High-Frequency Electromagnetic Currents of Transition Region of an Impedance Circular Cylinder Illuminated by an Obliquely Incident Plane Wave

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A Fock-type representation of the surface fields can be obtained for the transition region of the circular cylinder with impedance boundary conditions when it is illuminated by an obliquely incident plane wave. First, the eigen function expansions for the surface fields are derived using a standard method. Then, the Watson transformation is applied to derive expressions which are uniform in the transition region. This current supplements the PO current approximation near the transition region. The correction to the PO currents is expected to improve upon the accuracy of the prediction of the scattered field. The validity of the Fock-type representation was verified numerically by comparing with the exact series solution.

Keywords: Impedance Circular Cylinder, Obliquely Incident, Transition Current, Physical Optics

1. Introduction

The physical optics (PO) approximation is known to be a useful approach for predicting a diffracted or radiated field, since it reduces the determination of the field to quadratures. Through the use of a radiation integral the PO also overcomes the drawback of the geometrical optics (GO) at caustics and shadow boundaries. However, the PO approximation does not always provide reliable solutions for the high-frequency fields, as the method still relies upon the GO approximation to determine the PO surface current. The surface currents obtained by the GO are in error near shadow boundaries on the surface of the scatterer. Within this so-called transition region, the PO current considerably deviates from the exact total current and this is clearly the main reason for errors in the PO predictions of the field scattered from smooth convex surfaces. Thus the accuracy of the computed scattered fields can be improved by accounting for the contribution to the scattered fields of the difference between the total and PO currents. The difference current, we call it transition current, can be derived from the exact solution to the canonical shape that conforms to the shape of the general scatterer in the vicinity of the shadow boundary.

Starting from a canonical problem of diffraction of a plane electromagnetic wave incident normally to the axis of an infinite circular cylinder with impedance boundary conditions, Osipov, et al, demonstrated that the transition current becomes particularly important for the accurate determination of the near fields, side-lobe fields, fields near null and the cross-polarized components of the scattered fields [1,2].

The purpose of the present paper is to derive the transition current of an impedance circular cylinder illumi-

nated by a plane wave incident obliquely to the cylinder axis. The analysis focuses on the development of a Fock-type asymptotic representation for electromagnetic field tangent components on the surface of the scatterer. This expression is very important to the high-frequency analysis since the use of the series expression is limited to some small values of ka , where ka is the normalized radius of the cylinder. For a perfectly conducting cylinder surface, the role of transition current was analyzed by finding the far field contribution attributed to the PO and transition currents when a plane wave is incident normally to the cylinder, and the solution is generalized to that for arbitrary angles of incidence by a simple substitution method. For an impedance cylinder it is difficult to apply the substitution method to obtain the field for an arbitrary angle of incidence from the corresponding two dimensional solution. In contrast to the papers [3,4] which treated the derivation of incremental length diffraction coefficients corresponding to an equivalent line source, we are concerned with the analysis of the real transition currents in a vicinity of the penumbra region on the surface of the scatterer. Once the transition currents, or the associated electromagnetic field tangent components, which are the same, are found, they can be directly inserted in the radiation integral to provide an improved solution to the problem of scattering from a smooth convex non-metallic body at high-frequencies. The use of the real currents in the radiation integral allows one to obtain more accurate field representations, valid not only far from the cylinder but also at finite distance, including the surface of the cylinder. The validity of the Fock-type representation of the surface fields is verified numerically by comparing with the series expression for various values of radius ka and surface impedance. The

Fig. 1. Geometry of the problem

agreement is fairly good.

2. Statement of the Problem

The geometry of the problem and the associated coordinates (ρ, ϕ, z) are shown in Fig. 1, where the radius of the cylinder is a , angle of incidence is θ_0 , and wave vector $\mathbf{k}^i$ of an incident wave lies in the xz -plane. The incident wave is given by

$$\mathbf{E}^i = (A_i \mathbf{i}_\theta + B_i \mathbf{i}_\phi) \cdot \exp[jk(x \sin \theta_0 + z \cos \theta_0)] \quad (1)$$

$$\mathbf{H}^i = Y_0(-A_i \mathbf{i}_\phi + B_i \mathbf{i}_\theta) \cdot \exp[jk(x \sin \theta_0 + z \cos \theta_0)],$$

$$\mathbf{k}^i = \mathbf{i}_x \sin \theta_0 + \mathbf{i}_z \cos \theta_0 \quad (2)$$

