A New Modulation Strategy for Unbalanced Two Phase Induction Motor Drives Using a Three-Leg Voltage Source Inverter

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This paper proposes a new modulation scheme providing unbalanced output terminal voltages of a standard three-leg voltage source inverter (VSI) for unsymmetrical type two-phase induction motors. This strategy allows a control method of the output voltages with typically constant V/Hz for a main winding and with voltage boost to compensate magnitude of current for an auxiliary winding. Harmonic voltage characteristics and the motor performance are investigated under a wide range of operating conditions. Practical verification is presented to confirm correctness and capabilities of the proposed technique. All results are compared to those of a conventional two-leg half bridge topology. The results show that the simulation results well agree with the experimental ones, and also the proposed scheme is superior to the conventional drive.

Keywords: unsymmetrical two phase induction motor, harmonic voltages, unbalanced output terminal voltage, pwm voltage source inverter

List of Symbols

- \( \hat{V}_D \) = main winding voltage
- \( \hat{V}_Q \) = auxiliary winding voltage
- \( \hat{I}_m \) = main winding current
- \( \hat{I}_a \) = auxiliary winding current
- \( \hat{E}_{fm} \) = forward induced voltage of main winding
- \( \hat{E}_{fa} \) = forward induced voltage of auxiliary winding
- \( \hat{E}_{ba} \) = backward induced voltage of auxiliary winding
- \( a \) = turns ratio of auxiliary to main windings
- \( s \) = slip
- \( \omega_s \) = synchronous angular frequency (rad/sec)
- \( \phi \) = phase angle between the main and the auxiliary winding currents
- \( T_e \) = average electromagnetic torque
- \( T_{\text{pulse}} \) = amplitude of pulsating torque
- \( R_{1m}, X_{1m} \) = main winding resistance and leakage reactance
- \( R_{1a}, X_{1a} \) = auxiliary winding resistance and leakage reactance
- \( X_m \) = main winding magnetizing reactance
- \( R_2, X_2 \) = rotor resistance and leakage reactance referred to main winding
- \( \frac{R_2}{a^2}, \frac{X_2}{a^2} \) = rotor resistance and leakage reactance referred to auxiliary winding
- \( Z_f \) = forward impedance of main winding
- \( Z_b \) = backward impedance of main winding
- \( R_f, X_f \) = forward resistance and leakage reactance of main winding
- \( R_b, X_b \) = backward resistance and leakage reactance of main winding
- \( \frac{R_f}{a^2}, \frac{X_f}{a^2} \) = forward resistance and leakage reactance referred to auxiliary winding
- \( \frac{R_b}{a^2}, \frac{X_b}{a^2} \) = backward resistance and leakage reactance referred to auxiliary winding
- \( v_a, v_b, v_c \) = reference modulating function of control signal phase \( a, b, \) and \( c \), respectively
- \( v_{Q1} \) = fundamental main winding voltage
- \( v_{D1} \) = fundamental auxiliary winding voltage
- \( V_m \) = amplitude of modulating function
- \( V_{dc} \) = dc link voltage
- \( V_i \) = amplitude of additional modulating function
- \( V_t \) = amplitude of carrier signal
- \( M_1 \) = modulation index of additional modulating function
- \( M \) = modulation index
- \( \omega_r \) = reference angular frequency (rad/s)
- \( \omega_c \) = carrier angular frequency (rad/s)
- \( m_f \) = modulation frequency ratio

1. Introduction

Single phase induction motors have been widely used in residential and industrial applications. The most encountered single phase motor is actually a two-phase machine known as the permanent split capacitor motor (PSCM). The motors
are normally used in single speed drive applications. In case of desirable variable speed, the mechanical or electrical techniques are needed. These methods do not accomplish continuous speed control and energy saving viewpoint because of constant voltage and constant frequency sources. The reduction in the cost of the power electronic circuitry and importance of energy saving issue has made the use of a variable single phase drive system possible. The single phase induction motor has serious problems, when operating at low frequencies such as overheating, reduced pull-down torque and higher pulsating torque amplitude. Several publications have proved that, by varying frequencies and magnitude of the two-phase stator voltage, the torque ripple and audible noise can be reduced. Also, a 2-leg VSI provides the lower motor performance than a 4-leg VSI because of the quality of PWM scheme as well as dc link voltage utilization. Some publications have reported the 3-leg VSI drives with their own modulation strategies. A conclusion of almost the same performance between 3-leg and 4-leg topologies can be found in (13). However, the 4-leg VSI has main drawbacks in switching device counts. Additionally, the 3-leg topology is now available in a commercially compact Intelligent Power Module (IPM). To optimize between cost and performance of the drive system, 3-leg VSI is a good trade. Since each motor winding voltage of the 3-leg VSI has the same common node at an inverter branch, the normal regulating function cannot be succeeded for the unbalanced two-phase voltage control methodology. A specially modulating PWM strategy to accomplish wide range variable speed operation of commonly existing machines for the three-leg drive is needed.

