Investigation on the Electromagnetic Force Distribution in the Armature Teeth of a Permanent Magnet DC Motor

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The torque, to some extent, is an important performance index of electric motors. In many permanent magnet DC motors, the armature core is slotted with the winding embedded into the slots. What kind of force occurs on the armature teeth (or slot) and hence forms the torque is an interesting problem. In the past some relevant researches were reported, which concluded that all torque came from the force generated in the flanks of the armature teeth, and the force on the teeth surface facing the field magnet contributed nothing to the output torque. The objective of this paper is to investigate the overall electromagnetic force distribution in the armature teeth of a permanent magnet DC motor. An U-shaped core is employed as an analytical model which covers only one tooth pitch of the slotted armature, and the derivation of the force on the surfaces of the U-shaped core is obtained by means of the Maxwell stress method. Experimental results show that both tangential force on the teeth surface facing the field magnet and normal force in the flanks of the teeth contribute to the motion of a rotor. The analytical method and force expressions are verified by the experimental results.

Keywords: electromagnetic force, force distribution, Maxwell stress, normal force, tangential force, teeth

1. INTRODUCTION

The performance of electric motors has been significantly characterized by the force or torque that needs to be known as accurately as possible. In many permanent magnet DC motors, the armature core is slotted, with the winding embedded into these slots. Because of the large permeability of the armature, little flux links with the coils. The force can be viewed as a consequence of teeth acted upon by the PM field. In the past a few studies concerning the force distribution on the armature teeth were reported[1][2][3] and the closed-form solution was given[4]. However, these previous works concluded that all forces contributing to the motion of the rotor produced in the two flanks of the armature slot (this kind of force is called “armature torque” in the literature) and there is no tangential force on the teeth surface facing the field magnet. During the past two decades, with the advance of computer technologies coupled with the software package for numerical analysis, many researchers dealt with the torque computation of permanent magnet DC motors[5][6][7], yet, they mainly focused on the total torque of a full rotor. To our knowledge, the literature accurately dealing with the electromagnetic force distribution in the teeth is rare. Since not only the total force but also the local force is required in the design and optimization of electric motors, a better understanding of the force distribution in the teeth is helpful to improve the design and to meet the performance specification[8]. Therefore, it is meaningful, from practical points of view, to illustrate the distribution of the electromagnetic force in the armature teeth.

The paper is organized as follows. The first part theoretically points out that the traditional analytical methods are not suitable to investigate the electromagnetic force distribution in the armature teeth of a PMDC motor, hence introduces Maxwell stress method as a theoretical tool to conduct the analysis. The second part consists of the modeling of one tooth pitch of the armature in a permanent magnet DC motor by means of an U-shaped core. The third part describes the derivation of the force expressions for the surfaces of the U-shaped core and the experimental results.

2. THEORETICAL CONSIDERATIONS

Generally the method for the magnetic force calculation can be classified into two categories. The first contains Maxwell
stress method and energy method (virtual work principle). They have more a mathematical than physical meaning. The second contains the method of the equivalent magnetic current (EMC) or charge. They are very closely related to the macroscopic model of materials. Although with different methods the expressions for the forces are different, these models are equivalent representation of one reality, i.e., they are equivalent in calculating the global force (total force) of a magnetic body.

The expressions of magnetic forces in the flanks of an armature slot given in references [1], [3] and [4] are based on the formula below, which is derived by energy method:

\[
\tilde{f}_e = \left[ \tilde{T} \times \mu H \right] - \frac{1}{2} H^2 \text{grad} \mu + \tilde{H} \cdot \text{div}(-\tilde{J}_e) ,
\]

where \( \tilde{f}_e \) is volume force density, \( \tilde{H} \) is magnetic field intensity, \( \tilde{T} \) is electric current density, and \( \tilde{J}_e \) is permanent magnetization of the magnetic material. If there is no electric current and no permanent magnetization in the magnetic material, (1) becomes

\[
\tilde{f}_e = -\frac{1}{2} H^2 \text{grad} \mu .
\]