where

$$\mathbf{i}_\theta = \cos \theta_0 \mathbf{i}_x - \sin \theta_0 \mathbf{i}_z,$$

$$\mathbf{i}_\phi = \mathbf{i}_y \quad (3)$$

and Y_0 is an intrinsic admittance of free space. On the surface of the cylinder, the impedance boundary conditions

$$E_z^i + E_z^s = \zeta[H_\phi^i + H_\phi^s],$$

$$H_z^i + H_z^s = -\eta[E_\phi^i + E_\phi^s] \quad (4)$$

must be satisfied, where ζ and η are surface impedance and admittance of the scatterer, respectively, generally ζ is not equal $1/\eta$. Superscripts "i" and "s" refer to incident and scattered waves. We introduce the z -components of magnetic and electric vector potentials A_z and F_z , respectively, which relate to the electromagnetic field components by

$$\mathbf{E} = -j\omega \left[A_z \mathbf{i}_z + \frac{1}{k^2} \frac{\partial}{\partial z} \nabla A_z \right]$$

$$- \frac{1}{\epsilon} \nabla \times (\mathbf{i}_z F_z) \quad (5)$$

$$\mathbf{H} = -j\omega \left[F_z \mathbf{i}_z + \frac{1}{k^2} \frac{\partial}{\partial z} \nabla F_z \right]$$

$$+ \frac{1}{\mu} \nabla \times (\mathbf{i}_z A_z) \quad (6)$$

The potential functions A_z and F_z corresponding to the incident and scattered fields can be expressed as follows:

$$A_z^i = -\frac{jA}{\omega \sin \theta_0} \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n J_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad (7)$$

$$F_z^i = -\frac{jY_0 B}{\omega \sin \theta_0} \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n J_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad (8)$$

$$A_z^s = -\frac{j}{\omega \sin \theta_0} \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n a_n H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \quad (9)$$

$$F_z^s = -\frac{jY_0}{\omega \sin \theta_0} \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \quad (10)$$

where formula of the wave transformation

$$\exp(jkx) = \sum_{n=-\infty}^{\infty} j^n J_n(k\rho) \exp[-jn\phi] \quad (11)$$

is used. In the above equation, $J_n(x)$ and $H_n^{(2)}(x)$ are the Bessel and the second kind of Hankel function of order n and argument x , respectively. Symbols a_n and b_n stand for the expansion coefficients which are to be determined so that the total fields satisfy the required boundary conditions. The tangent magnetic components on the surface are given by

$$H_z^t = Y_0 A \frac{2\zeta_0 \eta_0}{\pi \Lambda} \cos \theta_0 \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka} \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi]$$

$$- Y_0 B \frac{2\eta_0}{\pi \Lambda} \sin \theta_0 \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n \frac{j\zeta_0 H_n^{(2)\prime}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \quad (12)$$

$$H_\phi^t = -Y_0 A \frac{2}{\pi \Lambda} \sin \theta_0 \exp[jkz \cos \theta_0]$$

$$\cdot \sum_{n=-\infty}^{\infty} j^n \frac{j\eta_0 H_n^{(2)\prime}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi]$$

$$-Y_0 B \frac{2\eta_0}{\pi \Lambda} \cos \theta_0 \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka} \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \dots (13)$$

$$\Delta(n) = \left[j\zeta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda) \right] \cdot \left[j\eta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda) \right] + \zeta_0 \eta_0 \frac{\cos^2 \theta_0}{\sin^2 \theta_0} \left[\frac{n}{ka} H_n^{(2)}(\Lambda) \right]^2 \dots (14)$$

where $\Lambda = ka \sin \theta_0$. The derivation of these equations is shown in Appendix A. Since E_z^t and E_ϕ^t are related to H_ϕ^t and H_z^t by Eq.(4) on the surface, these components can be omitted.