This paper describes the principle of generating PWM patterns for driving inverter fed two-phase squirrel cage induction motors using a 3-leg inverter system. This structure is less expensive than the 4-leg VSI and provides better performance in terms of harmonic distortion when compared with that of the 2-leg VSI. To maintain the 90 electrical degree difference of unbalanced type two-winding voltages with variable-voltage variable-frequency 3-leg VSI, the additional modulating function is required. To achieve a wide range of operation, the fundamental amplitude of the main winding voltage is kept constant at rated volt/Herz while that of auxiliary winding voltage is proportional to inverter frequency with constant boost voltage at "a" (turns ratio) times. The proposed PWM technique and the two-phase motor model are simulated to predict the harmonic voltage characteristics and the machine performance. To verify the validity of the proposed strategy, both simulation and experimental results of 3-leg VSI are compared with those of the conventional 2-leg VSI while the 4-leg VSI is not interested here because of more cost and switching devices. The results obviously show the performance improvement of the proposed method. The proposed modulation technique is appropriate for the drive of the ordinarily asymmetrical two-phase induction machine.

2. Unsymmetrical Two-Phase Motor Model

A two unsymmetrical stator windings motor formed in space quadrature can be illustrated as a similar single equivalent circuit for each winding including the induced voltage in the winding from the other winding’s flux as shown in Fig. 1 (12). Since the main and auxiliary windings are typically quite different, with a different number of turns, wire size and turns distribution, the turns ratio test should be made following Veinotte technique (12). Both stator resistances, \( R_{lm} \) and \( R_{la} \), can be derived from DC test. The remaining parameters can be obtained by a standard test method. In the equivalent circuit, no attempt has been made to include the core losses resistance into the model. However, the core loss is summed with friction and windage losses called as rotational losses, and is utilized for calculating the precisely mechanical output power. The motor parameter values used in this work are given in Appendix. The forward and backward impedances drawn in Fig. 1 are:

\[
Z_f = R_f + jX_f = \frac{jX_m}{2} / \left( \frac{R_2}{2s} + \frac{jX_2}{2} \right) \quad \text{(1)}
\]

\[
Z_b = R_b + jX_b = \frac{jX_m}{2} / \left( \frac{R_2}{2(2-s)} + \frac{jX_2}{2} \right) \quad \text{(2)}
\]

\[
\hat{V}_D = \hat{I}_m (R_{lm} + jX_{lm}) + \hat{E}_{fm} - \frac{j\dot{E}_{fa}}{a} + \hat{E}_{bm} + \frac{j\dot{E}_{ba}}{a} \quad \text{(3)}
\]

\[
\hat{V}_Q = \hat{I}_m (R_{la} + jX_{la}) + j\dot{a}\hat{E}_{fm} + j\dot{a}\hat{E}_{ba} - \dot{a}\hat{E}_{bm} \quad \text{(4)}
\]

As indicated in (12), the developed average electromagnetic torque using this model is given by:

\[
T_e = \frac{1}{\omega_s} \left\{ \left[ I_m^4 + (aI_a)^2 \right](R_f - R_b) \right\} \quad \text{(5)}
\]

and a pulsating torque:

\[
T_{\text{pulse}} = \frac{1}{\omega_s} \left[ I_m^4 + (aI_a)^4 + 2(aI_mI_a)^2 \cdot \cos 2\phi \right]^{\frac{1}{2}} \quad \text{(6)}
\]

From Equation (5), it can be seen that the maximum average torque will be present when \( \phi = 90^\circ \). When considering Equation (6), at \( \phi = 90^\circ \), undesirable pulsating torque can also be eliminated at any motor slips if:

\[
I_m^4 + (aI_a)^4 - 2(aI_mI_a)^2 = 0 \quad \text{........................................... (7)}
\]

Thus:
Fig. 2. The relationship between inverter frequency and fundamental output voltage for each winding

\[
\left( \frac{I_m}{aI_a} \right)^2 + \left( \frac{aI_a}{I_m} \right)^2 = 2 
\]

\[\text{(8)}\]

giving:

\[
I_m = aI_a 
\]

\[\text{(9)}\]

