An optical pulse compressor using a normal dispersion fiber combined with a linearly chirped fiber grating (instead of a bulk grating pair) is analyzed. A set of modified design rules of the fiber-fiber grating compressor that governs the optimum fiber length; optimum dispersion and length of fiber grating are given.

Key word: Fiber grating, pulse compression, compression factor

2. Analytical model for fiber-fiber grating compressor

A fiber-bulk grating pair compressor is combined of a normal dispersion fiber with a bulk grating pair. The role of NDF is to impose a nearly linear, positive chirp on the pulse through a combined effect of self phase modulation (SPM) and normal group velocity dispersion (GVD), while the bulk grating pair provides anomalous GVD required for compression of positively chirped pulses. In general, the larger the positive chirped parameter C, the larger the compression factor.

For optimum compression, references [1] and [3] gave a set of semi-empirical relations from numerical simulations.

\[
\frac{z_{\text{opt}}}{z_0} = \frac{1.6}{N} \quad (1)
\]
\[
\frac{a_c}{T_{\text{FWHM}}} = \frac{1.6}{N} \quad (2)
\]
\[
\frac{T_{\text{comp}}}{T_{\text{FWHM}}} = \frac{1.6}{N} \quad (3)
\]
\[
P = N^2 P_1 \quad (4)
\]

where \(z_{\text{opt}}\) is the optimum length of the normal dispersion fiber; \(z_0\) is the soliton period; \(N\) is the normalized peak amplitude order, \(a_c\) is the bulk grating pair parameter which affects dispersion property of grating pair, \(T_{\text{FWHM}}\), \(T_{\text{comp}}\), \(P_1\), and \(P\) are the width before and after...
compressions, the first order peak power, and the required input peak power for the pulse compression respectively.

For fiber-fiber grating compressor, linearly chirped fiber grating replaces grating pair, the bulk grating pair parameter \( a_c \) must be replaced by:

\[ a_c = -\frac{1}{2} \phi_2 \]  \hspace{2cm} (5)

where \( \phi_2 = \frac{\partial^2 \phi}{\partial \omega^2} \) reflects the dispersion of fiber grating defined as follows.

The characteristics of linearly chirped fiber grating can be described by the coupled-modes equations [4]:

\[ \frac{dA^+}{dz} = k(z) \exp\left[-i \int_0^z B(z')dz'\right] A^- \]  \hspace{2cm} (6)

\[ \frac{dA^-}{dz} = k(z) \exp\left[i \int_0^z B(z')dz'\right] A^+ \]  \hspace{2cm} (7)

where \( A^+ \) and \( A^- \) are the amplitude of the forward- and backward-propagating modes along the \( z \) direction. \( k(z) \) is the coupling coefficient, which is related to the index modulation. \( B(z) \) is given by

\[ B(z) = 2\beta - \Omega(z) = 2(\beta_0 + \frac{Fz}{L_g^2}) - \Omega_0 = \frac{2\beta_0 - Fz}{L_g^2} \]  \hspace{2cm} (8)

where \( \beta \) is the propagating constant, \( \Omega(z) \) is the local spatial frequency of the grating, \( 2\beta_0 = \Omega_0 \) is the spatial frequency at the center of grating, \( F \) is the chirp coefficient, and \( L_g \) is the grating length. The boundary conditions are \( A^+(-L_g/2) = 1 \) and \( A^-(L_g/2) = 0 \).

We define reflective transfer function of fiber grating as

\[ R = \frac{A^+(-L_g/2)}{A^-(-L_g/2)} = r \exp(i\varphi) \]  \hspace{2cm} (9)

where \( r \) and \( \varphi \) are the absolute value and phase angle of \( R \), respectively. \( \varphi \) can be expanded by a Taylor series

\[ \varphi = \varphi_0 + \frac{\partial \varphi}{\partial \omega} \delta \omega + \frac{\partial^2 \varphi}{2 \partial \omega^2} \delta \omega^2 + \cdots \]  \hspace{2cm} (10)

where \( \frac{\partial \varphi}{\partial \omega} \) and \( \frac{\partial^2 \varphi}{\partial \omega^2} \) correspond to the time delay and dispersion of the fiber grating, respectively. By introducing the normalized detuning \( \Delta = \frac{\delta \beta}{L_g} \), the dispersion can be expressed as

\[ \frac{\partial^2 \varphi}{\partial \omega^2} = \frac{n_0^2 L_g^2}{c^2} \frac{\partial^2 \varphi}{\partial \delta \omega^2} \]  \hspace{2cm} (11)

where \( n_0 \) is the effective index at the center frequency, and \( c \) is light velocity in vacuum, respectively.