For the special case of a perfect conductor, these expressions are simplified when the limiting process $\zeta_0 \rightarrow 0$, $\eta_0 \rightarrow \infty$, and $\eta_0 \zeta_0 \rightarrow 1$ are applied. The result is

$$H_z^t = \frac{j2Y_0 B}{\pi ka} \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{\exp[-jn\phi]}{H_n^{(2)'}(ka \sin \theta_0)} \dots (15)$$

$$H_\phi^t = \frac{2Y_0 A}{\pi ka \sin \theta_0} \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{\exp[-jn\phi]}{H_n^{(2)}(ka \sin \theta_0)} + \frac{j2Y_0 B \cos \theta_0}{\pi (ka)^2 \sin^2 \theta_0} \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n \exp[-jn\phi]}{H_n^{(2)'}(ka \sin \theta_0)} \dots (16)$$

It is noted that the expression H_z^t and the first term of H_ϕ^t are same forms of normal incidence except that the argument ka of the Hankel function has been replaced by $ka \sin \theta_0$, and the second term of H_ϕ^t was obtained from H_z^t by performing differentiation with respect to ϕ . This is the reason that these expressions can be derived from the corresponding normal incident solution by the substitution method.

In contrast to a perfectly conducting cylinder, it is difficult to construct the solution with this technique. It is readily seen that the tangential components H_z^t and H_ϕ^t consist of two kinds of series given by

$$U(\phi) = \sum_{n=-\infty}^{\infty} j^n \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] = \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka \sin \theta_0} \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \quad (17)$$

$$V(\phi) = \sum_{n=-\infty}^{\infty} j^n \frac{H_n^{(2)'}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \dots (18)$$

3. High-Frequency Representations of the Surface Fields

When we consider the diffraction of the electromagnetic wave by a smooth obstacle, the equivalent radius of curvature varies over a wide range. Hence the approximate solution is essential to this kind of problem. In case the radius of the circular cylinder becomes very large compared with the wavelength, the series representations are inadequate and these are transformed into more useful expressions using the Watson transformation. The detail of the derivation is given in Appendix B.

(a) Perfectly Conducting Circular Cylinder

$$H_z^t = -Y_0 B \sin \theta_0 \cdot \exp \left[jkz \cos \theta_0 - jka \sin \theta_0 \left(\phi - \frac{\pi}{2} \right) \right] \cdot g \left[m \left(\phi - \frac{\pi}{2} \right) \right] \dots (19)$$

$$H_\phi^t = -Y_0 B \cos \theta_0 \cdot \exp \left[jkz \cos \theta_0 - jka \sin \theta_0 \left(\phi - \frac{\pi}{2} \right) \right] \cdot g \left[m \left(\phi - \frac{\pi}{2} \right) \right] - \frac{Y_0 B}{m} \exp \left[jkz \cos \theta_0 - jka \sin \theta_0 \left(\phi - \frac{\pi}{2} \right) \right] \cdot f \left[m \left(\phi - \frac{\pi}{2} \right) \right] \dots (20)$$

(b) Impedance Circular Cylinder

$$H_z^t = Y_0 A \frac{2\zeta_0 \eta_0}{\pi ka} \cos \theta_0 \exp[jkz \cos \theta_0] U(\phi) - Y_0 B \frac{2\eta_0}{\pi ka} \exp[jkz \cos \theta_0] \cdot [j\zeta_0 V(\phi) + \sin \theta_0 U(\phi)]$$

$$H_\phi^t = -Y_0 A \frac{2}{\pi ka} \exp[jkz \cos \theta_0] \cdot [j\eta_0 V(\phi) + \sin \theta_0 U(\phi)] - Y_0 B \frac{2\eta_0}{\pi ka} \cos \theta_0 \exp[jkz \cos \theta_0] U(\phi) \quad (21)$$

In the above equations, the functions $f(\xi)$, $g(\xi)$, $U(\phi)$, and $V(\phi)$ are defined in Appendix B.

The Fock-type expressions for the function $U(\phi)$ and $V(\phi)$ defined by Eq.(17),(18) and (A27)-(A30) are programmed and compared to various values of ka and surface impedance. Tab.1 shows some illustrating results for parameters $ka = 50$, $\theta_0 = 60^\circ$, $\zeta_0 = 0.5 - j0.5$, $\eta_0 = \frac{1}{\zeta_0}$, and $A = 1$, $B = 0$. It can be seen that the agreement is fairly good.

4. Conclusion

A Fock-type representation of the surface fields was obtained for the transition region of the circular cylinder with impedance boundary conditions when it is illuminated by an obliquely incident plane wave. The

Table 1. Comparison of the surface fields computed from series (exact) and asymptotic (Fock-type) expressions. The parameters are: $ka = 50$, $\theta_0 = 60^\circ$, $\zeta_0 = 0.5 - j0.5$, $\eta_0 = 1/\zeta_0$, $A = 1$, $B = 0$.