Equation (9) is demonstrated that the relative amplitude of the winding currents should be the inverse of the winding turns ratio. In order to achieve the relationship between both currents, the voltage ratio is approximately equal to

\[
V_D = aV_Q 
\]

\[\text{(10)}\]

Equations (5)–(10) are expressed that the optimum control method to operate an unsymmetrical two-phase motor under variable frequency control is to maintain the winding currents in quadrature with a ratio between the main and the auxiliary winding currents equal to the turns ratio. Many papers\(^{(6-8)}\) have proved better performance of this method. The relationship between inverter frequency and fundamental output voltage \(V_{D1}\) and \(V_{Q1}\) can be seen in Fig. 2 as solid and dashed lines, respectively. While the fundamental frequency varies up to rated value, volt/hertz of the main winding is kept constant at nominal value. The fundamental amplitude of the auxiliary winding voltage is proportional to inverter frequencies with a boost voltage level at \(a\) times \(V/\text{Hz}\) of the main winding in the frequency range of \(0 < f < \frac{f_{\text{rated}}}{a}\). When inverter frequency reaches at \(f_{\text{rated}}/a\), the auxiliary winding voltage will be maintained constant at a rated value for the reason of preventing machine saturation in the frequency range of \(\frac{f_{\text{rated}}}{a} \leq f \leq f_{\text{rated}}\).

### 3. Converter Modulation Strategy

The conventional two-leg half bridge inverter supplying a two-phase induction motor as shown in Fig. 3 consists of a single phase voltage doubler rectifier, two filter capacitors in series, and two legs of an IGBT pair. The one end of the main and auxiliary windings are connected to the two-leg inverter. The other ends are connected together with the center-tap of the filter capacitors. Their fundamental components of output voltage are shifted with 90 electrical degree differences. The principle of generating balanced two-phase PWM output voltages is based on a comparison between balanced two-phase modulating sinusoidal signals with 90 electrical degree phase shift compared to a common triangular wave, known as a sinusoidal pulsewidth modulation (SPWM) is shown in Fig. 4. PWM output voltages are classified as bipolar PWM.

Two modulating function signals with balanced output voltages for a two-leg system are:

\[
\begin{align*}
    v_a &= V_m \sin(\omega_0 t) \\
    v_b &= V_m \sin(\omega_0 t - 90^\circ) 
\end{align*}
\]

\[\text{(11)}\]

The amplitude and frequency of the output phase voltages can be controlled independently by varying a sinusoidal reference amplitude and a frequency of each modulating signal whilst the carrier signal is fixed with a constant frequency and amplitude. More details of PWM parameters such as modulation index, modulation frequency ratio can be found in\(^{(16)}\). Note that an unbalanced case in accordance with the control method as mentioned in Fig. 2 can also be easily obtained due to independent control for each modulating signal.

Fig. 5 indicates the proposed three-leg VSI so-called semifullbridge PWM inverter for a two-phase motor drive system. When compared to the conventional two-leg half bridge inverter, the third modulating function for the third leg is needed as given in Equation (12). With these modulating
New Modulation Strategy For Two-phase Induction Motor Drives

functions for balanced output voltages, the both winding voltages are always in quadrature.

\[ \begin{align*}
    v_a &= V_m \sin(\omega_r t) \\
    v_b &= V_m \sin(\omega_r t - 90^\circ) \\
    v_c &= V_m \sin(\omega_r t - 180^\circ)
\end{align*} \]

Unfortunately, unbalanced output voltages having 90 electrical degree phase differences cannot occur with independent control of those modulating function signals. In order to overcome this problem, the additional modulating function is required as follows:

Let

\[ v_{c1} = V_m \sin(\omega_r t - 180^\circ) \]

and additional modulating function

\[ v_{c2} = V_x \sin(\omega_r t - 45^\circ) \]

and let

\[ v_c = v_{c1} + v_{c2} \]

The modulating function for the third leg can be rewritten as:

\[ v_c = \frac{\sqrt{2}}{2} V_x (\sin(\omega_r t - \cos(\omega_r t)) - V_m \sin(\omega_r t) \cdots (13) \]

Thus

\[ \begin{align*}
    v_{ab} &= v_a - v_b = \sqrt{2} V_m \sin(\omega_r t + 45^\circ) \\
    v_{bc} &= v_b - v_c = \left( \sqrt{2} V_m - V_x \right) \sin(\omega_r t - 45^\circ)
\end{align*} \]

