases. This pipe, however, does not have a significant impact on the field at the location far away from the insulated down conductor.

Fig. 1. Sectional view of a HV coaxial cable

Fig. 2. π -type circuit model of section n

Fig. 3. Experimental setup for model validation

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This paper presents an analysis of lightning-induced magnetic fields in a building. The building of concern is protected by the lightning protection system with an insulated down conductor. In this paper a system model for metallic structure of the building is constructed first using the circuit approach. The circuit model of the insulated down conductor is discussed extensively, and explicit expressions of the circuit parameters are presented. The system model was verified experimentally in the laboratory. The modeling approach is applied to analyze the impulse magnetic fields in a full-scale building during a direct lightning strike. It is found that the impulse magnetic field is significantly high near the down conductor. The field is attenuated if the down conductor is moved to a column in the building. The field can be reduced further if the down conductor is housed in an earthed metal pipe. Recommendations for protecting critical equipment against lightning-induced magnetic fields are also provided in the paper.

Keywords : lightning protection system, impulse magnetic field, coaxial cable, building

1. Introduction

The lightning protection system (LPS) in a building is dedicated to intercept lightning discharges and to channel the discharge currents directly to ground via down conductors (DCs). Recently insulated down conductors, such as high-voltage coaxial cables have been increasingly used because of aesthetic and economic reasons⁽¹⁾. These down conductors are routed in electrical ducts within or on façade of a building. During a direct strike the down conductors generate substantial magnetic fields around the building. The potential interference to sensitive equipment in the building becomes of concern⁽²⁾. To protect the equipment effectively against lightning, it is necessary to analyze the impulse magnetic fields in the building.

Traditionally down conductors of the LPS in a building are made by reinforcing bars embedded in concrete columns and beams. These bars are interconnected each other, and form a conductive wire grid. Under a direct lightning strike this wire grid is modeled as a coupled electrical network. Lightning-induced impulse magnetic fields around the structure can be then analyzed using the circuit approach. In the past decades modeling of the wire-grid structure and analysis of the impulse magnetic fields have been carried out extensively⁽³⁾⁻⁽⁶⁾. However, the impulse magnetic fields associated with the insulated down conductors, which have been introduced in buildings recently, has not been addressed significantly.

This paper presents an analysis of lightning-induced impulse magnetic fields around a building using the circuit approach. The building of concern is protected by the LPS with an insulated down conductor. The paper starts with a description of the system model for metallic structure of the building. A circuit model of the

insulated down conductor is discussed, and formulas of its circuit parameters are derived. Experimental validation of the system model is also presented in the paper. With the modeling approach impulse magnetic fields in a full-scale building are characterized. Mitigation measures of the impulse magnetic fields is addressed. Finally, recommendations for protecting critical equipment against lightning-induced magnetic fields are provided.

2. System Model

During a direct strike the lightning current is discharged via the down conductors of a LPS in a building. The lightning energy, however, is not fully confined within the down conductors even if the insulated down conductor is adopted. Because mutual coupling, induced currents are generated within metallic structure of the building, such as interconnected reinforcing steel bars. The impulse magnetic fields around the building, therefore, are determined by both the injected lightning current and induced currents over the building structure.

A system model for the insulated down conductor, the metallic structure and others needs to be set up first. With the system model transient current distribution during a lightning strike can be obtained using EMTP or other software. The impulse magnetic field is then calculated with the Biot-Savart law.

2.1 HV Coaxial Cable Illustrated in Fig. 1 is the sectional view of an insulated down conductor (a HV coaxial cable). This down conductor consists of two concentric cylindrical shells, each of which is made by a layer of small copper wires. Unlike in power cables, these copper wires are laid straightly along the cable axis. The inner conductor (core) is dedicated to deliver the lightning current from an air terminal down to ground. The outer conductor (screening) is insulated from the core with dielectric material. It is connected to the core on the ground only.