ϕ	H_z (Exact)	H_z (Approx)	H_ϕ (Exact)	H_ϕ (Approx)	ϕ	H_z (Exact)	H_z (Approx)	H_ϕ (Exact)	H_ϕ (Approx)
70	0.19196	0.20139	0.53944	0.53680	96	0.08753	0.08420	0.15501	0.15734
71	0.18952	0.19760	0.52211	0.51945	97	0.08325	0.08001	0.14516	0.14761
72	0.18688	0.19370	0.50480	0.50216	98	0.07906	0.07594	0.13577	0.13832
73	0.18404	0.18968	0.48752	0.48495	99	0.07497	0.07197	0.12682	0.12946
74	0.18101	0.18555	0.47030	0.46783	100	0.07100	0.06813	0.11832	0.12102
75	0.17779	0.18131	0.45317	0.45084	101	0.06714	0.06441	0.11025	0.11300
76	0.17440	0.17699	0.43616	0.43398	102	0.06340	0.06081	0.10261	0.10539
77	0.17084	0.17257	0.41928	0.41729	103	0.05979	0.05735	0.09539	0.09819
78	0.16713	0.16807	0.40257	0.40079	104	0.05630	0.05401	0.08857	0.09138
79	0.16326	0.16349	0.38605	0.38450	105	0.05296	0.05081	0.08215	0.08495
80	0.15926	0.15886	0.36974	0.36843	106	0.04974	0.04774	0.07612	0.07889
81	0.15514	0.15416	0.35368	0.35262	107	0.04667	0.04480	0.07045	0.07319
82	0.15090	0.14943	0.33788	0.33708	108	0.04373	0.04200	0.06514	0.06784
83	0.14657	0.14465	0.32236	0.32184	109	0.04092	0.03933	0.06017	0.06282
84	0.14215	0.13986	0.30716	0.30690	110	0.03826	0.03679	0.05553	0.05811
85	0.13765	0.13504	0.29229	0.29230	111	0.03572	0.03438	0.05120	0.05372
86	0.13311	0.13023	0.27777	0.27805	112	0.03332	0.03209	0.04717	0.04961
87	0.12851	0.12542	0.26361	0.26415	113	0.03105	0.02993	0.04342	0.04578
88	0.12389	0.12063	0.24985	0.25064	114	0.02890	0.02788	0.03994	0.04222
89	0.11926	0.11587	0.23648	0.23752	115	0.02688	0.02595	0.03671	0.03891
90	0.11462	0.11115	0.22353	0.22479	116	0.02497	0.02413	0.03372	0.03583
91	0.11000	0.10648	0.21100	0.21248	117	0.02318	0.02242	0.03095	0.03297
92	0.10540	0.10187	0.19891	0.20059	118	0.02150	0.02082	0.02840	0.03032
93	0.10085	0.09733	0.18726	0.18913	119	0.01992	0.01931	0.02604	0.02788
94	0.09634	0.09286	0.17605	0.17810	120	0.01845	0.01790	0.02386	0.02561
95	0.09190	0.08848	0.16530	0.16750					

validity of the Fock-type representation was verified numerically by comparing it with the exact series solution. The application of this expression to predict the fields diffracted by various kinds of obstacles is under study.

(Manuscript received October 2, 98, revised April 9, 99)

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Appendix

A-1. The Surface Fields on the Impedance Cylinder

In this appendix, the detailed derivation of the surface fields on the impedance cylinder is shown. The explicit expressions for incident and scattered fields are given as follows:

$$E_z^i = -A \sin \theta_0 \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n J_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad \cdots (A1)$$

$$H_z^i = -Y_0 B \sin \theta_0 \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n J_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad \cdots (A2)$$

$$E_\phi^i = -\frac{A \cos \theta_0}{\sin \theta_0} \cdot \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{k\rho} J_n(k\rho \sin \theta_0) \exp[-jn\phi] - jB \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n J'_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad \cdots (A3)$$

$$H_\phi^i = -\frac{Y_0 B \cos \theta_0}{\sin \theta_0} \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{k\rho} J_n(k\rho \sin \theta_0) \exp[-jn\phi] + jY_0 A \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n J'_n(k\rho \sin \theta_0) \exp[-jn\phi] \quad \cdots (A4)$$

$$E_z^s = -\sin \theta_0 \exp[jkz \cos \theta_0] \cdot \sum_{n=-\infty}^{\infty} j^n a_n H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \quad (A5)$$