The expression (2) is purely a volume force density. Although total force acting on a magnetic body achieved by integrating (2) is expressed in the form of the integral of the surface force (for a linear magnetic material grad \( \mu \) is zero inside the body), the force includes the internal stress on the interior surface. So they merely represents the total force distribution. External surface force densities can not be explained by the total force, which means that (1) and (2) are not applicable to calculate the local force or to analyze the electromagnetic force distribution in the armature teeth. Moreover, when utilizing such an analytical method, the interior forces in the two slot flanks can not be canceled or neglected if electric current flows in the armature coil.

In this research, the Maxwell stress method is utilized. The principal advantage of using the Maxwell stress method is that the stress distribution is solely related to the external surface of boundary with different permeability, and can be predicted. The Maxwell stress method is based on the fact that the field distribution inside a closed surface in air remains unchanged if the external sources are removed and replaced by currents and poles on the surface. Thus the force, \( \tilde{F} \), on a region can be calculated by integrating the force density, \( \tilde{f} \), over a surface \( S \), enclosing the region:

\[
\tilde{F} = \frac{1}{\mu_0} \int_S \left( \tilde{B} (\tilde{B} \cdot \tilde{n}) - \frac{1}{2} \tilde{B}^2 \cdot \tilde{n} \right) ds ,
\]

where \( \tilde{n} \) is the normal vector to the surface. Instead of evaluating (4) directly, it is easier to further expand (4) into the normal and the tangential components on the air-iron boundary

\[
F_t = \frac{1}{\mu_0} \int_S B_t B ds ,
\]

\[
F_n = \frac{1}{\mu_0} \int_S (B_n^2 - B_t^2) ds ,
\]

where \( B_t \) is tangential component of the flux density, \( B_n \) is normal component of the flux density, \( F_t \) is tangential component of the electromagnetic force, \( F_n \) is normal component of the electromagnetic force, and \( \mu_0 \) is permeability of the free space. Formulæ (5) and (6) are the basic building blocks utilized in the derivation below.

### 3. ANALYTICAL MODEL

Typically, armature slot shape may be trapezoidal, rectangular or round. For simplicity, we consider the rectangular profile of the armature slot. In addition, the following two assumptions are made:

1. There is no magnetic flux saturation in the iron core of the analytical model.
2. The eddy-current on the surface of the iron core is negligible.

Fig.1 shows the introduction of the U-shaped core. To analyze the electromagnetic force in the teeth, the approach taken is first to introduce an analytical model which needs only to cover one tooth pitch of the armature because of periodicity.

Here an U-shaped core is employed to simulate a slot which can be imagined as being cutoff from the armature along the symmetrical lines of two adjacent teeth. The field magnet, the stator yoke and the coil are replaced by a homopolar linear DC motor. A rectangular coil, with one of its edges inserted in the slot of the U-shaped core, is fabricated on a mover. The general view of the model, together with a Cartesian reference frame, is shown in Fig.2, that would be used for the analysis below.

For the convenience of investigation, an U-shaped core with a deep slot is designed, so as to encourage flux to flow mostly from the magnet to the two limbs. This will result in least flux linking with the wire of the coil, and consequently reduction of force due to \( B_l \). It should be noted that, the coil with the U-shaped core on it will have preferred "detent" position along the stroke (x = 0), owing to the equal reluctance paths in the magnetic structure.

The most deviation of the analytical model from reality would happen to the two outer surfaces of the limbs, which...
are the undesired byproducts of the modeling. However, since the forces on the two surfaces have been considered, there would be no negative influence on the analytical results. From above it can be seen that the model is applicable to the analysis of electromagnetic force distribution in the armature teeth of permanent magnet DC motors.

The two-dimensional size of the analytical model is indicated in Fig. 3. The field magnet is NdFeB rare-earth material and the U-shaped-core is made of SS400 low carbon steel.