Traditionally a coaxial cable is modeled as a transmission line in transient analysis. The transmission line model, however, is not applicable to such concentric conductors installed in a building. It is known that electromagnetic fields in the building, which result from a lightning strike, are not in a TEM structure. The π -type

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Fig. 1. Section view of a HV coaxial cable

Fig. 2. π -type circuit model of section n

circuit model, thus, is adopted to represent these concentric conductors.

To take account of the propagation phenomena of a lightning current over a long conductor, the insulated down conductor is subdivided into a number of short sections. The length of each section is usually less than one tenth of the wavelength corresponding to the maximum frequency of the lightning transient current⁽⁷⁾. Each section has two concentric conductive elements, that is, core and screening. It is modeled as an equivalent π -type circuit, as shown in Fig. 2.

In Fig. 2, $Z_{1n,1n}$ and $Z_{2n,2n}$ ($Y_{1n,1n}$ and $Y_{2n,2n}$) represent respectively self impedance (susceptance) of the core and the screening in section n . $Z_{1n,2n}$ ($Y_{1n,2n}$) is mutual impedance (susceptance) between the core and the screening. Matrices Z (impedance) and Y (susceptance) for sections m and n are given for illustration, as follows:

$$Z = \begin{bmatrix} Z_{1n,1n} & Z_{1n,2n} & Z_{1n,1m} & Z_{1n,2m} \\ Z_{2n,1n} & Z_{2n,2n} & Z_{2n,1m} & Z_{2n,2m} \\ Z_{1m,1n} & Z_{1m,2n} & Z_{1m,1m} & Z_{1m,2m} \\ Z_{2m,1n} & Z_{2m,2n} & Z_{2m,1m} & Z_{2m,2m} \end{bmatrix} \dots\dots\dots (1)$$

$$Y = \begin{bmatrix} Y_{1n,1n} & Y_{1n,2n} & Y_{1n,1m} & Y_{1n,2m} \\ Y_{2n,1n} & Y_{2n,2n} & Y_{2n,1m} & Y_{2n,2m} \\ Y_{1m,1n} & Y_{1m,2n} & Y_{1m,1m} & Y_{1m,2m} \\ Y_{2m,1n} & Y_{2m,2n} & Y_{2m,1m} & Y_{2m,2m} \end{bmatrix} \dots\dots\dots (2)$$

The entries of these matrixes are calculated by using the formulas given in Appendix.

2.2 Wire-Grid Structure Most modern buildings have a reinforced concrete (RC) structure which is comprised of RC columns, beams and others. The reinforcing bars inside these RC components are interconnected and form a complicated metallic structure. The bundle of reinforcing bars in a RC column or beam

Fig. 3. Conductor above the ground and its image

is usually modeled as a single reinforcing bar⁽³⁾⁽⁷⁾. Thus the complicated metallic structure in a building is simplified to be a conductive wire grid.

Each branch of the wire grid is divided into short elementary sections in order to take account propagation phenomena, as described in Section 2.1. Similar to the down conductor, each section is modeled as a lumped π -type circuit, which consists of transverse resistance, and transverse inductance and longitudinal capacitance. There exists mutual coupling between two sections in the wire grid, or one section in the wire grid and one section in insulated down conductor. Such mutual coupling is represented by mutual impedance and susceptance in the circuit model. Thus, a highly-coupled lumped electrical network is used to represent the building structure for electromagnetic analysis during a lightning strike.

Both impedance and susceptance of the conductive branches are determined by geometric parameters and material properties of the conductors. They are calculated with the Neumann's formula and the average potential method⁽⁸⁾.

2.3 Influence of the Earth In the model for both the down conductor and the wire grid the influence of the earth is included using the method of images, i.e. replacing the earth with a set of conductor images. As illustrated in Fig. 3, the image of a conductor is located at the equivalent height of l' below the ground. Height l' is expressed as

$$l' = l + d_p \dots\dots\dots (3)$$

where l is the height of the conductor above the ground, and d_p is the skin depth of the earth.