$$H_z^s = -Y_0 \sin \theta_0 \exp[jkz \cos \theta_0]$$

$$\begin{aligned}
& \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \quad (A6) \\
E_\phi^s &= -\frac{\cos \theta_0}{\sin \theta_0} \cdot \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n a_n \frac{n}{k\rho} H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \\
& -j \exp[jkz \cos \theta_0] \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)'}(k\rho \sin \theta_0) \\
& \cdot \exp[-jn\phi] \dots\dots\dots (A7) \\
H_\phi^s &= -Y_0 \frac{\cos \theta_0}{\sin \theta_0} \cdot \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n b_n \frac{n}{k\rho} H_n^{(2)}(k\rho \sin \theta_0) \exp[-jn\phi] \\
& +jY_0 \exp[jkz \cos \theta_0] \sum_{n=-\infty}^{\infty} j^n a_n H_n^{(2)'}(k\rho \sin \theta_0) \\
& \cdot \exp[-jn\phi] \dots\dots\dots (A8)
\end{aligned}$$

where superscripts "i" refers to the incident wave and "s" refers to the scattered field, respectively. Enforcement of the boundary condition yields the equation for the unknown expansion coefficients a_n and b_n .

$$\begin{aligned}
& [j\zeta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] a_n \\
& -\zeta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} H_n^{(2)}(\Lambda) b_n \\
& = -A [\sin \theta_0 J_n(\Lambda) + j\zeta_0 J_n'(\Lambda)] \\
& +B\zeta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda) \dots\dots\dots (A9) \\
& \eta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} H_n^{(2)}(\Lambda) a_n \\
& + [j\eta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] b_n \\
& = -A\eta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda) \\
& -B [\sin \theta_0 J_n(\Lambda) + j\eta_0 J_n'(\Lambda)] \dots\dots\dots (A10)
\end{aligned}$$

where $\Lambda = ka \sin \theta_0$, $\zeta_0 = Y_0 \zeta$ and $\eta_0 = Z_0 \eta$. The solution of these equations is given by

$$\begin{aligned}
a_n &= \frac{1}{\Delta(n)} \left\{ -[A [\sin \theta_0 J_n(\Lambda) + j\zeta_0 J_n'(\Lambda)] \right. \\
& -B\zeta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda)] \\
& \cdot [j\eta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] \\
& -[A\eta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda) \\
& +B [\sin \theta_0 J_n(\Lambda) + j\eta_0 J_n'(\Lambda)]] \\
& \cdot \zeta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} H_n^{(2)}(\Lambda) \} \dots\dots\dots (A11) \\
b_n &= \frac{1}{\Delta(n)} \left\{ [A [\sin \theta_0 J_n(\Lambda) + j\zeta_0 J_n'(\Lambda)] \right.
\end{aligned}$$

$$\begin{aligned}
& -B\zeta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda)] \\
& \cdot \eta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} H_n^{(2)}(\Lambda) \\
& -[A\eta_0 \frac{\cos \theta_0}{\sin \theta_0} \frac{n}{ka} J_n(\Lambda) \\
& +B [\sin \theta_0 J_n(\Lambda) + j\eta_0 J_n'(\Lambda)]] \\
& \cdot [j\zeta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] \} \quad (A12) \\
\Delta(n) &= [j\zeta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] \\
& \cdot [j\eta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)] \\
& +\zeta_0 \eta_0 \frac{\cos^2 \theta_0}{\sin^2 \theta_0} \left[\frac{n}{ka} H_n^{(2)}(\Lambda) \right]^2 \dots\dots (A13)
\end{aligned}$$

Using these results we can obtain the expressions for the surface fields

$$\begin{aligned}
E_z^t &= -A \frac{2\zeta_0}{\pi \Lambda} \sin \theta_0 \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n \frac{j\eta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)}{\Delta(n)} \\
& \cdot \exp[-jn\phi] \\
& -B \frac{2\zeta_0 \eta_0}{\pi \Lambda} \cos \theta_0 \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka} \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \dots\dots (A14) \\
H_z^t &= Y_0 A \frac{2\zeta_0 \eta_0}{\pi \Lambda} \cos \theta_0 \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka} \frac{H_n^{(2)}(\Lambda)}{\Delta(n)} \exp[-jn\phi] \\
& -Y_0 B \frac{2\eta_0}{\pi \Lambda} \sin \theta_0 \exp[jkz \cos \theta_0] \\
& \cdot \sum_{n=-\infty}^{\infty} j^n \frac{j\zeta_0 H_n^{(2)'}(\Lambda) + \sin \theta_0 H_n^{(2)}(\Lambda)}{\Delta(n)} \\
& \cdot \exp[-jn\phi] \dots\dots\dots (A15)
\end{aligned}$$