As can be seen in Equation (14), the two voltages, \( v_{ab} \) and \( v_{bc} \) equivalent to the fundamental auxiliary and main winding voltages, respectively, are in quadrature. The amplitude of both is 1.414 times the amplitude of each modulating function. This implies that this technique gives an advantage in terms of dc voltage utilization over the two-leg half bridge VSI. In order to better understand the principle of the proposed modulating functions, the phasor diagrams in Fig. 6(a) for the balanced and Fig. 6(b) for the unbalanced two-phase terminal voltage are illustrated. It is clear from the figure that with the additional modulating functions \( v_{c2} \), the unbalanced output terminal voltage with different magnitude and fixed 90° phase shift at any operating conditions can be obtained. The principle of generating unbalanced PWM output voltages based on the proposed modulation scheme for three-leg VSI is described in Fig. 7. PWM output voltages are classified as unipolar PWM. The fundamental output voltage of the auxiliary winding can be expressed as

\[ v_{D1} = \frac{\sqrt{2} M V_{dc}}{2} \sin(\omega_r t + 45^\circ) \cdots (15) \]

Also the fundamental output voltage of the main winding can be expressed as

\[ v_{Q1} = \left( \frac{\sqrt{2} M - M_1}{V} \right) \frac{V_{dc}}{2} \sin(\omega_r t - 45^\circ) \cdots (16) \]

where \( M = \frac{V_a}{V} \) and \( M_1 = \frac{V_b}{V} \). As can be seen in Equations (15) and (16), the fundamental of PWM output voltages for both windings are in quadrature and can be set by \( M \) and \( M_1 \) in accordance with the control method as mentioned in Fig. 2.

4. Simulation and Experimental Results

To verify correctness and features of the proposed modulation strategy, the experimental system is setup as demonstrated in Fig. 8. Asynchronously natural-sampled PWM signals are generated by comparing a triangular carrier with a sinusoidal reference modulating function generated from a microcontroller. 25 A, 1200 V IGBT IPM is used for the main power circuit. A 370-W 230-V 2.66-A 1375-rpm 50-Hz PSCM with turns ratio of 1.7 is used in the investigation. Along with the control methodology, the running capacitor is removed. The motor is connected with a 400 W DC generator to serve as a dynamic load. Shaft torques and speeds are
measured using torque-speed sensor. In order to detect the acoustic noise signal, a microphone is installed at a distance of 3 cm from the motor. Experimental results of voltage, current and power are recorded using a Power Harmonics Analyzer, Yokogawa Model PZ4000. Practical verification of the theoretical predictions derived from the mathematical and the machine model is corroborated. The motor parameters in the machine model are measured at 50 Hz utility supply as given in Appendix.

The effectiveness of PWM patterns in improving the quality of motor performance is demonstrated theoretically as well as experimentally. The motor is supplied by both 2-leg and 3-leg VSI at the same conditions. To inspect the harmonic voltage contents, the reference frequency of 20 Hz, and the triangular carrier frequency of 5 kHz are selected. The reason is that the inverter frequency is somewhat low (worst case) and in the voltage boost region, and the switching frequency is in the range of commercial drives available.

The fundamental peak voltage of the main winding is approximately 124.5 V. The fundamental peak voltage of the auxiliary winding is about 211.6 V. The load torque is fixed on 2.5 N·m. The calculated and measured rms output voltages versus functions of frequency are illustrated in Figs. 9–12 and Figs. 13–16, respectively. These figures confirm the precision of not only numerical analysis but also the drive implementation. A comparison of Figs. 13–14 and Figs. 15–16 also verify that although the sidebands voltage of 2-leg VSI are slightly less than those of 3-leg VSI, the harmonic magnitude at carrier frequency of 2-leg is remarkably higher than those of 3-leg, thus resulting in more motor losses.

The calculated amplitude of each harmonic voltage from the fundamental up to 25 kHz harmonic frequency is considered. This range covers the significant harmonic contents. The calculated harmonic voltages are applied to the equivalent circuit in Fig. 1. The calculated currents of each winding at the same time domain, then, are summed by the superposition technique as shown in Figs. 17–18 for 2-leg VSI and also Figs. 19–20 for 3-leg VSI. The actual current results as illustrated in Figs. 21–24 are obtained from the power harmonic analyzer. It is clear from the comparison that the 3-leg presents a better performance than the 2-leg in terms of THDi. This keeps in touch with harmonic voltage spectra characteristics.