To satisfy with the boundary conditions on the ground both the conductor and its image have to carry the equal but unlike charges, and equal and like currents⁽⁹⁾, as shown in Fig. 3. In the electrostatic field analysis for determining conductor capacitance the earth is treated as a perfectly conducting plane. Skin depth, therefore, is identical to zero. While in calculation of conductor impedance the earth is modeled as a lossy and homogeneous medium characterized by permeability μ_0 and conductivity σ ⁽¹⁰⁾. Its skin depth is given by

$$d_p = 1/\sqrt{j\omega\mu_0\sigma} \dots\dots\dots (4)$$

2.4 Earth Termination Both the down conductor and the wire grid are earthed on the ground via dedicated earth electrodes. It has been confirmed experimentally and analytically⁽¹¹⁾⁽¹²⁾ that earth soil is ionized during a lightning strike. The soil ionization increases the equivalent contact area of an electrode, subsequently reduces the earth resistance. According to the Ref.(13) earth resistance R_g of an electrode is calculated with the formula given by

Fig. 4. Experimental setup for model validation

$$R_g = 0.2613 \rho^{0.692} L^{-0.384} \left(\frac{I}{241 \rho^{0.215}} \right)^{-0.384} \quad (5)$$

where I is the stroke current through the electrode (kA), and L is the length of the electrode (m). ρ is the earth resistivity ($\Omega \cdot m$)

2.5 Lightning Stroke The air terminal of the LPS is intended to intercept lightning. A lightning stroke to the building, therefore, is modeled as a unidirectional current source applied to the air terminal. According to the Ref.(14) the waveform of the lightning source current is expressed by

$$i(t) = \frac{I_m}{\eta} \frac{(t/\tau_1)^n}{1 + (t/\tau_1)^n} e^{-t/\tau_2} \quad (6)$$

where I_m is the peak value of the lightning current. n is a current steepness factor. τ_1 and τ_2 are respectively the front and tail time constants. η is the correction factor of the current peak, and is given by:

$$\eta = e^{[-(\tau_1/\tau_2)(n\tau_1/\tau_2)^{1/n}]} \quad (7)$$

3. Model Validation

3.1 Experimental Setup A small-scale model of building structure was set up in the laboratory for validating the circuit models discussed previously. The scaled model included an insulated down conductor and a wire grid formed by a set of interconnected reinforcing bars, as illustrated in Fig. 5. The down conductor was erected vertically and stood on a metal plane. Its core and screening were bond together on the metal plane only. The down conductor was 2.5m long, and had mean diameters of 12.3mm and 24.1mm for its core and screening, respectively. The upper end of the down conductor was connected to an impulse current generator (ICG). The wire grid stood on the metal plane too. It was constructed with 260 bars, each of which was 0.5m long and had a diameter of 5mm.

In each test an impulse current from ICG was injected into the core of the down conductor. The impulse current had a nominal peak value of 11kA. Because of the inductances of ICG and the wire grid the rising time to the peak value of the impulse current from ICG was set to 7.8 μ s. The time to its half peak value was 17.6 μ s. During the test induced currents were generated in the branches of the wire grid as well as in the screening of the down conductor. The impulse currents and the impulse magnetic fields inside the structure were respectively measured using a Rogowski coil and a tailor-made magnetic field probe together with a Tektronix Digital Oscilloscope TDS7104. Both the Rogowski coil and the field probe had a sensitivity of 0.007V/A and 2.16V/mT, and a linearity error of less than 1.0% and 1.5% under

Table 1. Normalized peak values of the impulse currents in four branches

Test Configuration	Branch 1	Branch 2	Branch 3	Branch 4
(a) Measured	0.0%	13.1%	8.66%	4.46%
Calculated	0.0%	13.5%	8.98%	4.77%
(b) Measured	54.9%	13.7%	22.9%	18.3%
Calculated	54.2%	13.9%	21.7%	17.4%

Table 2. Normalized peak values of the impulse magnetic field B_x/I (mT/kA)

Test Configuration	Point 1*	Point 2**	Point 3***
(a) Measured	0.341	0.240	0.159
Calculated	0.349	0.237	0.162
(b) Measured	0.121	0.0695	0.0360
Calculated	0.117	0.0727	0.0380

* (0.0, 0.25, 0.75) ** (0.125, 0.25, 0.75) *** (0.25, 0.25, 0.75)

the 7.8/17.6 μ s testing current, respectively⁽¹⁵⁾. For simplicity, both impulse current and magnetic field were normalized with the peak value of the impulse current from ICG, and only the peak values of the normalized current and field are presented.