4. DERIVATION OF FORCES ON THE SURFACES OF U-SHAPED CORE

For the convenience of analysis, it is necessary to reduce the field problem to 2-dimension at x-y plane. Let $S_i$ denote each surface of the U-shaped core as shown in Fig. 4. The pattern of the flux distribution is also described. The direction of the current flowing in the coil is into the page. Besides, the nomenclature of some parameters that will appear in the derivation is given as follows:

- $B_{i(coil)}$: the flux density due to the coil on $S_i$,
- $B_{i(PM)}$: the flux density due to the PM on $S_i$,
- $F_{ix}$: x component of the electromagnetic force on $S_i$.

Refering to Fig. 4, the force in the x direction associated

$$B_{i(coil)} \quad x \text{ component of } B_{i(coil)}$$
$$B_{i(PM)} \quad x \text{ component of } B_{i(PM)}$$
$$B_{i(coil)} + B_{i(PM)} \quad x \text{ component of the resultant flux density due to the PM and the coil on } S_i$$
$$F_{ix} \quad x \text{ component of the electromagnetic force on } S_i. \quad (i = 1, 2, \ldots, 8)$$

$B_{i(PM)}$ and $B_{i(coil)}$ relate to the field magnet only. $B_{i(coil)}$ and $B_{i(PM)}$ come from the coil, by taking the PM as air ($\mu_0 \approx \mu_0$). With flux densities below saturation, $B_{x(PM+coil)}$ and $B_{x(PM)+B_{x(coil)}}$ are nearly the same, similarly $B_{y(PM+coil)}$ and $B_{y(PM)+B_{y(coil)}}$ are also nearly the same, which make it possible for the electromagnetic forces to be decomposed into various components that are independent of each other.
with $S_1, S_2, S_3, S_4$ is the normal force, and the one associated with $S_5, S_6, S_7, S_8$ is the tangential force. Thus the force in the $x$ direction, $F_x$, can be acquired by substituting appropriate flux density to equation (5) and (6), respectively. Then

$$F_x = \frac{1}{2\mu_0} \int \left[ (B_{1x}(PM) + B_{1x}(coil))^2 - (B_{2y}(PM) + B_{2y}(coil))^2 \right] ds .$$

(i=1, 2, 3, 4) (7)

Because of the large permeability of the core, $B_{2y}(PM + coil)$ in (7) is so small that its square is negligible. Thus (7) becomes

$$F_x = \frac{1}{2\mu_0} \int (B_{1x}(PM) + B_{1x}(coil))^2 ds$$

$$= \frac{1}{2\mu_0} \int B_{1x}(PM)^2 ds + \int B_{1x}(coil)^2 ds$$

$$+ \int 2B_{1x}(PM) \cdot B_{1x}(coil) ds , \quad (i=1, 2, 3, 4)$$

(8)

and

$$F_x = \frac{1}{\mu_0} \int B_{2x}(PM + coil) \cdot B_{2y}(PM + coil) ds$$

$$= \frac{1}{\mu_0} \int (B_{2x}(PM) + B_{2x}(coil))(B_{2y}(PM) + B_{2y}(coil)) ds$$

$$= \frac{1}{\mu_0} \int B_{2x}(PM)^2 ds + \frac{1}{\mu_0} \int B_{2x}(coil)^2 ds$$

$$+ \frac{1}{\mu_0} \int B_{2x}(PM) \cdot B_{2y}(coil) ds . \quad (i = 5, 6, 7, 8)$$

(9)

In equation (8) and (9), the first terms are the reluctance force which are independent of the coil current and always appear as detent force. The second terms produce zero force. The third term in (9) is very small. The final terms in both (8) and (9) contribute most of the output electromagnetic force.