We find that the semi-empirical equations must be modified when the linearly chirped fiber grating is used to replace the bulk grating pair, because the dispersion parameter of the fiber grating \( \phi_2 \) is oscillating [5], so the equation (5) can only give average equality.

The dispersion oscillation of the fiber grating influences compression quality of pulse. To solve such problem, we need to simulate actual case for fiber-fiber grating, not just a copy of bulk grating pair case, in order to obtain a set of semi-empirical equations for optimum compression. Our simulation model is based on the normalized nonlinear Schrodinger equation for NDF as follows and coupled-modes equations (6) and (7). In the case of GVD, the nonlinear Schrodinger equation is:

\[ i \frac{2}{\pi} \frac{\partial U}{\partial \xi} = \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 |U|^2 U \]  \hspace{2cm} (12)

To normalize amplitude, distance, and time separately, we have

\[ U = \frac{A}{\sqrt{P_0}}, \quad \xi = \frac{z}{z_n}, \quad \tau = \frac{T}{T_n}, \]  \hspace{2cm} (13)

respectively. In equation (13), \( T_n \) is the half width at 1/e point of input pulse. In general, the needed length for NDF is shorter than absorption length, therefore we neglect the loss of fiber in our numerical analysis.

In order to describe the property of compression quantitatively, we introduce compression factor \( F_c \) and quality factor \( Q_c \), as:

\[ F_c = \frac{T_{\text{comp}}}{T_{\text{FWHM}}}, \quad Q_c = \frac{\left| U_{\text{out}} \right|^2}{\left| U_{\text{in}} \right|^2 F_c} \]  \hspace{2cm} (14)

Obviously, the larger the \( Q_c \), the better the compressed pulse quality. But from (14) we can see that for a input pulse with given peak power, one improves the quality of pulse at the cost of compression factor.

3. New design rule for fiber-fiber grating compressor

Fig. 1 shows pulse shape and frequency chirp after transmission of different distances when \( N = 20 \). From Fig. 1, we see that under combined effect of GVD and SPM together, the shape of the expanded pulse has large differences for different transmission distance. The longer
the transmission distance, the shape of the pulse is more 
close to a square pulse, and the width of the expanded pulse 
is larger. Fig. 1 shows also that there exists oscillation at the 
tail of the pulse when transmission distance beyond 
\[ \frac{z}{z_0} > \frac{0.6}{N} \], and the oscillation become stronger with 
increasing distance.

![Fig. 1 The pulse shapes (a) and frequency chirps (b) after transmission length. Curve 1 represents the input pulse and chirp, and 2-5 represent the case where parameter $Nz/z_0 = 0.6, 0.9, 1.2, 1.8$, respectively.](image)

No matter of which transmission length cases, the 
pulse has approximated linear chirp in whole range of pulse 
width. A pulse with shorter transmission distance in NDF 
can produce a larger chirp parameter, in the meantime, the 
range of linear chirp becomes narrow. But the result is 
opposite for a pulse with longer transmission distance in 
NDF. When transmission distance is beyond 
\[ \frac{z}{z_0} > \frac{0.6}{N} \], 
oscillation appears at the tail of the frequency chirp, and the 
oscillation become strong with increasing distance. From 
the theory of compression of chirped pulse [1], we know 
that the parameter of chirp influences compression rate of 
pulse, while the range of the linear chirp influences the 
compression quality of pulse. On the one hand, when a 
pulse has a large chirp parameter after a comparatively 
short transmission distance, we can obtain a larger 
compression rate of pulse. On the other hand, the narrow 
range of linear chirp will influence on the compression 
quality. Therefore one needs consider all above factors of 
influences when choosing length of normal dispersion 
fibers.

Following numerical computation offers a design rule. 
For a known pulse and fiber parameter, one can determine 
the optimum length of NDF and optimum length of fiber 
grating, thereby obtain the optimum compression.