3.2 Result Comparisons Two configurations of the down conductor were tested. These configurations include (a) the down conductor was electrically insulated from the wire grid except on the metal plane, (b) the upper end of the DC screening was bonded to a corner of the wire grid, as illustrated in Fig. 4. It should be mentioned that configuration (b) was not recommended by the manufacturer, but was selected for model validation only.

Table 1 shows both calculated and measured peak values of the normalized currents in 4 branches. In configuration (a), the current in branch 1 was too small to measure because of electrical isolation of the screening. The current in branch 1, however, became significant in configuration (b) because of bonding at the upper end. The induced current was allowed to circulate between the screening and the wire grid. It is noted from the table that both measured and calculated currents match very well with an average error of less than 5%.

The impulse magnetic fields were measured at three points within the wire grid. Table 2 shows the normalized peak value of the x components of the impulse magnetic field for comparison. In both configurations, the measured and calculated results match well, with an average error of less than 5%. It is noted from the table that the impulse magnetic fields in configuration (b) are much less than those in configuration (a). As seen in Table 1, the DC screening carried a significant induced current. This current flowed in the opposite direction of the DC current, which led to a significant cancellation of the field within the wire grid.

4. Simulation Results

The modeling approach presented in previous sections was applied to analyze lightning-induced impulse magnetic fields in a full-scale reinforced-concrete building. The building of concern is 72m in length, 36m in width and 43.2m in height. It has 11 $\times$ 7 RC columns, which are equally spaced with the separation distance of 7.2m. These columns and beams were modeled as a wire grid. Each of these wires represented the reinforcing bar in an elementary column or beam of the building.

The building is protected by the LPS using an insulated down conductor. The down conductor runs vertically, and is connected to the air terminal on the roof. Both air terminal and down conductor are located either in the center or at the corner of the

building. The down conductor has the same dimensions as the one presented in Section 3. Impulse magnetic fields over an observation line in the building were calculated in a direct lightning strike. The observation line runs from one corner of the building (column *a*) to its opposite corner (column *f*) on 3/F, as seen in Fig. 5.

The lightning stroke to the building was simulated to be a current source. In simulation both front-time and tail-time constants τ_1 and τ_2 were taken to $8\mu\text{s}$ and $10\mu\text{s}$, respectively. The peak value of the current I_m was selected to be 160kA . The current has an effective peak value 100kA for $n=2$, and a waveform of $8/20\mu\text{s}$ (time to peak value/time to half peak value).

4.1 Down Conductor Mounted on a Wall The insulated down conductor is usually mounted on the wall either in an electrical duct or on façade. Fig. 6 shows the peak values of impulse magnetic fields over the observation line when the LPS is positioned at point *o* (dash line). It is noted that the impulse magnetic field is significantly high in the area between columns *c* and *d* (critical area). The peak value of the field reaches 49.5mT at the point 0.36m away from the down conductor. The field, however, is generally less than 1mT outside this area.

To reveal the shielding effect of the wire grid, impulse magnetic fields along the same observation line were also calculated by assuming there was no wire grid. The peak values of the impulse magnetic fields are superimposed on Fig. 6 (solid lines). The fields are solely determined by the discharge current in the down conductor. It is noted from the figures that the fields are generally less in the presence of the wire grid. Such field cancellation generally arises from the currents induced within the wire grid. The field cancellation becomes stronger on the middle floors (e.g., 3/F), as seen in the figures. As an induced current has a significant influence on the field near its carrier (e.g., column), local field enhancement around columns is also observed in the figures. The

Fig. 5. Observation lines on 3/F for field calculation

Fig. 6. Impulse magnetic fields in the case of DC in the center

field can be even greater than that in the absence of the wire grid at certain locations.