1) Electromagnetic force on $S_1$ and $S_2$

$$F_{1x} = \frac{1}{2\mu_0} \int B_{1x}(PM)^2 ds + \int B_{1x}(coil)^2 ds$$

$$+ \int 2B_{1x}(PM) \cdot B_{1x}(coil) ds ,$$

(10)

$$F_{2x} = -\frac{1}{2\mu_0} \int B_{2x}(PM)^2 ds + \int B_{2x}(coil)^2 ds$$

$$- \int 2B_{2x}(PM) \cdot B_{2x}(coil) ds .$$

(11)

As mentioned earlier, the core is in the center along the stroke, and the flux density is symmetrical about the $y$ axis, thus

$$B_{1x}(PM)(y) = B_{2x}(PM)(y),$$

$$B_{1x}(coil)(y) = B_{2x}(coil)(y).$$

(12)

(13)

when these conditions are exposed on (10) and (11), the superposition of $F_{1x}$ and $F_{2x}$ becomes

$$F_{1x} + F_{2x} = \frac{2}{\mu_0} \int B_{1x}(PM) \cdot B_{1x}(coil) ds .$$

(14)

2) Electromagnetic force on $S_3$ and $S_4$

Based on the same reason as 1), the superposition force of $F_{3x}$ and $F_{4x}$ can be readily deduced.

$$F_{3x} + F_{4x} = -\frac{2}{\mu_0} \int B_{3x}(PM) \cdot B_{3x}(coil) ds .$$

(15)

3) Electromagnetic force on $S_5$ and $S_6$

The tangential force on $S_5$ and $S_6$ is given by (16) and (17).

$$F_{5x} = \frac{1}{\mu_0} \int B_{5x}(coil) \cdot B_{5y}(PM) ds ,$$

(16)

$$F_{6x} = \frac{1}{\mu_0} \int B_{6x}( coil) \cdot B_{6y}(PM) ds ,$$

(17)

$$F_{5x} + F_{6x} = \frac{1}{\mu_0} \int B_{5x}( coil) \cdot B_{5x}(PM) ds$$

$$+ \frac{1}{\mu_0} \int B_{6x}(coil) \cdot B_{6y}(PM) ds .$$

(18)

4) Electromagnetic force on $S_7$ and $S_8$

Because of the large permeability of the core, it can be predicted that $B_{8y}(coil)$ is so small that it can be neglected. Moreover, at the deep bottom of the slot $B_{7y}(PM)$ is also very small. These two conditions lead to very small tangential force on $S_7$ and $S_8.$

5. EXPERIMENTAL RESULTS AND DISCUSSION

5.1 Measurement of the Flux Density

The formulae for calculating the electromagnetic force on each surface of the core are derived in the preceding section. Generally, to precisely obtain the force values by means of Maxwell stress method will face some problems, especially for motors with small air gap, because it is subject to the availability and the accuracy of the flux densities[13]. However, for the analytical model with relatively larger air-gap ($g=3\text{mm}$), such problems can be considerably attenuated.

The variation of the flux density due to the field magnet and that due to the coil in (14),(15) and (18) are independent of each other, which makes it possible to achieve these values by measurement. Here the Hall probe is used to detect the normal components of flux densities. Because of the probe dimension,
it is poorly suitable to measure the tangential component of flux densities. So a small delicate air-core search coil is employed to detect the tangential component of flux densities due to the coil current (equivalent ac current is fed). Before measurement, the search coil together with an amplifier is calibrated via a Hall probe by measuring an ac flux density. The parameters of the search coil and the measurement setup of tangential component of flux are shown in Fig. 5.

The method of measuring these flux densities is as follows. Firstly, the U-shaped core (free of coil) is placed at the center of the air-gap in the model, and then data of normal flux density due to the permanent magnet on S1, S2, S3, S4, S5, and S6 are taken by scanning the core surface with a Hall probe. The position of the Hall probe is measured with a linear displacement stage for a series of x, y, and z values at an interval of 0.25mm, respectively. Similarly, when the tangential components of the flux density due to the coil is detected, the core remains in the same place, and in the absence of the field magnet, the measurement of the flux density due to the current (flowing in the coil) is performed. A 50Hz ac current equivalent to the dc current is given to the coil. The search coil is placed directly over the core surface, with the axis of the search coil parallel to the core surface so that the tangential component of flux can link with the search coil. By substituting these measured values into the discrete equations of (14), (15), (18), the electromagnetic forces associated to the respective surface of the core are calculated.