We choose the fiber grating parameter as $k(L) = 3\pi$, 
$F = 40\pi$ [4], where $k(z) = k_0$ is the coupling coefficient 
of the grating, and $F$ is the chirp coefficient. Under such 
condition, $\left| \frac{\partial^2 \phi}{\partial \Delta^2} \right| = 0.032$, one can obtain grating length 
from (2), (5) and (11).

Fig. 2 shows a relationship of the compression factor 
with grating length, where pulse amplitude order $N$ is equal 
to 20, $T_0 = 100\, \text{ps}$, NDF length is 
$z/z_0 = 0.6/N$, 
$0.9/N$, 
$1.2/N$, 
$1.8/N$, respectively, and the fiber 
grating parameter is the same as above.

![Fig. 2 The pulse compression factor $F_c$ versus fiber grating 
length $L_g$ with the parameter $Nz/z_0 = 0.6, 0.9, 1.2, 1.8$ 
($N = 20$).](image)

From Fig. 2, we see that: for a given power ($N = 20$), 
the fiber gratings have optimum lengths, in which we can 
obtain the largest compression factor for different NDF 
lengths. From Fig. 1, we know that the pulse has the larger 
chirp parameter when its transmission distance in NDF is 
comparatively shorter (but larger than the length after which 
the combined effect of SPM and normal GVD can 
comparatively apparent), and it is expected that we can 
obtain comparably larger compression. But Fig. 2 shows 
that the largest compression factor appears at 
$z/z_0 = 0.9/N$ instead of $z/z_0 = 0.6/N$. This indicates 
that for a given power there exist the combined sets of 
optimum NDF length and fiber grating length for a largest 
compression factor.

The curves in Fig. 3 show the relationship between 
compression factor $F_c$ and pulse amplitude order $N$ with
different NDF lengths. In each case, we choose the optimum length of fiber grating, in order to have the maximum compressing factor. When \( N < 18 \), curve with \( z/z_0 = 1.2/N \) has an optimum compression factor \( F_c \). When \( N > 18 \), the curve with \( z/z_0 = 0.9/N \) has the optimum compression factor \( F_c \). But the compression factor between curve \( z/z_0 = 1.2/N \) and \( z/z_0 = 0.9/N \) has no obvious difference in a large range of \( N \) around 18. So we choose \( z/z_0 = 1.2/N \) as a common base for numerical simulation for \( N < 30 \). Now we obtained the semi-empirical equations for fiber-fiber grating compressor when \( N \gg 1 \) as:

\[
\begin{align*}
\frac{z_{\text{opt}}}{z_0} &\approx \frac{1.2}{N} \quad (15) \\
\frac{\varphi_2}{T_0} &\approx \frac{0.6}{N} \quad (16) \\
F_c &\approx \frac{N}{2.5} + 2.5 \quad (17) \\
P & = N^2 P_1 \quad (18)
\end{align*}
\]

Fig. 3 The compression factor \( F_c \) versus pulse amplitude \( N \) with \( z/z_0 = 0.6, 0.9, 1.2, 1.8 \).

Fig. 4 shows the pulse compression factor \( F_c \) versus fiber grating dispersion and pulse amplitude order \( N \), under the condition of \( z_{\text{opt}}/z_0 = 1.2/N, T_0 = 100 \text{ps} \).

Equation (17) is obtained from the curve fitting in Fig. 4(a), and equation (16) is from the curve fitting in Fig. 4(b). Numerical results show that in the region \( 4 \leq N \leq 30 \), Eqs. (15)-(18) can be used as a good design rule. In region \( 30 \leq N \leq 50 \), the factor in equation (15) should change to 0.9 instead of 1.2. For \( 50 \leq N \leq 80 \), the numerical factor in (15) should change to 0.6 and so on, in the meantime the other equations should also be modified.

