Parallel Ferroresonance Circuit Analysis by Chua-type Magnetization Model

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Various types of electrical apparatus using magnetic materials have been developed. Due to nonlinear properties of magnetic material, e.g., magnetic saturation, hysteresis, eddy current, etc.; the electrical apparatus occasionally exhibits complex responses that can not be anticipated and calculated easily. In the design of modern magnetic devices, prediction of various responses to the complex input signals is of paramount importance to prevent the troubles of devices. Nevertheless, any of the deterministic methodologies has not been yet proposed to do this mainly caused by the complex magnetization behaviors of inductors.

To clarify the regularity of chaotic behaviors in the up-to-date power magnetic devices, this paper carries out transient analysis in a parallel ferroresonant circuit exactly taking into account the magnetic hysteresis, saturation, aftereffects, and frequency dependence of ferromagnetic material properties. To extract the system regularity, the characteristic values of the state variable equations for the ferroresonant circuit are calculated in each step in the calculation period. It is revealed that the changes of characteristic values have no hysteretic properties, even though chaotic phenomenon is exhibiting.

To carry out transient analysis of ferroresonant circuit, we employ a Chua-type magnetization model representing dynamic constitutive relation between magnetic field \( H \) (A/m) and flux density \( B \) (T) as

\[
H = \frac{1}{\mu} B + \frac{1}{s} \left( \frac{dB}{dt} - \mu_r \frac{dH}{dt} \right)
\]

Equation (1)

where \( \mu, \mu_r, \) and \( s \) denote permeability (H/m), reversible permeability (H/m), and hysteresis parameter (Ω/m), respectively.

A significant feature of these parameters is that they are determined independently to the past magnetization histories because of the ideal magnetization curve approach, as pointed out by Bozorth. The permeability \( \mu \) and the reversible permeability \( \mu_r \) are obtained from the ideal magnetization curve and minor loops respectively when measuring the ideal magnetization curves.

When \( B=0 \), \( s \) is determined by:

\[
s = \frac{1}{H_c} \left( \frac{dB}{dt} - \mu_r \frac{dH}{dt} \right)
\]

Equation (2)

where \( \mu_r \) takes maximum value and the applied field \( H \) corresponds to coercive force \( H_c \). Thus, determination of parameter \( s \) in Eq. (2) requires the measurements of the \( dB/dt \) and \( dH/dt \). The validity of this model has been verified by the precise experiments of magnetization characteristics excepting for anisotropic materials and permanent magnets of typical materials.

Figure 1 illustrates \( dV_{out}/dt \) versus \( V_{out} \), exhibiting chaos-like behavior not tracing the same locus while the frequency of the driving voltage \( v \) is fixed at \( t=7.8 \) ms. Let us compare the series and parallel ferroresonant phenomena. At the beginning of resonance, either output response drastically increases. If the driving voltage is fixed when the ferroresonant mode is reached, we have nonlinear oscillation continuously. On Poincare diagrams, the parallel ferroresonance shakes \( dV_{out}/dt \) although the frequency of driving voltage is fixed. Further, the series ferroresonance has the same nature of small shaking in the current applied to the inductor. Since \( dV_{out}/dt \) in parallel ferroresonant circuit is associated with current, then these phenomena suggest that the chaos-like flicking is closely related to a condition of input term.
Parallel Ferroresonance Circuit Analysis by Chua-type Magnetization Model

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This paper studies the nonlinear response of a parallel ferroresonant circuit. To carry out a transient analysis in parallel ferroresonant circuit, we apply Chua-type magnetization model to an inductance exhibiting saturation and hysteretic nonlinear properties of ferromagnetic materials, deriving a state variable equation and solutions by the backward Euler method with automatic modification. The characteristic values of the state transition matrix are calculated in each calculation step of Euler method in order to extract the chaotic characteristics. As a result, it is clarified that the chaotic behavior in the ferroresonant circuit is greatly concerned with the magnetic aftereffect of ferromagnetic materials.

Keywords: Chua-type magnetization model, parallel ferroresonant circuit, chaotic phenomena, characteristic values

1. Introduction

Various types of electrical apparatus using magnetic materials have been developed. Due to nonlinear properties of magnetic material, e.g., magnetic saturation, hysteresis, eddy current, etc.; the electrical apparatus occasionally exhibits complex responses that can not be anticipated and calculated easily. In the design of modern magnetic devices, prediction of various responses to the complex input signals is of paramount importance to prevent the troubles of devices. Nevertheless, any of the deterministic methodologies has not been yet proposed to do this mainly caused by the complicated magnetization behaviors of inductors.