It is observed from the results that the simulated results give total harmonic distortion of current component slightly less than those of measured ones. This has proved the ac-
accuracy of the implemented circuits. The small error may be arisen from various factors. For example, the given simulation results have been analyzed from the ideal power electronic devices while the actual results have taken many effects such as voltage ripple of dc link voltage, blanking time in practical inverter legs, and voltage drop in devices, etc. into account. However, those are seen that the correlation is acceptable. The tested configuration of both winding motor currents with 2-leg and also 3-leg VSI are indicated in Figs. 25 and 26, respectively. These two figures confirm that the proposed modulation strategy dramatically provides the
lower harmonic voltage than the conventional system, especially at low operating frequency. Also, the output currents of 3-leg VSI under unbalanced output voltage are approximately a sinusoidal waveform and nearly 90 degree differences.

To study the influence of the decreasing harmonic voltage component on motor performance, the motor efficiency, torque-speed characteristics and acoustic noise are carried out. The variation of motor efficiency of 2-leg and 3-leg VSI with typical V/F for both windings and the proposed control method under the same fundamental voltage at fundamental frequency between 20 Hz and 50 Hz as a function of load torque are illustrated in Figs. 27–29. Only in Fig. 29, the motor under test fed by the two drive systems and sinusoidal supply at rated frequency of 50 Hz. This operating frequency provides the same voltage level of both windings for both drive systems. In case of sinusoidal source, the motor is utilized with a running capacitor of 8 $\mu$F in series with the auxiliary winding. It is clear from the figures that the proposed control method provides better efficiency than the conventional V/F control scheme for both windings. Also, the 3-leg VSI with the proposed technique gives the greatest motor efficiency at a wide range of speed, while the 2-leg VSI with
V/F control scheme for both windings provides the worst performance because of harmonic contents. It is monitored from the figures that the operation ranges of motor fed by inverter are narrower than those of sinusoidal supply. The 3-leg VSI with the proposed scheme, however, still obtains the best motor efficiency because the inverter structure can exactly control the quadrature two-winding voltages. Surprisingly, for the 50 Hz sinusoidal source, the efficiency is lower than both drive system at light load torque. This is because the running capacitor value is normally designed suitable for heavy load.

Additionally, at low frequency, the motor efficiencies for both drive systems are significantly different since at low frequency (low modulation index) the 2-leg VSI generates higher harmonic voltages, particularly, at order of \( m_f \). Fig. 30 demonstrates torque-speed motor profiles with various fundamental frequencies under the proposed control method, while Fig. 31 describes that relationship with constant V/F control scheme for both windings. When comparing these two figures, it can be evidently seen that unbalanced output voltage remarkably provides superior torque to constant V/F scheme for both windings and a wider range of operation.
The measured results for 2-leg and 3-leg are very close. Measured and simulated results are in fairly agreement. These show that both drive systems provide the same average torque. This means that harmonic voltages do not affect average torque. This agrees with the publications (4)-(8). The acoustic noise spectra are illustrated in Figs. 32–33. These have clearly shown that at the same frequency the noise components of 3-leg VSI are mostly attenuated. Consequently, the proposed technique can effectively diminish acoustic noise levels. This strongly confirms that harmonics voltages stress on motor mainly generate acoustic noise (13).

5. Conclusions

This paper has dealt with the analysis of a new modulation strategy for a three-phase voltage source inverter with unbalanced two-phase output voltages supplying an asymmetrical type two-phase induction motor. This technique allows in unbalanced output voltage control for both main and auxiliary windings with 90° electrical phase shift. Harmonic voltage contents have been investigated and compared to those of the two-leg voltage source inverter. A superior PWM quality over the conventional technique is achieved. The different schemes for obtaining a wide range of variable speed operation have been simulated and experimentally verified. The simulation results have shown very good agreement with the experimental results. All results confirm that the performance of the proposed modulation strategy is superior to the conventional scheme.

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References

Appendix

Parameters of motor used in experiments:
R_{lm} = 9.028 \, \Omega, \, R_{la} = 49.921 \, \Omega, \, R_2 = 15.283 \, \Omega, \, L_{1m} = L_2 = 43.4 \, \text{mH}, \, L_{1a} = 178.82 \, \text{mH}, \, L_{m} = 202.92 \, \text{mH}, \, \frac{N_s}{N_c} = a = 1.7

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