where Wronski's relation

$$J_n(x) H_n^{(2)'}(x) - J_n'(x) H_n^{(2)}(x) = -j \frac{2}{\pi x} \dots (A16)$$

is used to derive the above equation. Other tangential components E_ϕ^t and H_ϕ^t can be directly obtained from Eq.(A14),(A15) by using relation Eq.(4).

A-2. Fock-type Representation of the Surface Fields

From the discussion of Sec.2, the surface fields are obtained from four kinds of functions, $F(\phi)$, $G(\phi)$, $U(\phi)$, and $V(\phi)$ which are given by an infinite series. In this appendix, Fock-type representations of these functions are determined using Watson transformation.

(a) Functions $F(\phi)$ and $G(\phi)$:

$F(\phi)$ and $G(\phi)$ are defined as infinite series parts of Eq.(15),(16) and represented by Fock-type functions $f(\xi)$, $g(\xi)$ as follows:

$$\begin{aligned} F(\phi) &= \sum_{n=-\infty}^{\infty} j^n \frac{\exp[-jn\phi]}{H_n^{(2)}(ka \sin \theta_0)} \\ &= \frac{1}{2j} \int_C \frac{\exp[-j\nu(\phi - \pi)]}{\sin \nu\pi} \\ &\quad \cdot \frac{\exp\left[j\frac{\nu\pi}{2}\right]}{H_\nu^{(2)}(ka \sin \theta_0)} d\nu \dots\dots\dots (A17) \end{aligned}$$

where C is the contour which encircle the real axis counterclockwise. This can be transformed into

$$\begin{aligned} F(\phi) &= \frac{1}{2j} \int_{-\infty-j\epsilon}^{\infty-j\epsilon} \frac{\exp[-j\nu(\phi - \pi)]}{\sin \nu\pi} \\ &\quad \cdot \frac{\exp\left[j\frac{\nu\pi}{2}\right]}{H_\nu^{(2)}(ka \sin \theta_0)} d\nu \\ &\quad + \frac{1}{2j} \int_{-\infty-j\epsilon}^{\infty-j\epsilon} \frac{\exp[j\nu(\phi - \pi)]}{\sin \nu\pi} \\ &\quad \cdot \frac{\exp\left[j\frac{\nu\pi}{2}\right]}{H_\nu^{(2)}(ka \sin \theta_0)} d\nu \\ &= -j \int_{-\infty-j\epsilon}^{\infty-j\epsilon} \frac{\cos[\nu(\phi - \pi)]}{\sin \nu\pi} \\ &\quad \cdot \frac{\exp\left[j\frac{\nu\pi}{2}\right]}{H_\nu^{(2)}(ka \sin \theta_0)} d\nu \dots\dots\dots (A18) \end{aligned}$$

when $ka \sin \theta_0 \gg 1$, the main contribution comes from

$$\frac{\cos \nu(\phi - \pi)}{\sin \nu\pi} \simeq j \exp(-j\nu\phi) \dots\dots\dots (A19)$$

In a transition region, since the values near ka contribute most significantly to the above integrals when ka is very large, we can write

$$\nu = ka \sin \theta_0 + m\tau, \quad m = \left(\frac{ka \sin \theta_0}{2}\right)^{\frac{1}{3}} \dots\dots\dots (A20)$$

In this region, the Hankel functions may be approximated by

$$\begin{aligned} H_\nu^{(2)}(ka \sin \theta_0) &\simeq \frac{j}{\sqrt{\pi}} \frac{1}{m} w_2(\tau), \\ H_\nu^{(2)'}(ka \sin \theta_0) &\simeq -\frac{j}{\sqrt{\pi}} \frac{1}{m^2} w_2'(\tau) \dots\dots\dots (A21) \end{aligned}$$

Then, $F(\phi)$ and $G(\phi)$ are simplified as follows.