To clarify the regularity of chaotic behaviors in the up-to-date power magnetic devices, this paper carries out transient analysis in a parallel ferroresonant circuit exactly taking into account the magnetic hysteresis, saturation, aftereffects, and frequency dependence of ferromagnetic material properties(1)-(5). To extract the system regularity, the characteristic values of the state variable equations for the ferroresonant circuit are calculated in each step in the calculation period. It is revealed that the changes of characteristic values have no hysteretic properties, even though chaotic phenomenon is exhibiting.

2. Parallel Ferroresonant Circuit

2.1 Chua-type Magnetization Model

To carry out transient analysis of ferroresonant circuit, we employ a Chua-type magnetization model representing dynamic constitutive relation between magnetic field \( H \) (A/m) and flux density \( B \) (T) as

\[
H = \frac{1}{\mu} B + \frac{1}{s} \left( \frac{dB}{dt} - \mu_r \frac{dH}{dt} \right) \tag{1}
\]

where \( \mu, \mu_r \), and \( s \) denote permeability (H/m), reversible permeability (H/m), and hysteresis parameter (\( \Omega \)/m), respectively(3)(5). Figure 1 shows the measured curves giving these parameters for ferrite (TDK H5A). A significant feature of these parameters is that they are determined independently to the past magnetization histories because of the ideal magnetization curve approach, as pointed out by Bozorth(6). The permeability \( \mu \) and the reversible permeability \( \mu_r \) are obtained from the ideal magnetization curve and minor loops respectively when \( B=0 \), \( s \) is determined by

\[
s = \frac{1}{H_c} \left( \frac{dB}{dt} - \mu_r \frac{dH}{dt} \right) \tag{2}
\]

where \( \mu_r \) takes maximum value and the applied field \( H \) corresponds to coercive force \( H_c \). Thus, determination of parameter \( s \) in Eq. (2) requires the measurements of \( dB/dt \) and \( dH/dt \). The validity of this model has been verified by the precise experiments of magnetization characteristics excepting for anisotropic materials and permanent magnets of typical materials(7)(10).

2.2 Formulation

Consider a parallel ferroresonant circuit shown in Fig. 2. At first, the line integral of Eq. (1) along with flux path \( l \) yields magnetomotive force. Thus, the relation between the current \( i_1 \) and linkage flux \( \lambda \) of the inductor is given by

\[
N_i + \frac{\mu}{s} \frac{d\lambda}{dt} = \frac{l}{\mu AN} \lambda + \frac{l}{sAN} \frac{d\lambda}{dt} \tag{3}
\]

Moreover, a relation between the driving voltage source \( v \) and current \( i_1 \) is derived from the consideration of circuit connection and electromotive force \( dv/dt \)
Calculate $\Delta C$ second, substituting Eq. (3) into Eq. (4) yields the state equations having $3 \times 3$ square state transition matrix $a$

$$
\begin{align*}
    & \frac{d\lambda}{dt} = \begin{pmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \lambda \\ \frac{1}{s}N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{s}V_{\text{out}} \end{pmatrix} \\
    & \frac{d\mu}{dt} = \frac{1}{s}N \begin{pmatrix} 1 & \frac{1}{r} & 0 \\ 0 & 1 & \frac{1}{rC} \end{pmatrix} \frac{\lambda}{\mu} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{s}V_{\text{out}} \end{pmatrix}
\end{align*}
$$

or

$$
\frac{d\sum}{dt} = ax + b(t)
$$

where the elements $a_{21}$, $a_{22}$, ..., $u_2$ and $u_3$ in Eq. (7) are determined by Eqs. (5) and (6).

2.3 Backward Euler Method The backward Euler strategy to numerical solution of Eq. (7) makes it possible to carry out a transient analysis of ferroresonant circuit. As shown in Fig. 3, the calculation compares two solutions in each calculation step. The first one is one step solution with time step width $\Delta t$ ($s$), and another is two steps solution with $\Delta t/2$. Evaluating the difference between them reveals a relevant step-with for each of the calculations. Namely, if the difference is greater than a criterion listed in Table 1, then the same period is recalculated with the modified time step width $\Delta t = \Delta t/2$. The iteration with this modification is carried out until the criterion is satisfied.