Figure 7 shows peak values of the impulse magnetic fields when the LPS is installed at the corner of the building. In general these impulse magnetic fields have a similar pattern as those in the previous case. The fields generated from the down conductor are partially cancelled by the induced current. In this case, such field cancellation in the critical area is even more significant. The peak value of the field reaches 5.7mT at the point 0.36m away from column *a*. As the induced currents in columns *a* and *b* flow in opposite directions, the field cancellation in this area is enhanced.

4.2 Down Conductor Enclosed in a Pipe It is noted that impulse magnetic fields are significantly high in the area close to the down conductor. The situation can be improved if a conductive loop is constructed in this area to compensate the original field. In order to enhance field compensation it is necessary to use certain measures to increase the induced current in the loop. As seen in Fig. 1, the down conductor has a conductive screening. When the screening is isolated from the wire grid, there is almost no induced current on the screening. Thus, the screening has no shielding effect against the impulse magnetic fields. However, when the screening is bound to the wire grid at its both ends, the induced current will be significantly increased. This has been verified by the test presented in Section 4. Unfortunately the screening is not designed to carry the lightning current. The connection to the wire grid is not recommended by the manufacturer in practice.

To achieve a similar performance, a separate metal pipe is introduced to house the down conductor, and to carry the induced

Fig. 7. Impulse magnetic fields in the case of DC at the corner

Fig. 8. Impulse magnetic fields in the case of DC enclosed by a pipe in the center

Fig. 9. Impulse magnetic fields in the case of DC enclosed by a pipe at the corner

current. This pipe has a diameter of 0.2m and a thickness of 10mm. It is bonded to the wire grid on the ground as well as on the roof. As the induced current in the pipe flows in the opposite direction of the DC current, significant reduction of the impulse magnetic field is expected in the critical area around the DC. Figure 8 shows the peak values of impulse magnetic fields when the LPS is situated in the center of the building. Clearly the impulse magnetic fields become less in the critical area. The highest peak value is reduced to 2.75mT from 49.5mT on 3/F. However, this pipe does not have a significant impact on the field at a location away from the down conductor, as illustrated in the figures.

Figure 9 shows the peak values of impulse magnetic fields when the LPS is situated at the corner of the building. Similar to the previous case, the impulse magnetic fields are generally less in the critical area when the pipe is installed. On the 3/F the highest peak value is reduced to 3.8mT from 5.65mT.

5. Conclusion

This paper investigated the impulse magnetic fields arising from the discharge current in an insulated down conductor installed in a building. A system model for building structure during a direct lightning strike was constructed using the circuit approach. The π -type circuit model of the insulated down conductor was developed as well as the formulas for calculating its circuit parameters. To validate the system model laboratory tests on a small-scale structure were performed. Both experiment and simulation results matched well under two testing configurations.

The modeling approach was applied to analyze impulse magnetic fields around a full-scale building during a direct lightning strike. It is concluded that:

(1) The lightning-induced magnetic field around an insulated down conductor is significantly high, and can be as high as that generated from a standalone down conductor. It is highly recommended not placing sensitive equipment in the area near the down conductor.

(2) Reinforcing bars of columns and beams in the building can attenuate the impulse magnetic field if they are interconnected each other. Because of the induced currents in these bars the impulse magnetic field is reduced. It is recommended placing the down conductor next to a column or outside the building in order to achieve significant field compensation in the area around the down conductor.

(3) To enhance the field compensation further it is

recommended housing the down conductor with a metal pipe, and bonding both the metal pipe and metal structure of the building together. A large induced current will be generated in the pipe to compensate the field from the down conductor. The field can be reduced down to 10% of its value without the pipe in some cases. This pipe, however, does not have a significant impact on the field at the location away from the down conductor.

(4) Side flashing due to breakdown of the conductor insulation may occur under harsh conditions. Because of the side flashing the down conductor is accidentally connected to the wire grid. Both the down conductor and the wire grid now discharge the lightning current together. The approach proposed in this paper can be applied to determine the impulse magnetic field under such conditions. The location of the side flashing and the time when the side flashing occurs should be identified, and included in the system model.