$$\begin{aligned} F(\phi) &= -j\pi m^2 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot f\left[m\left(\phi - \frac{\pi}{2}\right)\right] \dots\dots\dots (A22) \end{aligned}$$

where

$$f(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp[-j\xi t]}{w_2(t)} dt, \dots\dots\dots (A23)$$

$$\begin{aligned} G(\phi) &= \sum_{n=-\infty}^{\infty} j^n \frac{\exp[-jn\phi]}{H_n^{(2)'}(ka \sin \theta_0)} \\ &= \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka \sin \theta_0} \frac{\exp[-jn\phi]}{H_n^{(2)'}(ka \sin \theta_0)} \\ &= j\pi m^3 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot g\left[m\left(\phi - \frac{\pi}{2}\right)\right] \dots\dots\dots (A24) \end{aligned}$$

where

$$g(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp[-j\xi t]}{w_2'(t)} dt \dots\dots\dots (A25)$$

The function $w_2(t)$ and $w_2'(t)$ are Fock-type Airy function defined below and its first derivative, respectively.

$$w_{1,2}(t) = \frac{1}{\pi} \int_{\Gamma_1, \Gamma_2} \exp\left[tz - \frac{1}{3}z^3\right] dz \dots\dots\dots (A26)$$

(b) Functions $U(\phi)$ and $V(\phi)$:

$$\begin{aligned} U(\phi) &= \sum_{n=-\infty}^{\infty} j^n \frac{H_n^{(2)}(ka \sin \theta_0)}{\Delta(n)} \exp[-jn\phi] \\ &= \sum_{n=-\infty}^{\infty} j^n \frac{n}{ka \sin \theta_0} \frac{H_n^{(2)}(ka \sin \theta_0)}{\Delta(n)} \\ &\quad \cdot \exp[-jn\phi] \\ &= -j\sqrt{\pi} m^3 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot \int_{-\infty}^{\infty} \frac{w_2(t)}{D_2(t)} \exp\left[-jmt\left(\phi - \frac{\pi}{2}\right)\right] dt \\ &= -j\sqrt{\pi} m^3 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot \int_0^{\infty} \frac{w_2(t)}{D_2(t)} \exp\left[-jmt\left(\phi - \frac{\pi}{2}\right)\right] dt \\ &\quad + \exp\left(j\frac{\pi}{6}\right) \sqrt{\pi} m^3 \\ &\quad \cdot \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \int_0^{\infty} \frac{w_1(t)}{D_1(t)} \\ &\quad \cdot \exp\left[jmt\left(\phi - \frac{\pi}{2}\right)\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right] dt \dots\dots\dots (A27) \\ V(\phi) &= \sum_{n=-\infty}^{\infty} j^n \frac{H_n^{(2)'}(ka \sin \theta_0)}{\Delta(n)} \exp[-jn\phi] \\ &= j\sqrt{\pi} m^2 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot \int_{-\infty}^{\infty} \frac{w_2'(t)}{D_2(t)} \exp\left[-jmt\left(\phi - \frac{\pi}{2}\right)\right] dt \\ &= j\sqrt{\pi} m^2 \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \\ &\quad \cdot \int_0^{\infty} \frac{w_2'(t)}{D_2(t)} \exp\left[-jmt\left(\phi - \frac{\pi}{2}\right)\right] dt \\ &\quad + \exp\left(-j\frac{\pi}{6}\right) \sqrt{\pi} m^2 \\ &\quad \cdot \exp\left[-jka \sin \theta_0 \left(\phi - \frac{\pi}{2}\right)\right] \int_0^{\infty} \frac{w_1'(t)}{D_1(t)} \end{aligned}$$

$$\cdot \exp \left[jmt \left(\phi - \frac{\pi}{2} \right) \left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right) \right] dt \quad (\text{A28})$$

$$D_1(t) = \left[\zeta_0 \exp \left(j \frac{\pi}{6} \right) w'_1(t) + m \sin \theta_0 w_1(t) \right] \cdot \left[\eta_0 \exp \left(j \frac{\pi}{6} \right) w'_1(t) + m \sin \theta_0 w_1(t) \right] + \zeta_0 \eta_0 m^2 \cos^2 \theta_0 w_1^2(t) \dots \dots \dots (\text{A29})$$

$$D_2(t) = \left[-j \zeta_0 w'_2(t) + m \sin \theta_0 w_2(t) \right] \cdot \left[-j \eta_0 w'_2(t) + m \sin \theta_0 w_2(t) \right] + \zeta_0 \eta_0 m^2 \cos^2 \theta_0 w_2^2(t) \dots \dots \dots (\text{A30})$$

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