We must point out that: when using fiber grating instead of bulk grating pair, the equations (1)-(4) used in references in [1] and [3] are not appropriate. Let us use Eqs. (1)-(4) [1, 3] to check pulse shape and chirp evolution after pulse passing NDF with \( N = 40 \). Fig. 5 (a) and (b) show pulse shape and chirp after transmission through the optimum fiber length from Eq. (1). Fig. 5 shows that the fiber length obtained from Eq. (1) is not the optimum length anymore. According to our analysis above, the optimum fiber length must be given by \( z_{\text{opt}}/z_c = 0.9/N \). Fig. 5 (c) and (d) show that: the pulse passing a NDF length under our design rule expands to a nearly square pulse, while the pulse becomes

Fig. 4 (a) pulse compression factor \( F_c \) and (b) grating dispersion \( \varphi_2 \) versus pulse amplitude \( N \) with parameter \( z_{\text{opt}}/z_0 = 1.2/N \) and \( T_0 = 100 \text{ps} \).

Fig. 5 (a) The pulse shape and (b) frequency chirps after transmission \( z_{\text{opt}}/z_0 = 1.6/N \) when \( N = 40 \). (c) The pulse shape and (d) frequency chirps after transmission \( z_{\text{opt}}/z_0 = 0.9/N \) when \( N = 40 \).
nearly linear chirp in whole pulse width, which is desired and ready for later pulse compression in fiber grating. This clearly proves that the equations (1)–(4) giving in references [1] and [3] need to be modified, and our design rule is feasible. Although above equations were obtained when the width of pulse is $T_0 = 100\,\text{ps}$, but it is also valid for all cases when $T_0 \geq 0.1\,\text{ps}$, because in the calculation we use NLS equation (12) with all parameters (including time) normalized. And in range of $T_0 \geq 0.1\,\text{ps}$ we do not have to consider (or add) the high dispersion terms and high nonlinear effect terms in normalized NLS equation (12).

When frequency response for fiber grating is not smooth, the compression quality of pulse would be influenced. For instance when $N = 20$, the compression factor $F_c$ for fiber-fiber grating is 10.5, quality factor $Q_c$ is 0.70; but for fiber-bulk grating pair $F_c = 12.5$, then $Q_c$ becomes 0.90. This indicates that frequency response property of fiber grating has large effect on the pulse compression. The apodization technology for fiber grating would smooths the dispersion frequency response curve [5].

Now we use obtained equations (15)–(18) to design a fiber-fiber grating compressor for a mode locking Nd:YAG laser. Parameters for the Nd:YAG laser are $\lambda = 1.32\,\mu\text{m}$, and $T_e = 60\,\text{ps}$. We want to compress such a pulse to 6ps ($F_c = 10$) through the fiber-fiber-grating compressor. Using dispersion-shift-fiber with zero-dispersion wavelength at 1.55μm, the GVD parameter for the dispersion-shift-fiber at $\lambda = 1.32\,\mu\text{m}$ is $\beta_2 = 20\,\text{ps}^2/\text{km}$, and its nonlinear coefficient is $\gamma = 10(\text{W} \cdot \text{km})^{-1}$ at 1.32μm. For desired compression factor $F_c = 10$, using the equation (15)–(18), one can estimate pulse amplitude order $N=19$ (corresponds to a pulse peak power 220mW), and $z_{op} = 17.9\,\text{km}$, $L_x = 2.55\,\text{cm}$. Fig.6 is numerical result for the pulse compression using designed fiber-fiber grating compressor. We get pulse with $F_c = 9.96$ and $Q_c = 0.73$ which is nearly desired compression factor.

There exists oscillation at tail of compressed pulse; this is caused by oscillation frequency response property of fiber grating dispersion. In order to increase the compression quality, a good dispersion property for fiber grating is necessary, and an apodized fiber grating is needed.

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{Fig6.png}
\caption{The pulse compression using designed fiber-fiber grating compressor at $N=19, L_x=2.55\,\text{cm}$. The curve 1, 2, and 3 stands for the initial pulse, the pulse after normal dispersion fiber and compressed pulse, respectively.}
\end{figure}

4. Conclusion

A linearly chirped fiber grating provides nearly linear dispersion that is required for compensate the linear frequency chirp produced by combined effects of normal dispersion and self-phase-modulation in NDF. So it is a good substitution of bulk grating pair for pulse compression. After this substitution, the design rule for fiber-fiber-grating compressor must be modified. We give a set of modified semi-empirical equations for the compressor. In the mean time, the shortcoming of semi-empirical equations giving in [1] and [3] is pointed out.

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