In the iterative calculation, the nonlinear parameters, shown in Fig. 1, $\mu$ and $\mu_0$ are treated as functions of flux density $B$ and $s$ as a function of $dB/dt$.

3. Results and Discussion

3.1 Ferroresonant Phenomenon Figure 4 shows the

![Parallel Ferroresonance Circuit Analysis](image)

Fig. 3. Flowchart of the calculation with the adaptive step size control

Finally, the state Eqs. (5) and (6) yield a system of state variable equations having $3 \times 3$ square state transition matrix $a$
calculated result of Eq. (7) employing the parameters in listed Table 1. As demonstrated in Fig. 4 (b), the experimentally obtained output voltage well agrees with calculated one. As shown in Fig. 5, the frequency of the driving voltage \( v \) is decreased from 3.0 to 1.441 kHz until time \( t = 7.8 \) ms in order to observe its ferroresonant process. Around this moment, the output voltage \( V_{\text{out}} \) drastically increases, exhibiting the typical ferroresonant phenomena.

### 3.2 Chaotic Behavior

Figure 6 illustrates \( dV_{\text{out}}/dt \) versus \( V_{\text{out}} \) obtained from Fig. 4 (b), exhibiting chaos-like behavior not tracing the same locus while the frequency of the driving voltage \( v \) is fixed at \( t = 7.8 \) ms. Let us compare the series and parallel ferroresonant phenomena. At the beginning of resonance, either output response drastically increases. If the driving voltage is fixed when the ferroresonant mode is reached, we have nonlinear oscillation continuously. On Poincare diagrams, the parallel ferroresonance shakes \( dV_{\text{out}}/dt \) although the frequency of driving voltage is fixed. Further, the series ferroresonance reported in Ref. (11) has the same nature of small shaking in the current applied to the inductor. Since \( dV_{\text{out}}/dt \) in parallel ferroresonant circuit is associated with current, then these phenomena suggest that the chaos-like flicking is closely related to a condition of input term of Eq. (7) or Eq. (8).

### 3.3 System Regularity

To consider the state of ferroresonant system in detail, we calculate the characteristic values of the state transition matrix \( a \) in Eq. (7) in each of the calculation steps. The characteristic value analysis could be applied to only the linear system. So that we essentially assume this nonlinear system to be a piecewise linear system in each of the calculation steps for solving Eq. (7).

Figure 6 shows the loci of characteristic values derived from the state transition matrix \( a \) in Eq. (7) assuming \( a \) to be linear in each calculation step. Since \( a \) is a 3×3 square matrix, we have three characteristic values. Fig. 7 shows time versus characteristic values, presenting that these are tracing on the regular loci. Meanwhile, the output voltage locus exhibits a chaotic flicking. One of the causes of this chaotic flicking is an equivalent coercive force \( H_c = (\mu_r/s)dH/dt \) in the Chua type magnetization model (1). This instantaneously leads to the positive real part of the characteristic values as shown in Fig. 7 (b).

Thus, this is the cause of the unstable chaos-like flicking. The characteristic values at the other timing are all on the left half plane on a complex coordinate system.

Let us consider the characteristic values by decomposed into the real and imaginary parts. Figs. 7 (b)-(d) express the characteristic values given as pure real number, complex number in real part, and complex number in imaginary part, respectively. Since the complex number characteristic values are always given as
complex conjugate pairs, then we omit one of them. The real parts in Figs. 7 (b) and (c) regularly oscillate with changes of amplitude after the resonance starts, affecting nonlinear output response in $V_{out}$ when resonance occurring. The imaginary part in Fig. 7 (d) oscillates in nearly constant amplitude and drastically increases after the rise of resonance. This gives that the state of resonance depends on the imaginary parts. Observing the characteristic values makes it possible to predict the ferroresonant phenomena.

4. Conclusions

In this paper, we have derived the state variable equations of the parallel ferroresonance circuit employing the Chua-type magnetization model, and carried out the transient analysis of the parallel ferroresonant circuit. The characteristic value analysis of the state transition matrices obtained in every calculation step of Euler method has elucidated that the cause of chaotic flicking is an equivalent coercive force $H_c = (\mu_r/\sigma) dH/dt$.

As described above, it is revealed that our approach employing the Chua-type magnetization model clarifies the precise processes of the parallel ferroresonance phenomenon.

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