The distribution of measured flux densities on each surface of the U-shaped core due to the permanent magnet and the coil are shown in Fig. 6 and Fig. 7, respectively.

**5-2 Result Analysis**

The computed normal force distributions on S1, S2, and tangential force distributions on S5, S6 are shown in Fig.8 and Fig.9, respectively, in the case of I=1A and z=0. It is found from Fig.8 that the normal forces on S1 and S2 are mainly generated near the corner of the tooth. Owing to the current flowing in the coil, the force on S1 is larger than that on S2, the direction of the two forces is opposite to each other. Thus there
is a resultant force. As can be seen from Fig.9, most of the tangential force on $S_5$, $S_6$ occurs near the flanks of the slot. The resultant electromagnetic forces in the $x$ direction on $S_1$, $S_2$, $S_3$, $S_4$, $S_5$, $S_6$, and force from $B_{li}$ are all demonstrated in Fig.10, which suggests that when the shape of armature slots is rectangular, of the overall effective output force (forces on $S_1$, $S_2$, $S_3$, $S_4$ and the coil), around 70% comes from the resultant of normal force on $S_1$ and $S_2$. The force on $S_3$ and $S_4$ accounts for about 15%. The proportion of this kind of force in actual DC motors is expected to be much larger because of large $B_{ix(coil)}$ due to the intricate slot shape and partial saturation of armature teeth. According to (16) and (17), the distribution of this part of force is the product of normal flux density due to the field magnet and the tangential field intensity due to the coil current on core surface, which can be well explained by the magnetizing current method[14]. The force directly exerted on the coil according to $B_{li}$ law is small. Although the resultant force on $S_1$ and $S_4$ plays a negative role in the overall output force, it is of little interest because in practical motors it does not exist.

The static force measuring setup is constructed to verify the analytical results. To measure the thrust, the side of the rectangular coil embedded in the slot of the U-shaped core are fixed in two sliders, which are supported by two rails with the aid of linear bearings. The sliders along with the coil and U-shaped core can slide freely along the rail. A contact is glued to the two slides. If no electric current flows in the coil, the coil would located in the center of the stroke because of the balanced reluctance force. The contact of the slider touches the load cell (Kyowa DPM-611A) mounted on a framework outside the stator yoke of the model. When the electric current is given to the coil, the force due to the current is applied to the load cell by means of the contact of the slider.

The measured and calculated values are shown in Fig.12. There is fair agreement between the measured and calculated forces as the current in the coil varies from 0 to 1A. The deviation is within 7%. This error is believed to be mainly due to measurement errors, e.g. positioning of the Hall probe. Another possible source of error is the cancellation of components of flux density in the process of calculation.

6. CONCLUSIONS

In this paper a method of analysis of the electromagnetic force distribution in the armature teeth of a permanent magnet DC motor is proposed. From the theoretical analysis and the experimental results, the following results have been obtained:

1) The analysis shows that both the tangential force on the teeth surface facing the field magnet and the normal force on the teeth flanks contribute to the output torque of the permanent magnet DC motors. The analytical method and conclusion in the previous literature[1][3][4] have proved to be incorrect.

2) The tangential force density on the teeth surface facing the field magnet is approximately the product of the normal component of flux density due to the field magnet and the tangential component of field intensity due to the coil, as determined by (18). Since a model with rectangular shape slot is chosen in our study, the tangential force only accounts for 15% of the total force. It is expected that in actual DC motors the proportion of this tangential force is much larger because of large $B_{ix(coil)}$ due to the intricate slot shape and partial saturation of armature teeth.

3) The measured values of the forces agreed well with calculated values, the discrepancy being 7%, which proved that the proposed analytical method and the force formulae derived are proper.

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