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Appendix

As seen in Fig. 1, an insulated down conductor consists of two concentric cylindrical conductors insulated by dielectric material. Such a down conductor is modeled as two thin conductive cylindrical shells separated by a layer of dielectric material characterized by the permittivity of ϵ_1 . app. Fig. 1 illustrates a section of the down conductor running vertically above the ground and its image. It contains two concentric conductive cylindrical elements, which are labeled by 1 and 2. These elements have the radii of R_a and R_b , respectively, and have the axial length of $L_n = l_{n+1} - l_n$. As the down conductor is rotationally symmetric with respect to its axis, both surface charges and axial currents are uniformly distributed along the circumferential direction.

1. Impedance

In calculating mutual impedance, the axial current in an element is treated as a surface current. The Neumann's surface integral formula⁽¹⁶⁾, then, is applied. For the case of the elements situated above the ground, mutual impedance $Z_{im,jn}$ between element i ($i=1$ or 2) in section m and element j ($j=1$ or 2) in section n becomes

$$Z_{im,jn} = \frac{j\omega\mu_0}{16\pi^3} [S_{im,jn} + S_{im,j'n'}] \dots\dots\dots (A1)$$

where $S_{im,jn}$ and $S_{im,j'n'}$ are the coupling coefficients associated with element i and its image, respectively. $S_{im,jn}$ is expressed by

$$S_{im,jn} = 2\pi \int_0^{2\pi} \left[d_{m,n+1} H\left(\frac{d_{m,n+1}}{A}\right) + d_{m+1,n} H\left(\frac{d_{m+1,n}}{A}\right) \right. \\ \left. - d_{m,n} H\left(\frac{d_{m,n}}{A}\right) - d_{m+1,n+1} H\left(\frac{d_{m+1,n+1}}{A}\right) \right] d\theta \dots\dots\dots (A2)$$

where $H(x) = \sinh^{-1}(x) + \sqrt{1+x^2}$. In Eq. (A2) $A^2 = R_i^2 + R_j^2 - 2R_i R_j \cos\theta$, and axial distances $d_{m,n} = |l_m - l_n|$. $S'_{im,jn}$ can be calculated with Eq. (A2) in which $d_{m,n}$ is substituted by $d'_{m,n} = |l_m + l_n|$.

Self impedance of a cylindrical element is comprised of internal impedance and external impedance. The external impedance can be calculated with Eq. (A1), in which both two elements coincide in space. The internal impedance of the element, however, is determined by the current distribution over its cross section area. For a cylindrical element with inner radius R_i and outer radius R_o , its internal impedance $Z_{in,ini}$ was derived in Ref. (17), and expressed by

$$Z_{in,ini} = -\frac{j\omega\mu L_n}{2\pi R_i} \cdot \frac{I_0(x_o)K_1(x_i) + I_1(x_i)K_0(x_o)}{I_1(x_i)K_1(x_o) - I_1(x_o)K_1(x_i)} \dots\dots\dots (A3)$$

where $I_n(x)$ and $K_n(x)$ are the first and second modified Bessel functions of order n with argument $x = R\sqrt{j\omega\mu\sigma}$, respectively. Both μ and σ are the permeability and conductivity of the element.

2. Susceptance

An electrostatic field analysis is performed to determine susceptance of an insulated down conductor. Because of the dielectric material bound charges appear on its dielectric surfaces when the down conductor is energized. Each section of the down conductor, therefore, is modeled as four charge-carrying concentric cylindrical elements with zero thickness. These

elements include the conductive surfaces 1 and 2 of the conductors at $r = R_a$ and R_b , the dielectric surfaces 3 and 4 of the insulator at $r = R_a$ and R_b as well, as illustrated in app. Fig. 1.

Assume that the charge density is constant over an element. The potential at a point can be expressed in terms of charge density using a free-space potential formula. The average potential on a conductive element in a system of N sections, therefore, is given by

$$\bar{V}_{i,m} = \frac{1}{16\pi^3\epsilon_0} \sum_{j=1}^4 \sum_{n=1}^N \frac{1}{L_m L_n} (S_{im,jn} - S_{im,j'n'}) Q_{jn} \dots\dots\dots (A4)$$

on surface i

where $\bar{V}_{i,m}$ is the average potential on an inner ($i=1$) or outer ($i=2$) conductive element in section m ($m=1 \dots N$). Q_{jm} is the free charge ($j=1$ or 2), or the bound charge ($j=3$ or 4). Both $S_{im,jn}$ and $S_{im,j'n'}$ are given in Eq. (A2).

The boundary condition of electric field on a dielectric surface is applied to eliminate those bound charges in Eq. (A4)⁽¹⁸⁾. The electric field in the radial direction (r) is given by $-\partial V/\partial r$, in which V is the potential at the point of interest. The average radial field components on both positive and negative sides of element i ($i=3$ or 4) are obtained by integrating over its surface. The equations, which arise from the boundary condition on elements $i=3$ and $i=4$ in section m , are given by

$$0 = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^4 \sum_{n=1}^N (T_{im,jn} - T_{im,j'n'}) \frac{Q_{jn}}{L_n} \\ + \frac{Q_{1m}}{2\epsilon_0 L_m} - \left(\frac{\epsilon_0 + \epsilon_1}{\epsilon_0 - \epsilon_1} \right) \frac{Q_{3m}}{2\epsilon_0 L_m} \text{ on surface 3} \dots\dots\dots (A5)$$

$$0 = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^4 \sum_{n=1}^N (T_{im,jn} - T_{im,j'n'}) \frac{Q_{jn}}{L_n} \\ - \frac{Q_{2m}}{2\epsilon_0 L_m} + \left(\frac{\epsilon_0 + \epsilon_1}{\epsilon_0 - \epsilon_1} \right) \frac{Q_{4m}}{2\epsilon_0 L_m} \text{ on surface 4} \dots\dots\dots (A6)$$

where $T_{im,jn}$ and $T_{im,j'n'}$ are the coupling coefficients of the electric field associated with element i in section n and its image. Both $T_{im,jn}$ and $T_{im,j'n'}$ are given by.

$$T_{im,jn} = \int_0^{2\pi} \frac{R_i - R_j \cos\theta}{A} \left[H'\left(\frac{d_{m,n+1}}{A}\right) + H'\left(\frac{d_{m+1,n}}{A}\right) \right. \\ \left. - H'\left(\frac{d_{m,n}}{A}\right) - H'\left(\frac{d_{m+1,n+1}}{A}\right) \right] d\theta \dots\dots\dots (A7)$$

app. Fig. 1. Cylindrical element of the down conductor and its image

where $H'(x) = \sqrt{x^2 + 1}$. Parameters A and d_{ij} in Eq. (A7) are defined previously in Eq. (A2). $T_{im,j'n'}$ can be calculated with Eq. (A7) in which $d_{m,n}$ is substituted by $d'_{m,n}$ defined in Eq. (A2). The second or third item of Eqs. (A5),(A6) results from a singular integral in which both the source and observation elements coincide in space. With Eq. (A4) and Eqs. (A5),(A6), a matrix equation for both free and bound charges is assembled, as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} \quad \text{.....(A8)}$$

where both V_n and Q_n are the potential and charge vectors on the elements in section n . Sub-matrix P_{nm} is formulated using the formulas given in Eq. (A4) and Eqs. (A5),(A6). Equation (A8) may be expanded if other conductors are presented in the vicinity of the down conductor, such as the wire grid of a building.

Inverting Eq. (A8) as well as collecting the equations for the free charge vectors yield,

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{.....(A9)}$$

where K_{ij} is an $N \times N$ sub-matrix of P^{-1} . Susceptance matrix Y for N sections or $2N$ elements of the down conductor, then, is expressed by

$$Y = j\omega \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad \text{.....(A10)}$$

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